Business Cycle during Structural Change: Arthur Lewis' Theory from a Neoclassical Perspective.*

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Abstract

We document that business cycles change during the process of development. In countries with large declining agricultural sectors, aggregate employment is uncorrelated with GDP. During booms, agricultural employment declines even though agricultural labor productivity increases relative to other sectors. We construct a unified theory of business cycles and structural change consistent with the stylized facts. The theory focuses on the simultaneous decline and modernization of agriculture. As capital accumulates, agriculture becomes increasingly capital intensive as traditional agriculture is crowded out. We estimate the model and show that it accounts well for both structural transformation and business cycle fluctuations in China.

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1 Introduction

The nature of economic fluctuations changes throughout economic development (see, e.g., Acemoglu and Zilibotti (1997), Aguiar and Gopinath (2007)). China is a point in case. The country has experienced a profound economic transformation with the share of employment in agriculture falling from about 2/3 in 1980 to about 1/3 today. During this time, structural change has systematically accelerated during economic booms and stagnated during recessions, implying procyclical employment in nonagriculture and countercyclical employment in agriculture. Aggregate employment is instead virtually uncorrelated with GDP. Labor productivity in agriculture is also noteworthy: the relative labor productivity in agriculture increases in booms as workers leave the rural sector.

These features are shared by other countries at a comparable stage of development. Extending the work of Da Rocha and Restuccia (2006) beyond OECD countries, we show that the correlation between agricultural employment and aggregate GDP varies systematically with the relative size of the agricultural sector. While employment is procyclical in industrialized countries, it is acyclical in economies with a large agricultural sector. In poorer economies, employment in agriculture is negatively correlated with employment in the rest of economy, while such a correlation is zero or even positive in the industrial world. Finally, downswings in agricultural employment are associated with upswings in the relative productivity and capital intensity of the agricultural sector in developing countries. No such pattern is observed in mature economies where the agricultural sector is typically very small and its behavior shows no significant correlation with aggregate GDP.

To explain these observations, we propose a neoclassical theory of growth and structural change subject to stochastic productivity shocks. In our theory, the same technological forces drive the structural transformation and business cycle fluctuations. The key drivers are sector-specific TFP growth (as in Ngai and Pissarides (2007)) and endogenous capital accumulation (as in Acemoglu and Guerrieri (2008)). Investments, capital deepening, and TFP growth cause both reallocation from agriculture to nonagriculture and *modernization* of agriculture. To capture the latter, we assume that the rural sector comprises two subsectors, modern and traditional agriculture. Modern agriculture uses capital which is instead not an input of the traditional sector. Over the process of development, agriculture shrinks as a share of total GDP and becomes more productive—not only in an absolute sense but also relative to the rest of the economy. In the long run, the equilibrium converges to an asymptotic balanced growth path where the agricultural sector is small, modernized, and highly productive.

The mechanism in our theory is reminiscent of the classical contribution of Lewis (1954), where the existence of a labor-intensive sector makes the supply of labor to nonagriculture very elastic. In Lewis' theory, labor supply is infinitely elastic for as long as the traditional sector exists. While in his model the elasticity falls discretely as soon as the last worker moves out of the traditional sector, in our model, the elasticity of labor supply declines gradually during the process of modernization of agriculture. A poor economy behaves like a Lewis economy; then, as it develops, the economy becomes more and more similar to a standard neoclassical economy.

Structural change has implications for business cycles. When the agricultural sector is large and predominantly traditional, the economy responds to sector-specific shocks by reallocating workers towards the more productive sectors with limited effects on wages and relative prices. As the agricultural sector declines, this margin of adjustment is muted. Wages and prices respond more to productivity shocks leading to larger swings in labor supply. We show that these theoretical predictions are in line with stylized facts about both structural change and business cycles across countries.

We estimate the growth model using data from China. The key parameters are the elasticities of substitution between the output of the agricultural and nonagricultural sector and, within agriculture, between the two agricultural subsectors. We estimate both elasticities to be significantly larger than unity. Our estimate of the elasticities of substitution, which is based on a structural model, differs from earlier estimates from Herrendorf et al. (2013) which estimates a production function with three sectors, agriculture, manufacturing, and services, and find a low elasticity of substitution close to Leontief when using value-added data. We check the robustness of our results with the aid of a flexible reduced-form specification that nests the three-sector model of Herrendorf et al. (2013) but allows (following Krusell et al. (2000)) for the elasticity of substitution between agriculture and nonagriculture to differ from that between manufacturing goods and services. We find an elasticity of substitution between the output of the agricultural and nonagricultural sector larger than unity, while our estimate confirms their earlier finding that the elasticity of substitution between manufacturing goods and services is very low. Our result is robust across data from China, Japan, and the US.

After estimating the deterministic model, we introduce stochastic shocks. We show that our model fits quantitatively well salient features of the business cycles of industrializing economies, most notably China. Among other things, the model explains why positive TFP shocks in nonagriculture cause a temporary acceleration of the process of structural change. Such shocks trigger an increase in total investments and a reallocation of capital and labor out of agriculture. Labor productivity increases more in agriculture than in the rest of the economy, as most of the temporary reallocation of labor away of the agriculture is drawn from the traditional sector. Therefore, agriculture experiences a sharp increase in capital intensity and average labor productivity during booms. Out-of-sample predictions also confirm that as structural change progresses the business cycle properties of the model become increasingly similar to those of advanced economies.

Our research contributes to the literature on structural change pioneered by Baumol (1967). This literature includes, among others, Kongsamut et al. (2001), Gollin et al. (2002), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), Buera and Kaboski (2009), Alvarez-Cuadrado and Poschke (2011), Herrendorf et al. (2013), Boppart (2014), Comin et al. (2015), Alder et al. (2018), Santana et al. (2021), and Leon-Ledesma and Moro (2020). In our model, exogenous TFP growth and capital accumulation induce transition away from agriculture. The properties of the transition are similar to those of Acemoglu and Guerrieri (2008), except that in their model there is no traditional sector. The closest theoretical contribution in the literature is the recent paper by Alvarez-Cuadrado and Poschke (2011), which studies the properties of two-sector models assuming a constant elasticity of substitution between capital and labor within each sector but allowing elasticities to differ across sectors. In contrast, in our model this elasticity changes over the process of development due to the reallocation between traditional and modern agriculture. While our main focus is on the business cycle implications of the theory, Alvarez-Cuadrado and Poschke (2011) do not consider economic fluctuations.

Our paper is also related to the debate on the effects of technological progress and demand factors on structural change and industrialization. The traditional development literature suggests that productivity growth in agriculture combined with Engel's law released resources from agriculture and generated industrialization (*labor push*), cf. Nurkse (1953), Rostow (1960), and, more recently, Gollin et al. (2002). An alternative view—the *labor pull* theory—is that fast productivity growth in manufacturing attracted workers from agriculture to manufacturing (cf. Mokyr (1976)). Our model can be consistent with either of these two views depending on parameters. Under the assumption that agricultural and industrial production are gross substitutes, which is the outcome of our structural estimation for China, it is in line with the labor pull theory. We document that our results are in line with the common narrative about episodes of business cycles in recent Chinese history.

Our work also adds to the existing business cycle literature by incorporating endogenous structural change. The classical multi-sector model focus on the stable economic structures or abstract from growth, for example, Benhabib et al. (1991), Horvath (2000), Boldrin et al. (2001), etc. We extend the standard model to account for the business cycles properties at different stages of development. Related research on this theme includes Da Rocha and Restuccia (2006), Aguiar and Gopinath (2007), and Moro (2012).[1] Recent studies on the properties of the process of structural change in China include, among others, Cheremukhin et al. (2017) and Brandt and Zhu (2010). Yao and Zhu (2021) and Chang et al. (2016) study properties of the Chinese business cycle. Yao and Zhu (2021) construct a two-sector model of the Chinese economy in which consumers have non-homothetic preferences and show that in such a model the size of the agricultural sector affects the nature of business cycles. In their model, there is no capital. Our structural estimation allows for both capital accumulation and nonhomothetic preferences of the Stone-Geary type, but in our model the latter do not appear to have a large quantitative effect relative to technological factors. The process of modernization of agriculture is instead at the core of the recent papers by Chen (2020) and Boppart et al. (2019), although the focus of their papers is very different from ours. Productivity in Chinese agriculture is also the focus of Adamopoulos et al. (2021) who study the importance of factor misallocation in China, especially, the role of land institutions that constrain the growth of the more productive farms. Finally, our paper provides a novel numerical technique for solving stochastic equilibrium models with unbalanced growth. This is related to the recent work of Rubini and Moro (2019).

The remainder of the paper is organized as follows. Section 2 presents a set of stylized facts about business cycles and structural change across countries, zooming on China and the US. Section 3 lays out the model. Section 4 describes the estimation of the growth model. Section 5 performs a quantitative analysis of the business cycle properties of the model. Section 6 compares the results with additional international evidence. Section 7 concludes. An appendix includes proofs and additional material referred in the text.

[1] In particular, Da Rocha and Restuccia (2006) documents that among OECD countries, economies with larger agricultural sectors have smoother aggregate employment fluctuations and that agricultural employment is less correlated with aggregate GDP. Relative to their paper we show that this pattern holds up when extending the sample of countries to include a large number of countries, including very poor countries that are predominantly agrarian. Moreover, we document several additional salient differences in business cycle properties between poor and rich countries and show that business cycle properties of China are representative of countries with a similar share of agriculture.

2 Evidence on Modernization of Agriculture and Business Cycles

The process of economic development is associated with a significant downsizing of the agricultural sector. In the US, a third of the workforce was employed at farms in 1900. This employment share fell below 2% by 2000. Today, the average employment in agriculture is 4.6% in OECD countries, which compares with 31.6% in non-OECD countries (World Development Indicators 2017). The cross-country correlation between the employment share of agriculture and log GDP per capita is -0.84.

2.1 Modernization of Agriculture

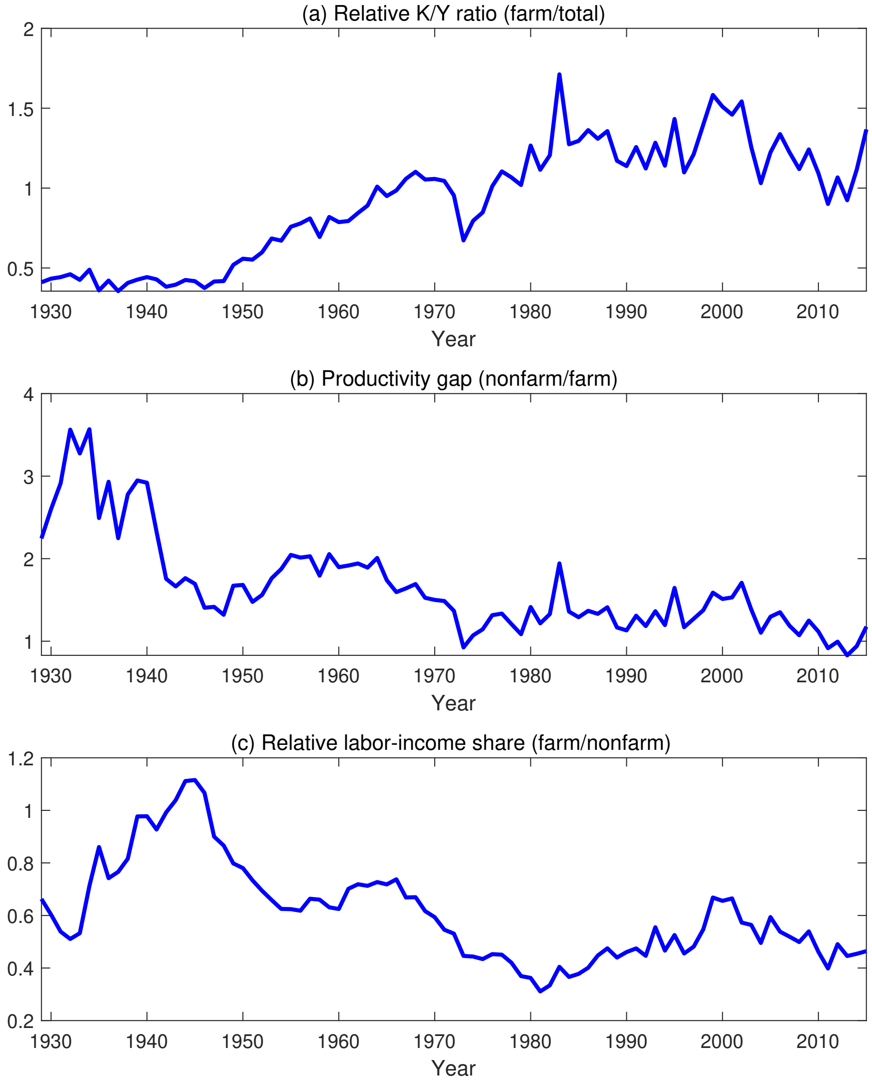
As the employment share of agriculture declines, both capital intensity and labor productivity grow faster in agriculture than in the rest of the economy (see Chen (2020)).

Relative Capital Deepening. While capital deepening is pervasive over the development process, it is especially pronounced in agriculture: both the capital-output and the capital-labor ratio grow faster in agriculture than in the nonfarm sector. Figure 1 (panel a) shows the time series for the US of the *relative K-Y ratio,* defined as the ratio between the capital-output (K-Y) ratio in agriculture and the aggregate K-Y ratio. The relative K-Y ratio increased from about 40% in the pre-war period, to about 120% since the 1980s. Figure 2 (panel a) plots the relative K-Y ratio against the employment share of agriculture for the period 1995-2016 across countries. Across countries, the relative K-Y ratio is significantly lower in developing than in industrialized countries. For example, it is very low in Sub-Saharan African countries, where agriculture is very labor intensive and employs a large proportion of the labor force. Since measurement error of agricultural capital may be especially severe in very poor countries, in panel (b) we drop African countries from the sample. Restricting the sample strengthens the results: the regression coefficient becomes more negative and significant.

Since data are available over a 22 year panel, we can also study the within-country comovement of employment shares in agriculture and relative K-Y ratios. To this aim, we split the sample for each country into two observations, the average for the period 1995-2005 and the average for the period 2006-2016. Then, we regress the relative K-Y ratio on the employment share of agriculture and a full set of country dummies. Panel a in Appendix Table 7 shows a set of regression results. Since the relationship is nonlinear, we regress the logarithm of the K-Y ratio on the logarithm of the employment share. We run both pooled regressions and regressions with country fixed effects. The results confirm a negative (in most cases, highly significant) relationship between employment in agriculture and relative K-Y ratios. The coefficients of the different regressions are similar in magnitude, indicating a strong consistency in the patterns across and within countries.[2]

Productivity Gap. Capital deepening and technical change raise labor productivity over time in all sectors of the economy. However, productivity grows faster in agriculture than in the rest of the economy. To show this, define the *productivity gap* to be the average labor productivity in

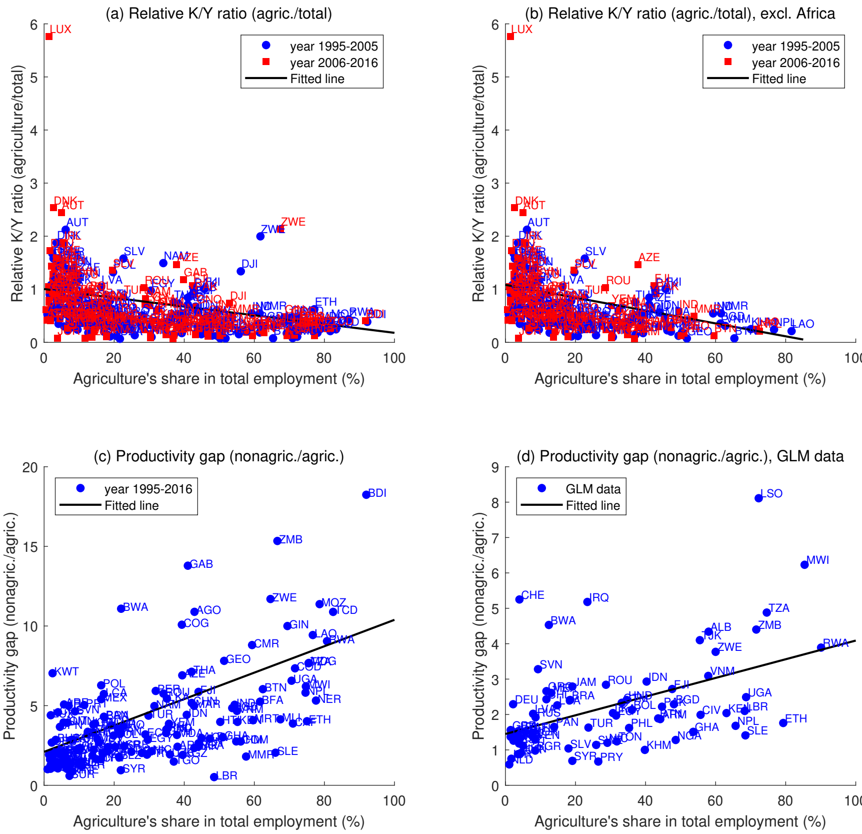
[2]The regressions in column (3) and (4) do not include time dummies and identify the effect of interest out of both global trends and cross-country deviations. The regressions in columns (5) and (6) include time dummies, filtering out common trends as in standard panel regressions. The magnitude of the coefficients is similar in all specifications, although the statistical significance drops in panels (5)-(6).



Note: Panel a plots the farm capital-output ratio divided by the total capital-output ratio in the US. Panel b plots the Labor productivity gap over time in the US. The labor productivity gap is measured as the nonfarm value added per worker divided by the farm value added per worker. Panel c plots the ratio of the labor income share in the farm sector over the labor-income share in the non-farm sector in the US. These labor-income shares are measured as the compensation of employees divided by the value added excluding proprietors' income. Sources: Capital stocks by sectors 1929-2015 are from the U.S. Bureau of Economic Analysis (BEA) Fixed Asset Tables 6.1 "Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods". The value added by sector come from the National Income and Product Accounts (NIPA) Table 1.3.5. Employment by sector is from the NIPA Table 6.8A, 6.8B, 6.8C, and 6.8D. Proprietors' income by sectors come from the NIPA Table 1.12. Compensation of employees by sectors come NIPA Table 6.2A, 6.2B, 6.2C, and 6.2D.

nonagriculture relative to agriculture. Figure 1 (panel b) shows that the productivity gap declines over time in the United States.[3] Figure 2 (panel c) documents a similar pattern across countries: the productivity gap is especially high in poor countries with a large agricultural sector (in line with the

[3]Labor productivity is measured as value added per worker in current prices.



Note: Panel a and b plot the relative capital-output ratio (agriculture vs. total) $\left(K^G/Y^G\right)/(K/Y)$ against the average agricultural employment share across countries. Each country has two observations: the average for the period 1995-2005 and the average for the period 2006-2016. Panel b excludes African countries. Panel c plots labor productivity gap across countries. The labor productivity gap is measured as the nonagriculture value added per worker divided by the agriculture value added per worker. The horizontal axis shows the average employment share of agriculture over the sample period for each country. Panel d shows the same relationship using the data in Gollin et al. (2014) who control for sectoral differences in hours worked and human capital per worker. Source: FAO (K^G is measured by the net capital stock; Y^G is value added, both at current prices) and Penn World Table (capital stock and real GDP at current national prices from detailed Penn World Table 9.1). Agriculture's employment share comes from ILO modeled estimates. Agriculture's value added output share comes from FAO extracted UNSD AMA data.

findings of Restuccia et al. (2008) and Gollin et al. (2014)). The same pattern emerges from fixed effect regressions exploiting the within-country variation over time (see panel b in Appendix Table 7.) Panel d shows the same relationship using the cross-country data in Gollin et al. (2014) which controls for sectoral differences in hours worked and human capital per worker. The regression coefficient is in both cases significantly positive.

The behavior of the labor income share in agriculture relative to the aggregate labor-income share mirrors that of the productivity gap. Figure 1 (Panel c) shows that the labor-income share in the farm sector relative to that of the nonfarm sector fell over time in the US since the end of World War II. We could not find comparable data across developing countries.

2.2 Business Cycles

Consider, next, economic fluctuations. We start by comparing China and the US. Sectoral employment and aggregate employment in China are from the Statistic Year Book by the China National Bureau of Statistics. From 1990 and onward NBS estimates aggregate employment based on a labor force survey, while employment before 1990 is based on annual administrative data. To make the series consistent, we apply the correction proposed by Holz (2006) revising the official employment data before 1990 using the 1982 and 1990 population census. In addition, we follow Brandt and Zhu (2010) and construct an adjusted series for agricultural employment given by total rural employment minus rural employment in private enterprises and individual enterprises minus employment in Township and Village Enterprises (TVEs). Employment in nonagriculture is total employment net of agricultural employment.[4]

Panels a and b in Figure 3 compare the business cycle of China with that of the US. While aggregate employment is volatile and highly procyclical in the US, aggregate employment is acyclical and relatively smooth in China. The data are annual and are filtered using an HP smoothing parameter of 6.25 (Ravn and Uhlig (2002)). These patterns are robust to using alternative filters such as first differencing the data.

Interestingly, employment fluctuations are closely associated with the process of structural change. Consider panel c in Figure 3. Until 1960, NBER recessions in the US were associated with a slowdown and at occasion even a reversal of the industrialization process. Namely, the employment share of agriculture used to fall in booms and increase in downturns. The cyclicality of employment in agriculture declines over time and ceases to be noticeable after 1960. China today looks similar to the US of yesteryear. Panel d in Figure 3 shows that structural change—measured by the decline in agricultural employment—accelerates during periods of high growth and slows down or halts during periods of low growth in China.

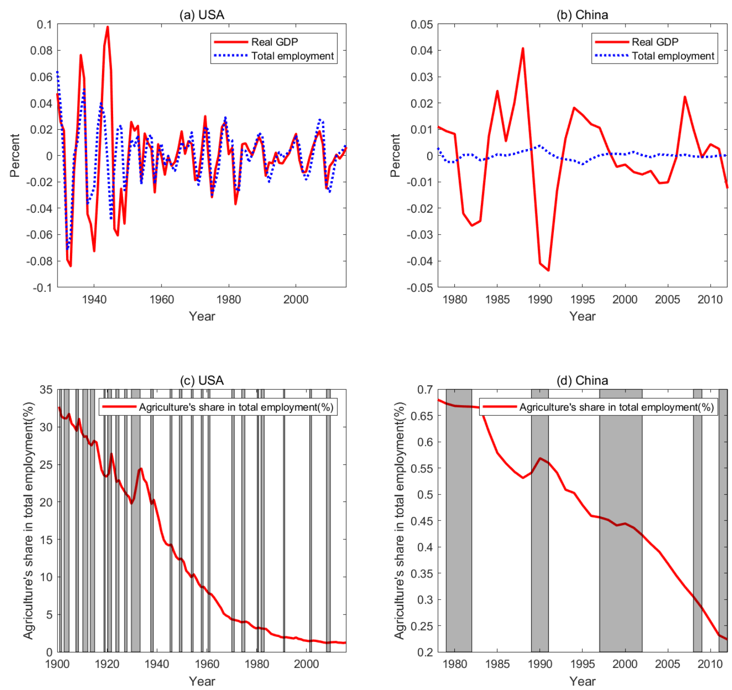
Panel b of Figure 4 documents that agricultural employment is volatile and countercyclical in China. There is no such pattern in the contemporary data for the US, where the correlation is positive rather than negative. In contrast, the cyclical pattern of nonagricultural employment is similar in the US and in China: Panels c and d of Figure 4 shows that nonagricultural employment is highly procyclical and roughly as volatile as GDP in both China and the US. It follows that agricultural and nonagricultural employment are strongly negatively correlated in China.

The stylized facts documented above are consistent with international data.[5] We use sector-level employment data from the International Labor Organization (ILO) (see Appendix A). Panel a in Figure

[4]According to Brandt and Zhu (2010), the NBS series for sector-specific employment, based on working status one week before the survey is conducted, exaggerates the extent of agricultural employment. The reason is that NBS includes non-agricultural workers in rural private enterprises and rural individual enterprises (i.e., firms employing less than eight employees) in their measure of agricultural workers. Appendix Figure 15 plots the revised agricultural employment series against the official data. The adjusted series has a larger decline and somewhat larger fluctuations in agricultural labor. See Appendix A.2 for more details.

[5]Some of the findings documented in this section echo those in Da Rocha and Restuccia (2006). They show that in OECD countries with a large agriculture, employment in agriculture is negatively correlated with employment in nonagriculture. Moreover, the aggregate employment is relatively smooth and uncorrelated with GDP. We extend the analysis to nonOECD countries. To the best of our knowledge, our paper is the first emphasizing the stylized fact in Panel 3 on the cyclicalty of the output gap.

Figure 3: Employment fluctuations: China vs. USA



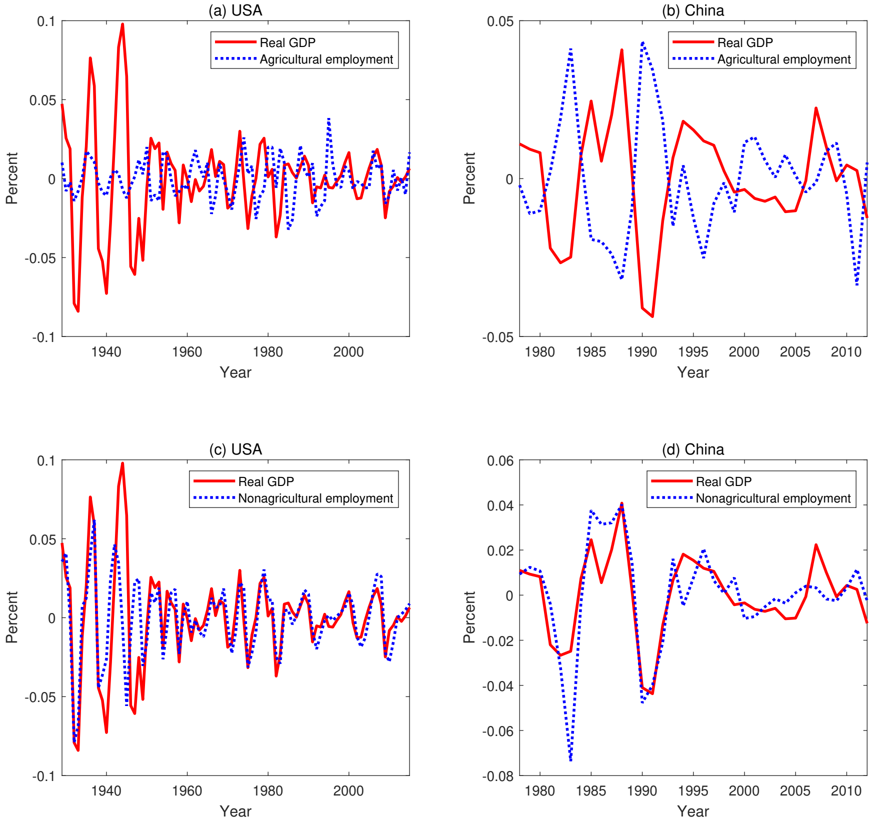
Note: Panel a and b plot the volatility of HP-filtered total employment and real GDP in the US (left panel) and China (right panel). Source: NBS with adjustments proposed by Brandt and Zhu (2010) and Holz (2006). Panel c and d plot the agriculture's share in total employment over the business cycles. The left panel plots the farm employment share in the US. Grey ranges denote recessions as classified by the NBER. The right panel plots the agriculture employment share over time in China. Grey ranges denote recessions, defined as years when the real GDP growth rate was below the average growth rate (9.7 percent during 1978-2012).

5 shows that the time-series correlation of the HP-filtered employment in agriculture and nonagriculture is positive (on average) for countries with a small agricultural sector like the US and negative for countries with a large agricultural sector like China.

Panel (c) in Figure 5 shows that the correlation between aggregate employment and GDP declines strongly with the employment share of agriculture, being large and positive for industrialized countries like the US and negative for countries with a large agricultural sector. This is in line with the US-China contrast discussed above.

However, panel d shows that in the cross section of countries the volatility of employment relative to GDP is only weakly negatively correlated with the employment share of agriculture, and this relationship is not statistically significant This figure also shows that when it comes to the volatility of employment, China is an outlier relative to countries at the same level of development.

Next, we highlight the dynamics of the productivity gap. We have already shown that the productivity gap falls throughout structural change, (see panel b of Figures 1-2). Panel c in Figure 5 shows a similar pattern at business-cycle frequencies. Namely, in countries with large agricultural sectors, the



Note: Panel a and b plot the HP-filtered log agricultural employment vs HP-filtered log real GDP in the US 1929-2015 and in China 1978-2012. Panel c and d plot the HP-filtered log nonagricultural employment vs HP-filtered log real GDP in the US and in China. We use a smoothing parameter 6.25 for the HP filter Ravn and Uhlig (2002). Source: The US employment by sectors is from the NIPA Table 6.8A, 6.8B, 6.8C, and 6.8D. The sectoral employment data for China are from NBS with standard adjustments described in the text.

productivity gap is negatively correlated with employment in nonagriculture, while this correlation is close to zero in countries with a small agricultural sector. For instance the correlation is -0.54 for China. We conclude that relative output per worker and relative employment (in agriculture) move in opposite directions. This happens both during the process of structural change and over the business cycle in countries that are undergoing structural change away from agriculture.

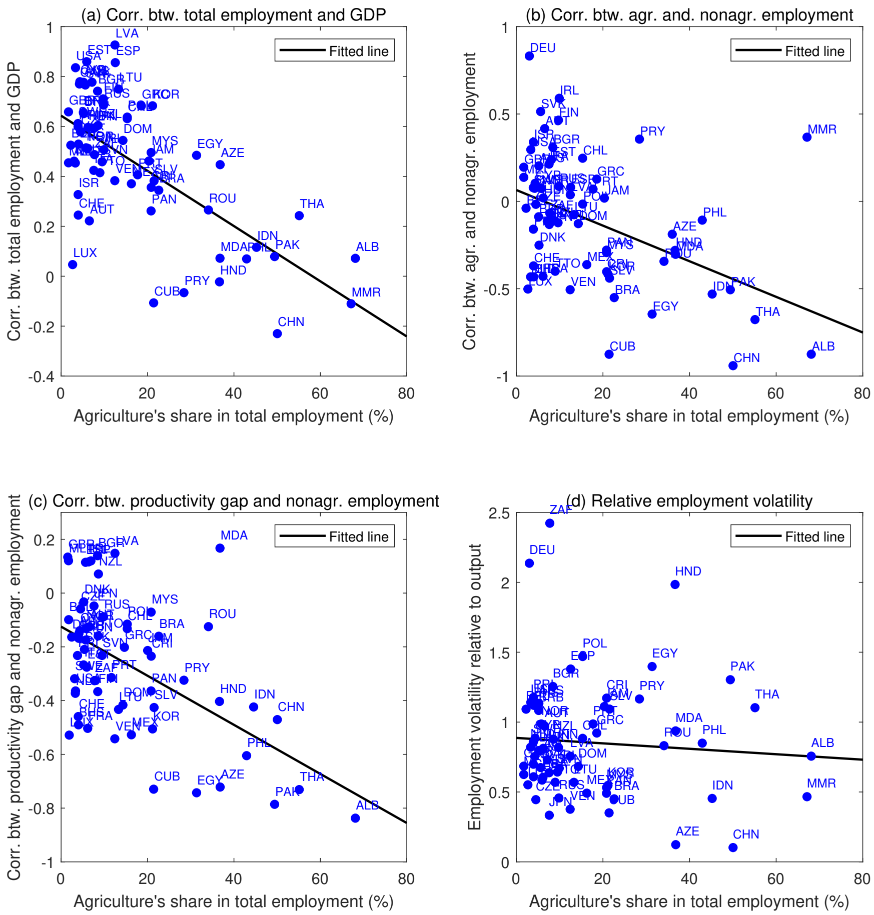
‘ Finally, Appendix Figure 13 shows that consumption volatility relative to GDP volatility declines over the process of development, consistent with the evidence in Aguiar and Gopinath (2007).

In summary, the characteristics of the business cycles change systematically across different stages of the process of structural change. In countries with a large agricultural sector, we observe:

1. Aggregate employment is less correlated with GDP;

2. Employment in agriculture is countercyclical;

3. The labor productivity gap is negatively correlated with employment in nonagriculture;



Note: The figure is a cross-country scatter plot of business cycle statistics. The sample period is 1970-2015 and some countries have fewer observations. We keep the countries that have at least more than 15 years of consecutive observations in order to calculate the business cycle statistics. Panel a plots the time series correlation of HP-filtered log nonagricultural employment and HP-filtered log agricultural employment in a sample of 66 countries. Panel b shows the correlation between the HP-filtered log total employment and HP-filtered log real GDP in a sample of 66 countries. Panel c plots the time series correlation of the HP-filtered log productivity gap and the HP-filtered log nonagricultural employment in a sample of 63 countries. Panel d shows the volatility of total employment relative to the volatility of GDP in a sample of 66 countries. We use a smoothing parameter 6.25 for the HP filter Ravn and Uhlig (2002). The x-axis denotes the mean agriculture's share in total employment over the sample period for each country. Appendix A describe how we constructed the data.

4. Consumption is more volatile relative to GDP (cf. Aguiar and Gopinath (2007)).

These observations (especially, point 3) suggest a systematic relationship between business cycles and the drivers of structural change. In developing and emerging economies, recessions are times of slowdown and even reversal of structural change: People stop leaving farms or even move back to rural areas, and recessions have a sullying effect on the productivity of agriculture.[6]

3 A Model of Business Cycle with Structural Change

In this section, we present a dynamic general equilibrium model in the spirit of Acemoglu and Guerrieri (2008) which describes the process of growth and structural change in an economy with a declining agriculture. We first derive an analytical characterization of the equilibrium.[7] Then, we estimate the key structural parameters of the deterministic version of the model using data for China. Finally, we introduce uncertainty and productivity shocks, and show that the stochastic version of the estimated model is consistent with salient business cycle features of China.

3.1 Environment

3.1.1 Production

The consumption good, assumed to be the numeraire, is a CES aggregate of two goods,

$$Y = F\left(Y^G, Y^M\right) = \left[\gamma \left(Y^G\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(Y^M\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{1}$$

We label the sectors producing the two goods *agriculture* (superscript G) and *nonagriculture* (superscript M, as in "manufacturing"), respectively. We denote by $\varepsilon > 0$ the elasticity of substitution between the two goods.[8]

The technology of nonagriculture is described by the Cobb Douglas production function

$$Y^M = \left(K^M\right)^{1-\alpha} \times \left(Z^M H^M\right)^{\alpha}, \tag{2}$$

where $H^M = hN^M$ is the labor input. N^M denotes the number of workers and h denotes the number of hours worked by each of them. K^M denotes capital and Z^M is a productivity parameter (henceforth, TFP).

Agriculture is a CES aggregate of two subsectors producing imperfect substitutes with an elasticity

[6] This pattern is documented in a recent empirical study for China. Using data from Northern Jiangsu, Zhang et al. (2001) document that farm employment increases during recessions and decreases during booms. Thus, the agricultural sector stabilizes total employment in China.

[7] In the text, we summarize the main findings. The technical analysis including proofs of all propositions and lemmas are provided in Appendix B.

[8] The technology parameters γ and ε can alternatively be interpreted as preference parameters, reflecting the relative weight and the elasticity of substitution between goods produced by the agricultural and nonagricultural sector.

of substitution $\omega > 0$. More formally,

$$Y^G = \left[\left(Y^{AM} \right)^{\frac{\omega-1}{\omega}} + \left(Y^S \right)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}. \tag{3}$$

Modern agriculture (superscript AM) uses a Cobb-Douglas technology with capital and labor (in the quantitative section we also introduce land in agriculture):

$$Y^{AM} = \left(K^{AM} \right)^{1-\beta} \left(Z^{AM} H^{AM} \right)^{\beta}. \tag{4}$$

Traditional agriculture (superscript S, as in "subsistence") does not use any capital:

$$Y^S = Z^S H^S. \tag{5}$$

Note that the presence of a traditional sector implies a variable elasticity of substitution between capital and labor in agriculture. When $\omega > 1$, the elasticity of substitution is larger than unity and declines as the economy develops. For large K^{AM}, the elasticity of substitution approaches unity.

We assume the TFPs Z^M, Z^{AM}, and Z^S to grow at the exponential rates g^M, g^{AM}, and g^S, respectively. In the quantitative analysis, introduce stochastic disturbances to the TFP levels. All goods are produced competitively. Both capital and labor are perfectly mobile across sectors. Capital depreciates at the rate δ.

3.1.2 Preferences

Agent's preferences are described by a logarithmic utility function.

$$U = \int_0^{\infty} \left(\theta \log c + (1-\theta) \log (1-h) \right) \times e^{-(\rho-n)t} dt. \tag{6}$$

Here, $c \equiv C/N$ denotes the consumption per capita, $1-h$ is leisure, and ρ is the time discount rate. Population grows at the exogenous rate $n < \rho$. In the analytical section, for simplicity, we assume an inelastic labor supply by setting $\theta = 1$ (hence, $H^i = N^i$). We introduce an endogenous labor-leisure choice below where we estimate the model and study economic fluctuations. When we introduce uncertainty, (6) is replaced by an expected utility function with unit relative risk aversion. We suppress time indexes when it is not a source of confusion.

The representative household maximizes expected utility subject to a set of period budget constraints $Nc + \dot{K} = WN + RK + Tr$, where $K = K^M + K^{AM}$ and $N = N^M + N^{AM} + N^S$ denote the aggregate capital stock and number of workers, respectively. W denotes the after-tax wage that is equalized across sectors in equilibrium; R denotes the equilibrium (gross) interest rate, and Tr is government transfers.[9]

[9] For analytical convenience, we write and characterize equilibrium in the deterministic model using a continuous-time representation. When we introduce stohastic shocks, we switch to discrete time representation.

3.1.3 Misallocation

Since in the data we often observe persistent labor wage differences across agriculture and nonagriculture (see, e.g., Gollin et al. (2014)), we introduce an exogenous wedge causing overemployment in agriculture. Formally, we assume that the government taxes wages in nonagriculture at the rate τ and rebates the tax proceeds to the representative household as lump-sum transfers $Tr = \tau W^M N^M$, where W^M denotes the nonagriculture pre-tax wage. In equilibrium, $W^{AM} = W^S = (1-\tau) W^M = W$.

The wedge τ is a stand-in for a variety of labor mobility frictions such as the *hukou* system and the land assignment policy in rural China—see, e.g., Ngai et al. (2018). The wedge distorts resource allocation preventing the equalization of the marginal product of labor across sectors. This misallocation increases the agricultural employment and reduces the equilibrium after-tax wage W. The wedge implies that sectoral productivity shocks that reshuffle labor and capital from agriculture to nonagriculture have productivity- and welfare-enhancing effects. We return to this in Section 5.

3.2 Competitive Equilibrium

We characterize the recursive competitive equilibrium as the solution of a distorted planning problem. More formally, the planner maximizes (6) subject to the resource constraint

$$\dot{K} = F\left(Y^G, Y^M\right) - \tau \bar{W} N^M - \delta K - C + Tr, \tag{7}$$

and to the technological constraints implied by eq. (1)–(5). Here, $\bar{W}$ denotes the marginal product of labor in manufacturing and $Tr = \tau \bar{W} N^M$. When solving the problem, the planner takes $\bar{W}$ as parametric. This results in too small a share of the labor input being allocated to nonagriculture relative to first best. Note that, in the decentralized equilibrium, $W^M = \bar{W}$.

It is useful to introduce some normalizations.

Notation 1 *Define:*

$$\begin{aligned}
c &\equiv \frac{C}{N}, \ \chi \equiv \frac{K}{N}, \\
\kappa &\equiv \frac{K^M}{K}, \ \nu^M \equiv \frac{N^M}{N}, \ \nu^{AM} \equiv \frac{N^{AM}}{N}, \ \nu^S \equiv \frac{N^S}{N} \\
\upsilon &\equiv \frac{\left(Y^{AM}\right)^{\frac{\omega-1}{\omega}}}{\left(Y^{AM}\right)^{\frac{\omega-1}{\omega}} + \left(Y^S\right)^{\frac{\omega-1}{\omega}}}.
\end{aligned}$$

χ is the aggregate capital-labor ratio, the key endogenous state variable of the economy. κ is the share of the aggregate capital stock used in nonagriculture. Since the traditional sector does not use capital, $1-\kappa$ is the corresponding share in modern agriculture. $\nu^i \equiv N^i/N$ denotes the employment share in sector $i \in \{AM, M, S\}$ and υ captures the share of agricultural output produced by the modern sector.

We characterize equilibrium in two stages. First, we solve the static problem defined the maximized current output per capita subject to the wedge τ and a given aggregate stock of capital per worker

χ and the TFP levels. Then, we solve the dynamic equilibrium involving capital accumulation and technical progress.

3.2.1 Static Equilibrium

Let $\boldsymbol{x} \equiv \left(\kappa, \nu^S, \nu^{AM}, \nu^M\right)$ and $\boldsymbol{Z} = \left(Z^M, Z^{AM}, Z^S\right)$. Then, the competitive equilibrium maximizes output per worker y subject to the above-mentioned distortion:

$$y\left(\chi, \boldsymbol{Z}\right) = \max_{\boldsymbol{x}} f\left(y^G\left(\boldsymbol{x}, \chi, \boldsymbol{Z}\right), y^M\left(\boldsymbol{x}, \chi, \boldsymbol{Z}\right)\right) - \tau \bar{W} \nu^M + Tr. \tag{8}$$

The maximization is also subject to the technology constraint

$$f\left(y^G, y^M\right) = \left(\gamma\left(y^G\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(y^M\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{9}$$

where

$$\begin{aligned} y^G &= y^G\left(\boldsymbol{x}, \chi, \boldsymbol{Z}\right) = \left(\begin{array}{c} \left(\left(Z^{AM}\right)^{\beta} \times \left(\left(1-\kappa\right)\chi\right)^{1-\beta} \times \left(\nu^{AM}\right)^{\beta}\right)^{\frac{\omega-1}{\omega}} \\ + \left(Z^S \nu^S\right)^{\frac{\omega-1}{\omega}} \end{array} \right)^{\frac{\omega}{\omega-1}}, \\ y^M &= y^M\left(\boldsymbol{x}, \chi, \boldsymbol{Z}\right) = \left(\left(Z^M\right)^{\alpha} \times \left(\kappa\chi\right)^{1-\alpha} \times \left(\nu^M\right)^{\alpha}\right), \end{aligned}$$

and to the resource constraints $\kappa \in [0,1]$ and $\nu^M + \nu^{AM} + \nu^S = 1$.

The static allocation attains constrained production efficiency, conditional on the wedge τ. This requires the equalization of the marginal product capital across nonagriculture and modern agriculture and of the marginal product of labor across the three sectors.

Proposition 1 *Given χ and $\boldsymbol{Z}$, a static competitive equilibrium is characterized by a set of policy functions $\kappa = \kappa\left(\chi, \boldsymbol{Z}\right)$, $\upsilon = \upsilon\left(\chi, \boldsymbol{Z}\right)$, $\nu^M\left(\kappa\left(\chi, \boldsymbol{Z}\right), \upsilon\left(\chi, \boldsymbol{Z}\right)\right)$, $\nu^{AM}\left(\kappa\left(\chi, \boldsymbol{Z}\right), \upsilon\left(\chi, \boldsymbol{Z}\right)\right)$, and $\nu^S\left(\kappa\left(\chi, \boldsymbol{Z}\right), \upsilon\left(\chi, \boldsymbol{Z}\right)\right)$ satisfying the First Order Conditions of the program (8) – see eq. (22), (23), (24), (25) Appendix B – and the technology constraint (9).*

Next, we characterize some properties of the policy functions, focusing on the case $\varepsilon > 1$ which is consistent with our estimates below.

Capital deepening: In the general case, the comparative statics of κ with respect to χ is involved and possibly nonmonotonic. However, we can establish sharp monotonicity properties for the special case in which the two elasticities, ω and ε, are close to each other. In this case, capital deepening triggers a relative decline of agriculture (i.e., increasing κ) and modernization (i.e., increasing υ) at all stages of economic transition. The analysis of the first-order conditions of the program (8) (see Appendix B) leads to the result that

$$\lim_{\omega \to \varepsilon} \frac{\partial \ln \kappa\left(\chi, \boldsymbol{Z}\right)}{\partial \ln \chi} = \frac{\left(\varepsilon - 1\right)\left(\beta - \alpha\right)\left(1-\kappa\right)}{1 + \left(\varepsilon - 1\right)\left(\left(\beta - \alpha\right)\left(\kappa - \nu^M\right) + \nu^S\left(1-\beta\right)\right)}, \tag{10}$$

which is a generalization of a result in Acemoglu and Guerrieri (2008). If $\varepsilon > 1$ and $\beta > \alpha$, this derivative is necessarily positive. Building on this result, we establish the following lemma.[10]

Lemma 1 *Suppose $\beta > \alpha$, and $\varepsilon > 1$. Then, there exists $\bar{x} > 0$ such that, if $\|\omega - \varepsilon\| < \bar{x}$, then, both $\kappa(\chi, \boldsymbol{Z})$ and $\upsilon(\chi, \boldsymbol{Z})$ are monotone increasing in χ. Moreover, ν^M, ν^M/ν^{AM}, and ν^{AM}/ν^S are monotone increasing in χ while ν^S is monotone decreasing in χ.*

Lemma 1 establishes the intuitive result that, under appropriate conditions, capital deepening (holding technology constant) yields an increase in the shares of capital and labor in the economy that are allocated to nonagriculture, and a decline of traditional agriculture. The assumption that $\beta > \alpha$ is sufficient for Lemma 1 to hold true. This assumption has no bearing on the long-run properties of the model in Proposition 2 below. Note that we will not impose any restriction on ε nor on β in our quantitative analysis.

TFP differences: Next, we study some comparative statics of the sectoral TFPs. First, we note that neutral technical change (i.e., a proportional increase in all sectoral TFPs) leaves κ and υ unaffected, i.e., $\kappa(\chi, \boldsymbol{Z}) = \kappa(\chi, \lambda\boldsymbol{Z})$ and $\upsilon(\chi, \boldsymbol{Z}) = \upsilon(\chi, \lambda\boldsymbol{Z})$ for any $\lambda > 0$. The following lemma focuses on the effect of unbalanced technical progress in nonagriculture vs. modern agriculture. This comparative static is informative about the properties of the balanced growth equilibrium discussed below.

Lemma 2 *Suppose $\varepsilon > 1$. Let $z_A \equiv \left(Z^{AM}\right)^\beta / \left(Z^M\right)^\alpha$. Then, both κ and ν^M/ν^{AM} are decreasing in z_A.*

Lemma 2 establishes the intuitive result that, if agriculture and nonagriculture are substitutes, a TFP increase in nonagriculture relative to modern agriculture induces a reallocation of both capital and labor towards nonagriculture.

3.2.2 Dynamic Equilibrium

In this section, we characterize the dynamic equilibrium. Using the equivalence between the distorted planning solution and the competitive equilibrium, we characterize equilibrium as the solution of the following program:

[10] The lemma is established using eq. (10) together with the fact that the capital-labor ratio in modern agriculture is proportional to that in nonagriculture. More formally, the First Order Conditions imply that

$$\frac{\kappa}{\nu^M} = \frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{1-\kappa}{\nu^{AM}}, \tag{11}$$

which in turn implies that capital accumulation triggers capital deepening in both sectors. Since, by eq. (10), $\partial\kappa/\partial\chi > 0$, (11) implies that $\partial\left(\nu^M/\nu^{AM}\right)/\partial\chi > 0$. Moreover, capital deepening in agriculture implies that modern agriculture accounts for a growing share of the total agricultural production, i.e., $\partial\upsilon/\partial\chi > 0$. Finally, employment in the traditional sector must fall ($\partial\nu_S/\partial\chi > 0$), since, on the one hand, capital deepening increases the productivity of modern agriculture and, on the other hand, modern and traditional agriculture are substitutes ($\omega > 1$).

$$\max_{[c_t,\boldsymbol{x}_t,\chi_t]_{t=0}^{\infty}} U=\int_0^{\infty} \log\left(c_t\right)\times e^{-(\rho-n)t}dt,$$

subject to the resource constraint

$$\dot{\chi}_t=f\left(y^G\left(\boldsymbol{x}_t,\chi_t,\boldsymbol{Z}_t\right),y^M\left(\boldsymbol{x}_t,\chi_t,\boldsymbol{Z}_t\right)\right)-\left(\delta+n+g^M\right)\chi_t-c_t,$$

where f is given by (9), and to the exogenous law of motion of TFPs, $\dot{Z}_t^M/Z_t^M=g^M$, $\dot{Z}_t^{AM}/Z_t^{AM}=g^G/\beta$, and $\dot{Z}_t^S/Z_t^S=g^G$. The problem is subject to a vector of initial conditions $(\chi_0,\boldsymbol{Z}_0)=\left(\bar{\chi}_0,\bar{\boldsymbol{Z}}_0\right)$.

The set of static equilibrium conditions allows us to write the present-value Hamiltonian as

$$H\left(c_t,\chi_t,\boldsymbol{Z}_t,\xi_t\right)=e^{-(\rho-n)t}\log\left(c_t\right)+\xi_t\left(y\left(\chi_t,\boldsymbol{Z}_t\right)-\left(\delta+n\right)\chi_t-c_t\right), \tag{12}$$

where c_t is a control variable, $(\chi_t,\boldsymbol{Z}_t)$ is the state vector, and ξ_t is a dynamic Lagrange multiplier. The production function $y\left(\chi_t,\boldsymbol{Z}_t\right)$ subject to distorted productive efficiency (static equilibrium) can be expressed as

$$y\left(\chi_t,\boldsymbol{Z}_t\right)=\eta\left(\kappa\left(\chi_t,\boldsymbol{Z}_t\right),\upsilon\left(\chi_t,\boldsymbol{Z}_t\right)\right)\times\left(\chi_t\kappa\left(\chi_t,\boldsymbol{Z}_t\right)\right)^{1-\alpha}\left(\nu^M\left(\kappa\left(\chi_t,\boldsymbol{Z}_t\right),\upsilon\left(\chi_t,\boldsymbol{Z}_t\right)\right)\right)^{\alpha}, \tag{13}$$

where

$$\eta\left(\kappa\left(\chi_t,\boldsymbol{Z}_t\right),\upsilon\left(\chi_t,\boldsymbol{Z}_t\right)\right)=(1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}\left(1+\frac{1-\alpha}{1-\beta}\frac{1-\kappa\left(\chi_t,\boldsymbol{Z}_t\right)}{\kappa\left(\chi_t,\boldsymbol{Z}_t\right)}\frac{1}{\upsilon\left(\chi_t,\boldsymbol{Z}_t\right)}\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{14}$$

Note that $\kappa\left(\chi,\boldsymbol{Z}\right)$, $\nu^M\left(\chi,\boldsymbol{Z}\right)$, and $\upsilon\left(\chi,\boldsymbol{Z}\right)$ are the policy function consistent with the static equilibrium. Thus, eq. (13) yields the technology constraint subject to the static equilibrium conditions.

Appendix B (in particular, Proposition 3 and Corollary 1) provides a complete characterization of the dynamic equilibrium in terms of a system of Ordinary Differential Equations (ODE), a set of initial conditions, and a transversality condition. The equilibrium can be represented by an autonomous system of ODE (eq. (34)–(37) in the appendix) in the endogenous variables c_t,κ_t,υ_t, and χ_t, and the exogenous law of motion $\dot{Z}_t^M/Z_t^M=g^M$, where the initial conditions are given by χ_0 (i.e., the initial capital stock), the initial state of technology, and κ_0 and υ_0 are set at the levels that satisfy the static equilibrium conditions at time zero $\kappa_0=\kappa\left(\chi_0,\boldsymbol{Z}_0\right)$ and $\upsilon_0=\upsilon\left(\chi_0,\boldsymbol{Z}_0\right)$. A standard transversality condition pins down the (unique) optimal choice of the control variable c_0. This dynamic system fully characterizes the unique equilibrium trajectories of all variables in the economy.

Proposition 2 characterizes the long-term properties of the model. We provide conditions under which the economy converges to an asymptotic balanced growth path (ABGP) where the agriculture is fully modernized and the share of nonagriculture in total GDP is unity.

Proposition 2 *Let $k^M=\kappa\chi$ and $k^{AM}=(1-\kappa)\chi$. Then, there exists an Asymptotic Balanced Growth*

Path (ABGP) such that

$$\begin{aligned}
\frac{\dot{c}_t}{c_t} &= \frac{\dot{\chi}_t}{\chi_t} = \frac{\dot{k}_t^M}{k_t^M} = g_M; \\
\kappa_t &= \nu_t^M = 1; \; \frac{\dot{\kappa}_t}{\kappa_t} = \frac{\dot{\upsilon}_t}{\upsilon_t} = 0; \\
\frac{\dot{k}_t^M}{k_t^M} &= g^M, \; \frac{\dot{k}_t^{AM}}{k_t^{AM}} = g^M - (\varepsilon - 1)\left(\beta g^M - g^G\right); \\
\frac{\dot{N}_t^M}{N_t^M} &= n, \; \frac{\dot{N}_t^{AM}}{N_t^{AM}} = n - (\varepsilon - 1)\left(\beta g^M - g^G\right); \\
\frac{\dot{N}_t^S}{N_t^S} &= \frac{\dot{N}_t^{AM}}{N_t^{AM}} - (\omega - 1)(1 - \beta)\, g^M.
\end{aligned}$$

Along the ABGP,

$$\left(\frac{c}{\chi}\right)^* = \left(\frac{g^M + \delta + \rho}{1 - \alpha}\right) - \left(g^M + \delta + n\right), \tag{15}$$

$$\left(\frac{\chi}{Z^M}\right)^* = \left(\frac{(1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}(1-\alpha)}{g^M + \delta + \rho}\right)^{\frac{1}{\alpha}}. \tag{16}$$

If either (i) $\varepsilon > 1$, $\omega > 1$, and $\beta g^M \geq g^G$, or (ii) $\varepsilon < 1$, $\omega > 1$, and $\beta g^M \leq g^G$, then, the ABGP is asymptotically stable, i.e., given a vector of initial conditions $\left(\chi_0/Z_0^M, \kappa_0, \upsilon_0\right)$ close to the ABGP, the economy converges to the ABGP.

The ABGP features a vanishing agriculture. Sufficient conditions for this to happen are that either the elasticity of substitution between nonagriculture and agriculture be larger than unity and technical progress be faster in nonagriculture than in agriculture or that, alternatively, $\varepsilon < 1$ and $\beta g^G \geq g^M$. Note that these conditions do not involve any ranking of capital intensity in nonagriculture vs. modern agriculture (i.e., whether β is smaller or larger than α). The parameter β only matters in determining whether capital accumulation in modern agriculture remains positive in the ABGP. The ABGP features full modernization of agriculture: the traditional sector vanishes both as a share of value added and as a share of total agricultural employment. This is due to the combination of a high elasticity of substitution ($\omega > 1$) and technical progress being faster in modern than in traditional agriculture.

Our theory bears predictions about the labor income shares and the productivity gap. To highlight them, we must map the planner's allocation to its decentralized counterpart. Denote by $LIS^j \equiv W^j H^j / P^j Y^j$ for $j \in \{G, M\}$ the labor income share in sector j. The labor share in nonagriculture is constant, owing to the Cobb-Douglas production function. The labor share in agriculture declines as long as the labor-intensive traditional sector shrinks, consistent with Figure 1 above. More precisely, $LIS^G = \beta\upsilon + 1 - \upsilon$, implying that the labor income share in agriculture declines from unity—when capital is very low and the agriculture is dominated by the traditional sector—to β when agriculture is fully modernized.[11] The theory also predicts a declining productivity gap in line with the data in Figures 1 and 2. The two predictions are two sides of the same coin. More formally, the productivity gap is $APL_t^M/APL_t^G = \left((1-\tau)\times LIS_t^M/LIS_t^G\right)^{-1}$. Note that the existence of a traditional agricultural sector is crucial to explain the empirical regularity that this ratio falls with economic development. In an economy where agriculture is fully modernized (i.e., $\upsilon = 1$), the productivity gap is constant, i.e., $APL_t^M/APL_t^A = (1-\tau)^{-1}\beta/\alpha$.

3.3 Capital Accumulation in the Lewis Model

A particular case for which a global characterization of the equilibrium dynamics is available is when the output of traditional and modern agriculture are perfect substitutes ($\omega \to \infty$). This model is interesting on its own as it close resembles the seminal contribution of Lewis (1954). In addition, it illustrates why the dynamics of κ and υ are in general nonmonotone. Intuitively, when ω is large, labor in the traditional sector is a good substitute for capital in modern agriculture. When capital is scarce, it is then efficient to allocate its entire stock to nonagriculture and defer the modernization of agriculture to a later stage in which capital is more abundant.

The technical analysis is deferred to Appendix B. Here, we summarize the main qualitative features of a transition induced by capital accumulation in the case where $\varepsilon > 1$. The dynamic equilibrium evolves through three stages. In the first stage, (*Early Lewis* stage), capital is very scarce, all agricultural production takes place in the traditional sector ($\upsilon = 0$) and all capital is allocated to the manufacturing sector ($\kappa = 1$). Capital accumulation brings about a steady increase in the relative price of agricultural goods and a growth in the real wage. The interest rate decreases over time.

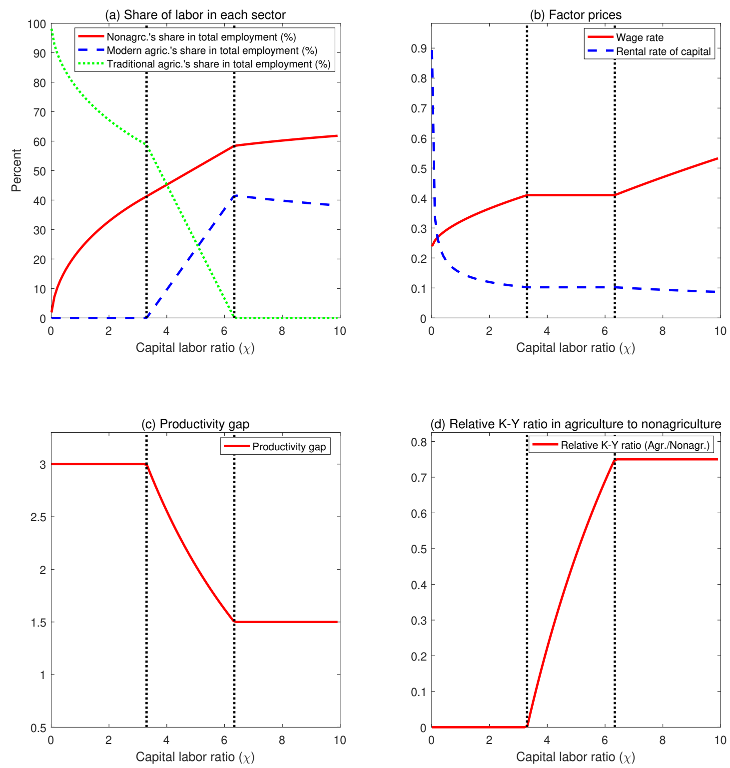
At some point, the increase in the capital stock and the growing price of the agricultural good make it efficient to activate modern agriculture allocating part of the capital stock to this subsector. We enter then the *Advanced Lewis* stage, when the share of capital in modern agriculture (κ) increases in χ. Employment increases in both nonagriculture and modern agriculture and declines in traditional sector—thus, υ increases. The sectoral capital-labor ratios and the relative factor price remain constant over time. Intuitively, during this stage of development capital accumulation does not trigger capital deepening in either of the two sectors employing capital. Any increase in capital is matched by a reduction in employment in the traditional sector—with no effect on the marginal product of labor because production there is linear in labor. Thus, the equilibrium factor prices remain constant as long as there is a labor force reserve in the traditional sector. The mechanism of this transition is reminiscent of Ventura (1997) and Song et al. (2011).

When the labor force reserve in traditional agriculture is exhausted, the economy enters the third stage (*Neoclassical* stage). In this stage, agricultural production takes place exclusively in modern farms using capital ($\upsilon = 1$). Since $\varepsilon > 1$ and nonagriculture is more capital intensive than agriculture, the output share of nonagriculture keeps growing. Wages grow and the interest rate falls like in the

[11] One could obtain a declining labor share by assuming an aggregate CES production function in agriculture.with a high elasticity of substitution between capital and labor, like in Alvarez-Cuadrado et al. (2017). However, such alternative model would feature, counterfactually, an ever declining labor share that would converge to zero in the long run. In our model, like in the data, the labor share in agriculture declines but remains bounded away of zero.

transition of a standard neolclassical growth model.

Figure 6: Equilibrium in the Lewis model



Note: The figure illustrates the allocations and prices as a function of capital per worker, χ, in a version of the Lewis model ($\omega \to \infty$ and no technical change).

Figure 6 displays the equilibrium dynamics during an economic transition featuring a growing capital stock over time, assuming no technical progress. We do not spell out here equilibrium law of motion of capital for which we refer the reader to the analysis in Section 3.2.2. Each panel shows the aggregate capital labor ratio χ on the horizontal axis. Panel a plots the share of labor in each sector. The labor share in the traditional sector (ν^S) starts high and declines with χ in both the Early and Advanced Lewis stages. The labor share in nonagriculture increases throughout the entire transition. The labor share in modern agriculture is nonmonotone: it is zero in the Early Lewis stage, takes off in the Advanced Lewis stage, and declines again in the neoclassical stage.

Panel b plots the factor price dynamics. During the Early Lewis stage, the interest rate falls and wages increase with capital accumulation. Wages and interest rates are flat during the Advanced Lewis stage. Eventually, the interest rate resumes its fall and wages resume their increase during the Neoclassical stage.

Panel c plots the productivity gap, which tracks the dynamics of υ. The gap is constant during the

Early Lewis stage, when $\upsilon = 0$. It falls in χ during the Advanced Lewis stage, when the traditional sector declines.It is constant again in the Neoclassical stage, when $\upsilon = 1$.

Finally, panel d plots the relative capital-output ratio $\frac{K^G}{P^G Y^G} / \frac{K^M}{P^M Y^M}$. The ratio stays at zero during the Early Lewis stage. It increases during the Advanced Lewis stage. Eventually, it becomes constant during the Neoclassical stage.

4 Estimating the Model

Our quantitative analysis proceeds through the following two steps:

1. We estimate the deterministic model with constant productivity growth in each sector. To this aim, we first calibrate some parameters externally, and then estimate the structural parameters and initial conditions to match moments of the structural change of China between 1985 and 2012.[12]

2. We introduce productivity shocks. We estimate stochastic processes for the three TFP shocks and simulate the model. Then, we evaluate the ability of the stochastic model to account for the business cycle properties of China. Finally, we use the estimated model to forecast how the Chinese business cycle will evolve as economic development progresses further.[13]

In this section, we estimate the model. We generalize the theory in the following dimensions. First, we introduce an endogenous labor supply choice as in eq. (6). Second, we introduce land as a fixed factor in modern agriculture.[14] We could add land also in the traditional sector. However, in the spirit of Lewis (1954), we retain the property of a traditional sector working as a labor force reserve at a constant marginal cost. Third, we relax the assumption that g^{AM} and g^S are constant and assume instead a constant TFP growth rate in agriculture as a whole, in line with standard assumptions in two-sector models like Acemoglu and Guerrieri (2008). Namely, we assume g^G to be time-invariant. We also assume g^S to be constant over time and estimate g^G and g^S from the data. In this specification, g_t^{AM} is obtained residually from the equation $g^G = \beta \upsilon_t g_t^{AM} + (1 - \upsilon_t) g^S$. This model nests the theoretical model in the previous section as a particular case if $g^S = g^G$.[15] Finally, we switch to discrete time because it is more convenient to analyze economic fluctuations.

[12] We choose the year 1985 as our initial period because it was a turning point in internal migration policy. In earlier years restrictions on labor mobility between rural and urban areas were very severe. These restrictions were relaxed in January 1985, following the issuance of the "Ten Policies on Further Active Rural Economy" from the CPC Central Committee and the State Council.

[13] We decided to use China instead of a fully industrialized economy because it is difficult to find data for periods in which one such economy had a large agricultural sector. For instance, the available time-series for the US cover a period when the employment in agriculture is already quite low (20.3% of the total US employment in 1929) and the activities captured by our traditional agricultural sector are arguably negligible. In contrast, China has a large and declining share of employment in agriculture ranging from 57.9% in 1985 to 22.4% in 2012. As late as in 2004 more than 10% of China's agricultural output was produced without machine equipment (source: Fixed Point Survey of Research Center for Rural Economy (RCRE)).

[14] We assume $Y^{AM} = \left(K^{AM}\right)^{1-\beta-\beta_T} \left(Z^{AM} H^{AM}\right)^{\beta} T^{\beta_T}$, where T is land and β_T is the output elasticity of land.

[15] One could alternatively estimate the model under the assumption that g^{AM} and g^S are constant over time. This would imply a time-varying TFP growth rate in agriculture as a whole. In particular, if $g^{AM} > g^S$, TFP growth in

We use a mixture of calibration and estimation methods. More precisely, we set some parameters to exactly match some empirical moments and then we estimate the remaining parameters based on a joint set of moment conditions using the Simulated Method of Moments.

Calibrated parameters: We set $\alpha = 0.5$ to match the average labor-income share in the nonagricultural sector in China (see Bai and Qian (2010)). Then, we estimate g^M using standard growth accounting based on a Cobb Douglas production function – as in the model – to match the trend of real GDP in the nonagricultural sector of China. This yields $g^M = 6.4\%$. Capital is assumed to depreciate at a 5% annual rate. We set $\tau = 0.64$ to match an average wage gap between rural and urban workers of 2.78 over the sample period according to the Rural and Urban Household Surveys. We set $\theta = 0.76$ such that the hours worked in the steady state to be 0.5. We set the discount factor $\frac{1}{1+\rho}$ to be 0.96. We set the annual population growth rate (n) to 1.4%, which is the annualized growth rate of the working-age population in China 1985-2012. This parameter has no significant effect on any of the results. Finally, without loss of generality we normalize $Y_{1985} = 1$.[16]

Estimated parameters: We estimate the following parameters using the Simulated Method of Moments:

$$\left\{\varepsilon, \omega, \gamma, \beta, \beta_T, g^G, g^S, Z_{1985}^M, Z_{1985}^{AM}, Z_{1985}^S\right\}.$$

We target (the natural logarithm of) the following observations:

(i) the share of agricultural employment in total employment;

(ii) the share of capital in agriculture relative to the total capital stock;

(iii) the ratio of real output in agriculture to total GDP;

(iv) the relative value added share of agriculture, evaluated at current prices (i.e., the expenditure share of agricultural goods);

(v) the capital/output ratio;

(vi) the productivity gap between agriculture and nonagriculture (adjusted for rural-urban wage differences). We calculate this gap as the ratio of labor productivity in nonagriculture to agriculture times the ratio of wages in agriculture to nonagriculture. Recall that in the model this gap equals $(1 - \upsilon(1-\beta))/\alpha$, i.e., the ratio of the labor-income share in agriculture to the labor-income share in nonagriculture.[17]

(vii) Finally, we target the overall 1985–2012 change in the productivity gap.[18]

agriculture would accelerate over time throughout transition. This alternative model (that we explored in a previous version of the paper) fits the data less accurately. In our model, capital deepening already ensures that the productivity of agriculture grows during the transition.

[16] Note that it would be redundant to estimate weights on modern versus traditional agriculture in the production function (3) because these weights would be subsumed in the sector-specific TFP levels.

[17] We could alternatively have targeted the empirical labor-income shares directly. However, such data are available only up until 2003. From 2004 onwards, the labor income shares are not comparable to their counterparts in the pre-2004 period. See Bai and Qian (2010) for details. As it turns out, the overall change in the ratio of labor income shares is comparable to the change in the labor productivity gap: over the 1985-2003 period these ratios fall by 13 and 14 log points, respectively.

[18] The reason for adding the overall change in the productivity gap as an explicit moment is that obtaining a sufficiently large change in the productivity gap over the 1985–2012 period is central for the mechanism in our business-cycle analysis. For robustness we also estimate a version of the model without targeting this last moment.

For the moments (i)-(vi) we target the initial, middle, and final observation along the transition or, more precisely, 1985, 2012, and the averages for 1995-2004. This procedure yields 19 moment conditions. The estimation is based on equal weights on the moments (i)-(vi) and give moment (vii) a weight of three to make it comparable to the other moments.

In the estimation of the benchmark model we impose the constraint that the elasticity of substitution between modern and traditional agricultural good, ω is at least as large as the elasticity of substitution between agriculture and non-agriculture, ε. For robustness, we also estimate an unconstrained model where we allow $\varepsilon > \omega$.

When calibrating the model, we also make sure that the consistency of expectation of the TFP growth holds. We can directly identify the non-agriculture TFP growth rate from the data, however, we need the help of the model to tell the split of labor between modern-agriculture and traditional agriculture to identify the TFP levels in the two sectors. Since agents have rational expectation, we require that the expected future traditional/modern agriculture TFP levels should be consistent with the data after using the model implied employment composition.

We measure real output in agriculture and nonagriculture following the same approach as China's National Bureau of Statistics (NBS). To be consistent with the empirical data, we start the model in 1980 and base the 1985-1990 growth rates using 1980 as the base year, then update and chain the output levels using 1990 relative prices, etc.

Nonhomothetic preferences: We also estimate a more general version model allowing for non-homothetic Stone-Geary preferences, where the agricultural good is a necessity. To this end, we reinterpret the goods M and G as final goods that are allocated to consumption (denoted C^M and C^G) and investment (denoted X^M and X^G), where $Y^G = C^G + X^G = Nc^G + X^G$ and $Y^M = C^M + X^M = Nc^M + X^M$. Then, we rewrite the utility function as

$$u\left(c^G, c^M, h\right) = \theta \log\left(\left[\gamma\left(c^G - \bar{c}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(c^M\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\right) + (1-\theta)\log(1-h).$$

We continue to assume that the investment good is a CES aggregate of the agricultural and nonagricultural goods,

$$\left[\gamma\left(X^G\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(X^M\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} = I.$$

The aggregate resource constraint (7) can be written as

$$\Delta K = I - \delta K - \tau \bar{W} H^M + Tr. \tag{17}$$

We denote the total expenditure by $Y = P^G Y^G + P^M Y^M$, the total consumption expenditure as $C = P^G C^G + P^M C^M$, and the total investment expenditure as $I = P^G X^G + P^M X^M$. The constant $\bar{c}$ is an additional parameter we estimate. This model nests the baseline model as a particular case with $\bar{c} = 0$.

Estimation results:

Table 1: Estimated parameters.

		Benchmark	Robustness		
	Calibrated Parameters	$\varepsilon \leq \omega$	Unconstrained Estim.	Stone-Gary	$\varepsilon = 0.5$
n	popul. growth rate (15+)	1.4%	1.4%	1.4%	1.4%
θ	pref. weight on consumption	0.76	0.76	0.76	0.76
$\frac{1}{1+\rho}$	discount factor	0.96	0.96	0.96	0.96
δ	capital deprec. rate	5%	5%	5%	5%
τ	labor wedge	0.64	0.64	0.64	0.64
α	labor share in nonagr.	0.50	0.50	0.50	0.50
g_M	nonagr. TFP growth rate	6.4%	6.4%	6.4%	6.4%
	Estimated Parameters	$\varepsilon \leq \omega$	Unconstrained Estim.	Stone-Gary	$\varepsilon = 0.5$
$\bar{c}$	Subsist. level in food cons.	–	–	0.005	0.133
ε	ES btw agric. and nonagric. cons.	4.07	6.70	4.34	0.50
ω	ES btw modern and trad. agr.	4.07	1.40	4.94	1.28
γ	weight on agric. goods	0.52	0.58	0.56	0.0018
$1-\beta-\beta_T$	capital's income share in modern agr.	0.21	0.21	0.22	0.18
β	labor's income share in modern-agric.	0.17	0.35	0.17	0.16
$\frac{g^G}{\beta+\beta_T}$	TFP growth rate in total agric.	6.0%	6.2%	6.1%	5.7%
g^S	TFP growth rate in trad. sector	4.7%	1.3%	5.0%	0.5%
Z^S_{1985}	initial TFP level in trad. agr.	1.74	1.97	1.62	0.39
Z^{AM}_{1985}	initial TFP level in modern-agr.	0.92	1.29	0.86	0.38
Z^M_{1985}	initial TFP level in nonagr.	2.62	2.84	2.90	1.39
	J-statistic	0.808	0.491	0.805	1.008

Table 1 summarizes the results of the estimation. The first column displays the benchmark model, which features homothetic utility (no subsistence level of agricultural goods). As it turns out, the constraint $\varepsilon \leq \omega$ is binding.

The second column reports the results from an unconstrained estimation, where we only target the moments (i)-(vi), omitting the additional weight on the change in the productivity gap. As can be expected, the unconstrained model implies estimates featuring $\varepsilon > \omega$. For the other parameters, the main difference relative to the benchmark estimation is that the labor share of modern agriculture is larger. As we shall see below, this implies that unconstrained model generates a smaller fall in the labor productivity gap.

In both models the estimated elasticities of substitution ε and ω are significantly larger than unity. This conclusion holds true even if we allow for a subsistence level in food consumption (column 3, labeled "Stone-Geary" with ε unrestricted). Note that the estimated subsistence level in agricultural consumption is very small, less than two percent of agricultural production in 1985. Since the predictions of this model are very similar to those of the benchmark model, we omit further discussion of the Stone-Geary model.

We find that for all models, $g^G > g^S$. This implies that TFP growth is larger in modern than in traditional agriculture ($\beta g_t^{AM} > g^S$) and that g_t^{AM} declines over time throughout transition and converges to βg^G in the long run. The finding that $g^G > g^S$ stems from the fact that the labor productivity gap between agriculture and nonagriculture is falling very fast, given the observed capital accumulation in agriculture. Recall that capital deepening in agriculture contributes to mitigating the agricultural productivity gap, i.e., to an increasing trend in the average labor productivity in agriculture relative to nonagriculture. However, if we were to maintain that $g^G = g^S$, the transition from traditional to modern agriculture would be too slow and the reduction in the productivity gap over the 1985-2012 period would be too small compared to the data.

The findings that $\varepsilon > 1$ and $g^M > g^G / \left(\beta + \beta^T\right)$ are consistent (in a model with land) with the the first case in Proposition 2. With Stone-Geary preferences (third and fourth columns), the estimates of ε fall slightly. The reason is that with homothetic preferences there is only a substitution effect: nonagricultural products turn relatively cheaper due to capital accumulation and faster technical progress causing a decline in the relative demand of agricultural goods. To match the declining expenditure share on agricultural goods observed in the data, the price elasticity of the relative demand must be sufficiently high. In contrast, with Stone-Geary preferences, the expenditure share in agriculture falls also because of an income effect. Therefore, the subsistence level tends to lower the estimated ε (even, possibly, turning it smaller than unity). However, the estimated subsistence level $\bar{c}$ turns out to be very small, corresponding to 1.6% of agricultural production in 1985. We conclude that allowing a positive subsistence level in agriculture ($\bar{c} > 0$) yields negligible changes in the estimated parameters. We leave it to future research to explore the robustness of this finding to more general classes of nonhomothetic preferences.[19]

Note that some existing papers find an elasticity of substitution between agriculture and nonagricultural goods smaller than unity—an issue to which we return in Section 6. To investigate this possibility we consider an alternative estimation of the model where we restrict ε to be equal to 0.5, implying complementarity between agricultural and nonagricultural goods. The results are in column (4). The constrained estimation yields very large estimates for the subsistence parameter, which now amounts to 81% of agricultural production in 1985. In this case, the TFP growth in modern agriculture is not sufficiently large to meet the conditions for the second case of Proposition 2 (convergence to an ABGP when $\varepsilon < 1$). Thus, while this constrained version of the model can be consistent with the decline in agriculture during the period 1985–2012, it predicts a future reversal whereby agriculture eventually takes over in the long run (unless the growth rates of sectoral TFPs were to change at some future time).

Note that the productivity growth rates are very high in both manufacturing and modern agriculture, reflecting the high growth rate of the Chinese economy. In an earlier version we assumed an exogenous future reduction of the TFP growth rate in both nonagriculture and modern agriculture gradually declining to 1.8% by 2085. The results are unchanged.

4.1 Accounting for Structural Change in China

In this section, we show that the benchmark model fits well the data along salient dimensions of the process of structural change.[20] Figures 7 and 8 plot the time series for the empirical targets against the

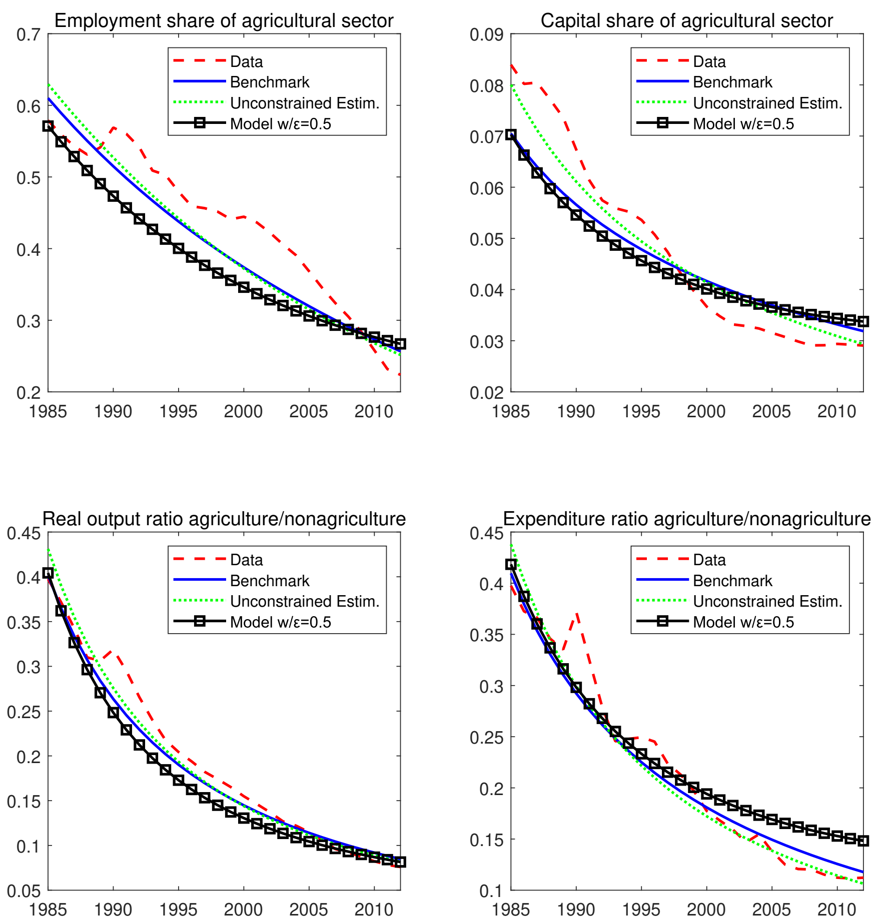
[19] A step in this direction is taken by Yao and Zhu (2021) who find a large role for nonhomothetic preferences. However, their model abstracts from capital accumulation that is the focal point of our analysis. Thus, the findings are difficult to compare.

[20] Using a somewhat larger value for ε, our model would also be consistent with structural change in the U.S. between 1800 and 1945, as documented by Alvarez-Cuadrado and Poschke (2010). See Appendix D.5 for details.

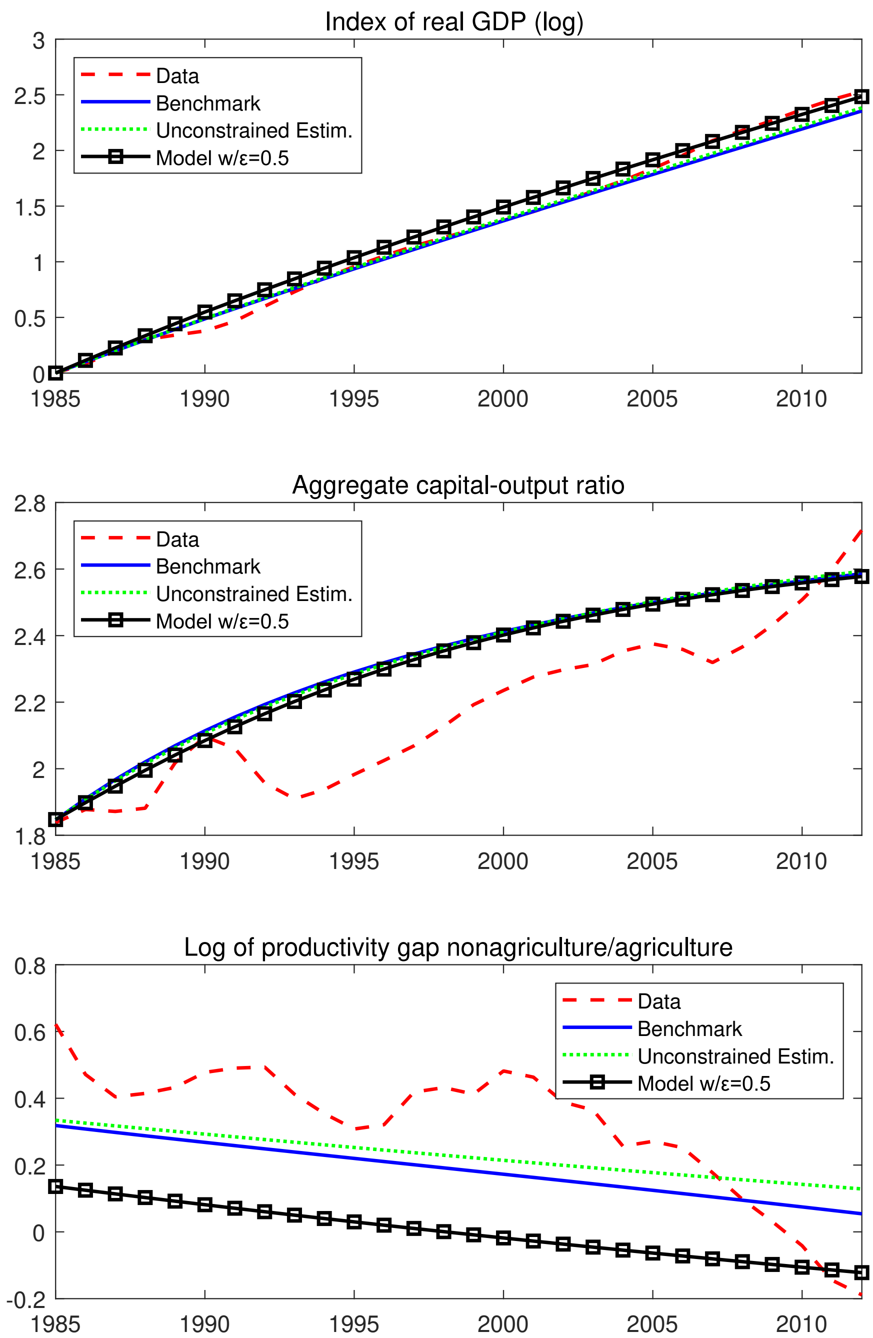
implications of the estimated model – with and without homothetic preferences. Figure 7 shows that the benchmark model predicts that the shares of employment, capital, value added, and expenditure in the agricultural sector relative to total should be falling over time, in line with the data.

Figure 8 displays an index of the log GDP per capita (index), the capital-output ratio, the aggregate labor supply, and the productivity gap measured by the output per worker in agriculture relative to total output per worker. The estimated benchmark model captures well the trend in GDP and capital-output ratio and the falling labor supply. The model can also capture the falling productivity gap, although the data features large swings.

Note that the $\varepsilon = 0.5$ economy can also fit most trends, although it predicts a too small decline in the ratio of expenditure on agriculture versus nonagriculture (Panel D of Figure 7) and a too low level for the labor productivity gap (Panel C of Figure 8).



Note: Structural change in model versus targeted empirical moments. Red dashed lines: data. Solid blue lines: benchmark model. Black lines: unrestricted model. Green lines: restricted $\varepsilon = 0.5$ model. Top left panel: agricultural employment as a share of total employment. Top right panel: the share of aggregate capital invested in agriculture. Bottom left panel: the agricultural value added as a share of aggregate GDP at current prices. Bottom right panel: the expenditure on agricultural goods as a share of aggregate GDP.



Note: Structural change in model versus targeted empirical moments. Red dashed lines: data. Solid blue lines: benchmark model. Black lines: unrestricted model. Green lines: restricted $\varepsilon = 0.5$ model. Top panel: log of real GDP. Middle panel: the capital-output ratio. Bottom panel: the labor productivity gap.

Finally, Figure 9 illustrates the gradual demise of the traditional agriculture implied by the model. The benchmark model predicts an increase in the employment share of modern over total agriculture from 86% in 1985 to 72% in 2012. The output share (corresponding to the variable υ in the model) exhibits a similar behavior (Panel B of Figure 9). The decline of the traditional sector is due to both relative TFP growth and fast capital accumulation. Note that we do not observe a direct distinction between traditional and modern agriculture in the data. For this reason, we cannot provide an explicit measure of fit of the data in this dimension. As discussed above, the transition from traditional to modern agriculture is instead identified by the change in the productivity gap—see the bottom panel of Figure 8. In this respect, our estimated benchmark model predicts one half of the magnitude of the decline in the productivity gap we observe in the data. In contrast, the unconstrained model predicts about one third of the empirical decline. Recall that we did not target explicitly the fall in the productivity gap (moment (vii)) when estimating model.

5 Business Cycle Analysis

In this section, we introduce shocks to the three sectoral TFPs. We assume that capital is allocated to the two sectors one period in advance, knowing $\boldsymbol{Z}_t$ and its stochastic process but not the realization of $\boldsymbol{Z}_{t+1}$.[21] Agents then decide their levels of consumption (c_t), savings, and labor supply (h_t).

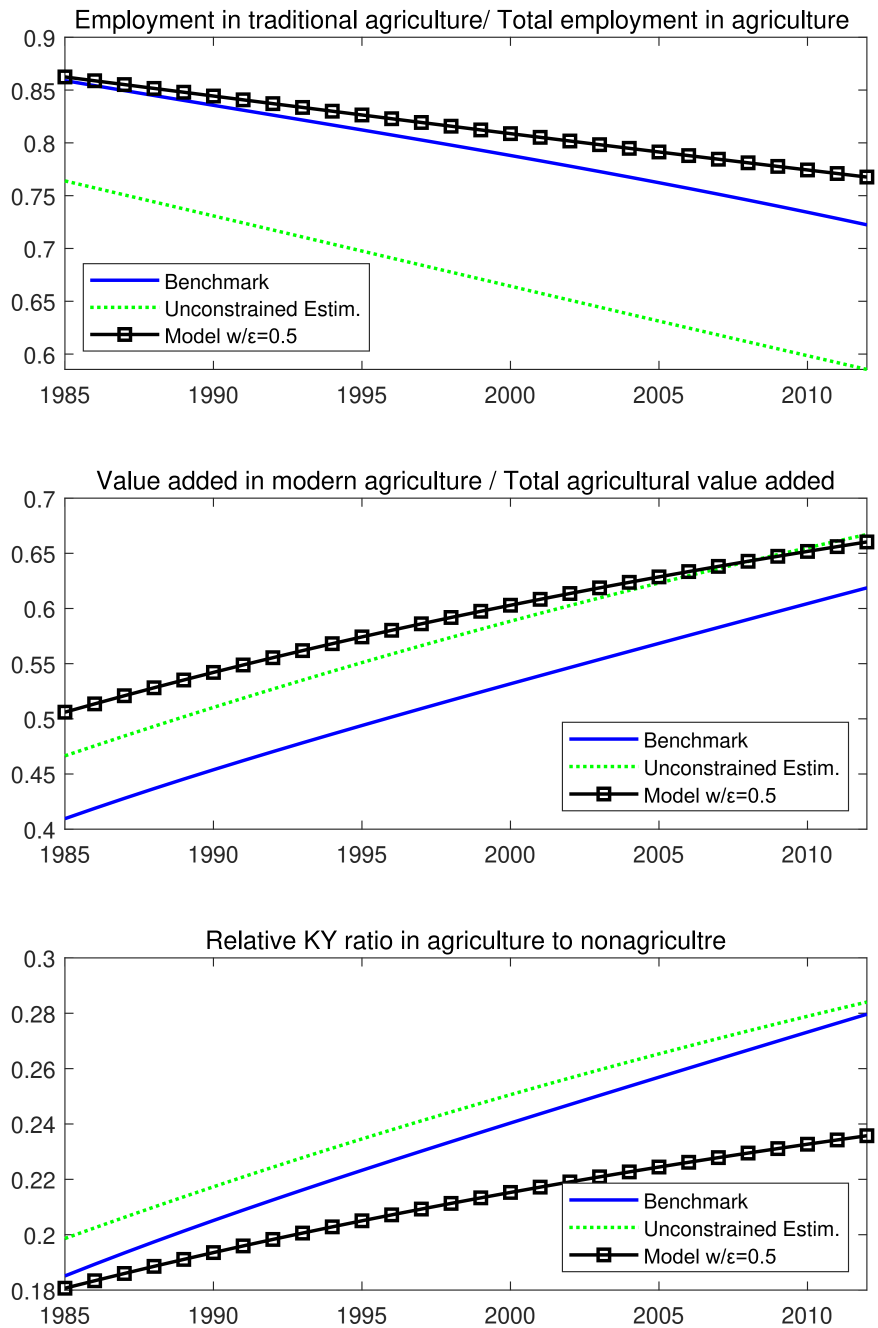
5.1 Estimating the Stochastic TFP Process

The estimation of the process for technology shocks is complicated by the fact that we do not separately measure traditional and modern agriculture in the data. We identify the sectoral TFP using two equilibrium conditions in the theory, subject to the distortion τ: equalization of the marginal product of capital in nonagriculture and modern agriculture and equalization of the marginal product of labor in traditional and modern agriculture. Conditional on the observed levels of capital, labor, and value added in agriculture and non-agriculture, these conditions uniquely identify the sectoral TFP sequence $\left\{\widetilde{Z_t^M}, \widetilde{Z_t^{AM}}, \widetilde{Z_t^S}\right\}_{t=1985}^{2012}$.

We decompose each true TFP into a trend and a cyclical component, assuming a deterministic trend. To this aim, define the cyclical component as $\tilde{z}_t^j \equiv \log\left(\widetilde{Z_t^j}\right) - \log\left(\bar{Z}_t^j\right)$, where $\bar{Z}_t^j = \bar{Z}_0^j \prod_t \left(1 + g_t^j\right)$ is the deterministic trend for $j \in \{M, AM, S\}$.

We assume that the observed sectoral TFP sequence $\tilde{z}_t^j$ consists of two parts: the true TFP level z_t^j and an i.i.d. measurement error ζ_t^j. Moreover, the cyclical component follows an autoregressive VAR(1) process,

[21] With this assumption, κ_t becomes predetermined. More formally, the state vector can be expressed as (κ_t, χ_t, z_t) and the vector of control variables as $\left(\chi_{t+1},\ \kappa_{t+1}, h_t, c_t, \upsilon_t, \nu_t^M, \nu_t^{AM}, \nu_t^S\right)$. A formal statement of the planning problem is deferred to the appendix.



Note: Structural change, according to the models. Solid blue lines: benchmark model. Black lines: unrestricted model. Green lines: restricted $\varepsilon = 0.5$ model. Panel A: share of agricultural employment working in traditional sector. Panel B: evolution of υ. i.e., value added in modern agriculture as a share of total agricultural value added, in current prices. Panel C: relative capital-output ratio in agriculture relative to nonagriculture.

$$\begin{bmatrix} z_{t+1}^M \\ z_{t+1}^{AM} \\ z_{t+1}^S \end{bmatrix} = \begin{bmatrix} \phi^M & 0 & 0 \\ 0 & \phi^{AM} & 0 \\ 0 & 0 & \phi^S \end{bmatrix} \cdot \begin{bmatrix} z_t^M \\ z_t^{AM} \\ z_t^S \end{bmatrix} + \epsilon_t. \tag{18}$$

The vector of innovations ϵ_t is governed by $\epsilon_t = A \cdot \tilde{\epsilon}_t$, where A is a 3×3 matrix and $\tilde{\epsilon}_t$ denotes a vector of orthogonal i.i.d. shocks. The off-diagonal elements in the matrix A capture correlation between the three TFP innovations ϵ_t^j.

We estimate the stochastic process in equation (18), including the persistence ϕ^j, the conditional variance, and the measurement error with GMM, using the covariance matrix of current and lagged values of the cyclical components (period t, $t-1$, and $t-2$). See Appendix C for details. The estimated results are shown in Table 2.

Table 2: The estimated TFP process

$j=$	ϕ^j	$\sigma\left(\varepsilon_t^j\right)$	$\sigma\left(\zeta_t^j\right)$	$corr\left(\varepsilon_t^j,\varepsilon_t^M\right)$	$corr\left(\varepsilon_t^j,\varepsilon_t^{AM}\right)$	$corr\left(\varepsilon_t^j,\varepsilon_t^S\right)$
			Case A: Benchmark			
M	0.70	0.0182	6.0e-04	1	0.631	-0.389
AM	0.51	0.0489	0.0042	0.631	1	-0.745
S	0.51	0.0480	0.0295	-0.389	-0.745	1
			Case B: Unconstrained Estim.			
M	0.70	0.0182	6.0e-04	1	0.592	-0.538
AM	0.46	0.1345	0.0458	0.592	1	-0.964
S	0.35	0.1443	0.0823	-0.538	-0.964	1
			Case C: Stone-Gary			
M	0.70	0.0182	6.0e-04	1	0.635	-0.333
AM	0.51	0.0463	0.0023	0.635	1	-0.675
S	0.58	0.0428	0.0244	-0.333	-0.675	1
			Case D: ε=0.5			
M	0.70	0.0182	6.0e-04	1	0.586	-0.546
AM	0.44	0.187	0.0734	0.586	1	-0.983
S	0.38	0.291	0.148	-0.546	-0.983	1

Note that innovations to nonagriculture and modern agriculture are positively correlated, whereas innovations to traditional agriculture are negatively correlated with innovations in the other sectors. Below we perform a sensitivity analysis where we impose that the three shocks in ϵ_t are orthogonal.

5.2 Simulating the Stochastic Economy

We simulate the model using the estimates in Table 1 after augmenting the model with the stochastic process for TFPs estimated above.

5.2.1 Methodology

Solving the model entails some challenges. We cannot approximate the economy around a balanced growth path. Instead, we proceed as follows. We first solve for a stochastic one-sector version of our model without agriculture, using standard methods. Proposition 2 implies that our three-sector model converges to such one-sector model. We assume that this convergence is approximately completed after 250 years. We then solve the stochastic economy recursively for each time period, back to period $t = 0$ using global methods.[22]

The stochastic process for $z_t \equiv [z_t^M, z_t^{AM}, z_t^S]'$ is approximated by a 27-state Markov chain with three realizations for each shock, using a standard method of Tauchen (1986). There are two continuous state variables, κ and χ. We approximate the next-period decision rules for (c_{t+1}, h_{t+1}) with piecewise linear functions over the state variable $(\kappa_{t+1}, \chi_{t+1}, z_{t+1})$. We solve for the optimal decisions on a grid with 75 grid points for κ and χ, adjusting the location of the grid along the transition.[23] Given decision rules for (c_{t+1}, h_{t+1}), the optimal control variables follow from the state and the optimality conditions. In particular, we solve for current-period optimal choices for $\left(\chi_{t+1}, \kappa_{t+1}, h_t, c_t, \upsilon_t, \nu_t^M, \nu_t^{AM}, \nu_t^S\right)$. The decision rules for $\left(\chi_{t+1}, \kappa_{t+1}, h_t, c_t, \nu_t^M, \upsilon_t\right)$ are approximated by piecewise linear functions over (κ_t, χ_t, z_t) and the decision rules for ν_t^{AM} and ν_t^S follow directly from the optimality conditions once the values for κ_t and ν_t^M are determined.

When simulating the economy, we start each sample economy five years before the initial period. Namely, we calculate a deterministic path that attains the empirical values for $\hat{\chi}_{1985}$ in 1985 (note that κ_{1985} does not match the empirical observation being instead equal to the level implied by the equilibrium path). Then, we start each sample from the implied (pseudo-)initial conditions χ_{1980} and $\kappa_{1980.}$ We simulate 1,000 versions of the economy and calculate statistics for the period 1985–2185.

5.2.2 Results

We start by comparing the empirical business-cycle fact for China with the corresponding statistics for our benchmark model (based on the benchmark parameterization of column 1 in Table 1). Both the empirical data and the simulated data are filtered to remove trends. The upper panel of Table 3 reports the business cycle statistics for China 1985-2012 based on an HP-filter with an HP parameter of 6.25. The lower panels report the corresponding statistics for the simulated economies over the 1985–2012 period. Appendix Table 8 reports the corresponding results based on a first difference filter. In the discussion, we focus on the HP-filtered data. As an inspection of the tables shows, using first differences to filter the data and the model yields similar results as the HP-filter. We conclude that the choice of filter is not important.

We complement the discussion of Table 3 with an analysis of the impulse-response functions to a one-standard deviation shock to each sector-specific TFP holding the other TFPs at the level implied by their deterministic trajectories.[24] Figure 10 shows the impulse response functions for employment,

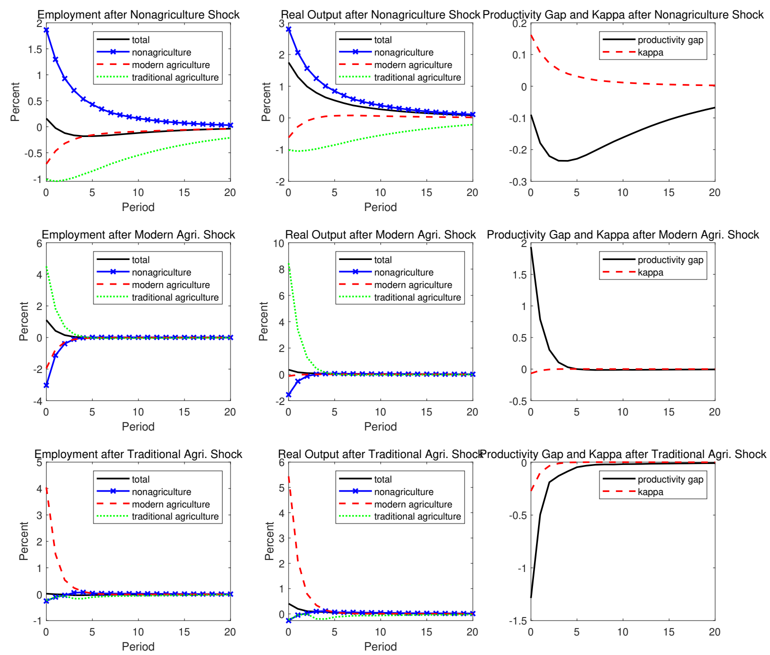
[22]Given the estimated TFP growth rates, the model with ε =).5 converges in the long run to an economy with only modern agriculture. Thus, for this economy we solve for a one-sector model with modern agriculture and assume convergence to this economy after 250 years.

[23]In period t the grid for χ_t is distributed from $0.90\bar{\chi}_t$ to $1.1\bar{\chi}_t$ where $\bar{\chi}_t$ denotes the deterministic trend. Similarly, the grid for κ_t distributed from $\bar{\kappa}_t - 0.025$ to $\bar{\kappa}_t + 0.025$. We verify that the realized path for (κ_t, χ_t) never hits the boundary of this range.

[24]The impulse response functions change over the process of structural change. In the appendix, we report the corresponding impulse response functions for year 2000. Relative to 1985, the productivity gap responds less to shocks because the traditional agriculture has a smaller size.

value added, and the productivity gap to sector-specific TFP shocks.

Figure 10: Impulse Response Functions in year 1985



Note: All graphs show impulse response as percentage deviation from the deterministic path in 1985. The three top panels show responses to a one-standard deviation change in nonagricultural TFP (Z^M). The three middle panels show responses to modern agricultural TFP (Z^{AM}). The three bottom panels show responses to traditional agricultural TFP (Z^S).

Productivity gap. The dynamics of the productivity gap are remarkably similar to its empirical counterparts, see column 7 of Table 3 ("APL^G/APL^M"). Namely, the benchmark model generates the right volatility of the productivity gap and the same correlations between the the productivity gap and output and the sector-specific labor supply. In particular, the productivity gap is countercyclical; it decreases when the employment in nonagriculture is high and decreases when the employment in modern agriculture is high. Intuitively, a positive productivity shock in nonagriculture attracts workers from agriculture and induces modernization in agriculture. Interestingly, a temporary boom is associated with a temporary acceleration of the process of structural change—cf. Lemma 2. As expected, the unconstrained model (Panel C) has a lower volatility of the productivity gap because it has a slower transition over time.

The impulse response functions in Figure 10 illustrate the mechanism. An increase in z^M (upper panels) triggers a shift of labor and capital to nonagriculture. In the period when the shock occurs, only labor adjusts (recall that capital is set one period in advance). Subsequently, capital goes up in the more productive sector, sourced from both modern agriculture and net capital accumulation. Labor comes from traditional agriculture and, to a lesser extent, from modern agriculture. Thus, as labor supply in agriculture falls, this sector becomes modernized: both the average capital intensity and the average labor productivity increase in agriculture. Modernization reduces the productivity gap (upper right panel of Figure 10). A shock to z^S (bottom panels) has a similar but opposite effect on the employment and output dynamics: a higher z^S causes labor to be reallocated to traditional agriculture and sourced from modern agriculture and nonagriculture. This in turn increases the relative size of traditional agriculture which, in turn, increases the productivity gap. Thus, shocks to z^M and z^S induce a positive comovement in the productivity gap and agricultural employment. In contrast, a positive shock to modern agriculture induces both an increase in total employment and modernization of agriculture. Overall, shocks to modern agriculture mitigate the countercyclicality of the productivity gap.

Structural change and sectoral productivity growth. Figure 11 shows the impulse responses for the agricultural employment share in response to TFP shocks. As discussed above, structural change accelerates in response to positive TFP shocks to z^M and slows down is response to TFP shocks in agriculture.

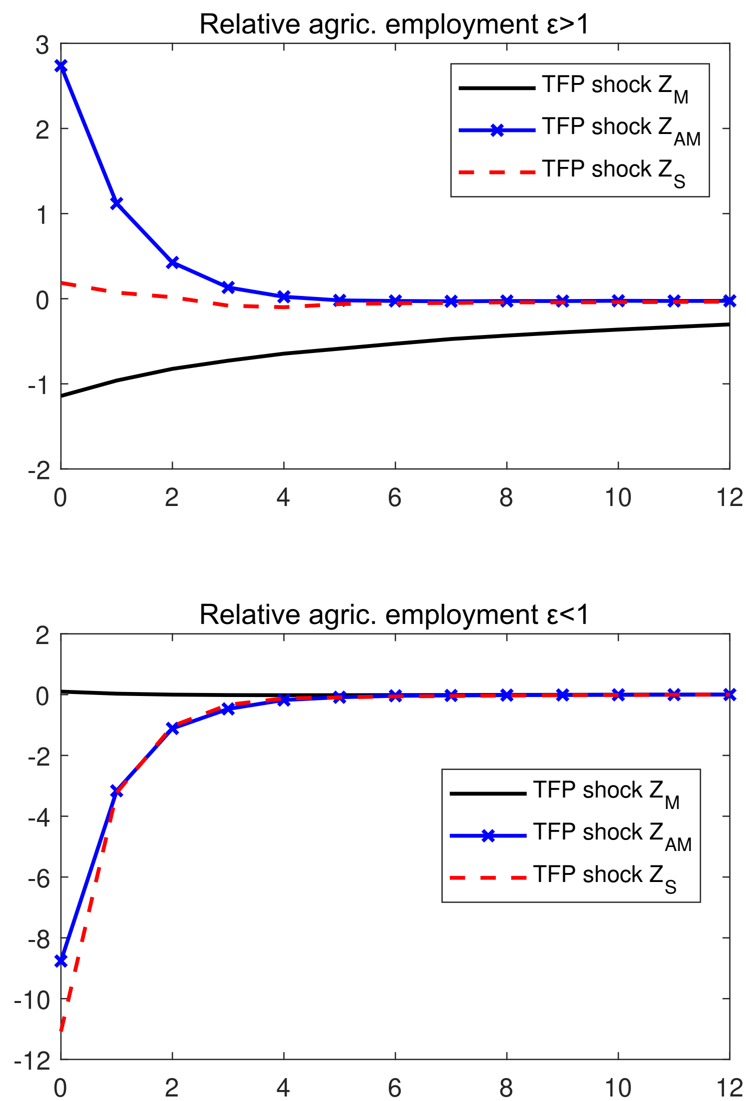
Our impulse-response functions conform with the common narrative about business-cycle fluctuations in contemporary China. For example, during the early 1980s China implemented large agricultural reforms through the so-called Household Responsibility System. This reform induced substantial productivity growth in agriculture (Lin (1988)). According to Figure 3, this period is associated with a slowdown in structural change.[25] Conversely, the period 2002–08 witnessed a remarkable TFP growth in manufacturing, associated with WTO accession and important market reforms (Song et al. (2011)), while TFP in agriculture stagnated (Wang et al. (2013)). In Figure 3, this period is characterized by an acceleration of structural change, which is in line with the prediction of our model after a positive TFP shock in manufacturing.

Sectoral labor supply and misallocation. Both in the model and in the data (see rows 3–4 in Panels A and B of Table 3) employment in nonagriculture ν^M is positively correlated with GDP, consumption, and investment whereas employment in agriculture ν^G is negatively correlated with GDP, consumption, and investment. The root of these asymmetries is misallocation: because of the wedge τ, the agricultural sector is suboptimally too large. Thus, reallocating resources away from agriculture has an efficiency-enhancing effect.

With this in mind, we can contrast the effect of a TFP shock in nonagriculture and in agriculture. When TFP in nonagriculture increases, both capital and labor move towards nonagriculture. Therefore, GDP increases because of both the direct effect of the productivity increase and the decline in misallocation. In contrast, when TFP increases in agriculture, the positive direct effect of an increase in TFP is dampened (and possibly offset) by the increase in misallocation. This dichotomy is clear in the second column of Figure 10: GDP increases sharply after a positive TFP shock in nonagriculture,

[25] Clearly, we do not mean to diminish the value of the agriculture reforms in the 1980s on the development of China. The Household Responsibility System was important in highlighting the salience of individual incentives, with crucial lessons for the subsequent reforms that increased productivity in the industrial sector. It also had dramatic effects on the living conditions of millions of people. These and other aspects are abstracted from in our stylized theory whose focus is positive rather than normative.

Figure 11: Impulse Response to TFP shocks in agriculture



Note: The graphs show impulse responses for the agricultural employment share, i.e., $\left(N^S + N^{AM}\right)/N$, as percentage deviation from the deterministic path. The top panel shows the employment dynamics in the estimated case ($\varepsilon = 4.3$) for each of the TFP shocks (Z^S, Z^{AM}, and Z^M). The bottom panel shows the corresponding responses in the case with a low elasticity ($\varepsilon = 0.5$).

while it hardly moves after TFP shocks to agriculture.[26] These predictions are again consistent with the recent history of China—see Figure 3. The early 1980s (positive TFP shock in agriculture) is associated with below-trend GDP growth whereas the period 2002–08 (positive TFP shock in nonagriculture) is associated with boom in GDP.

The existence of a labor reserve in the traditional sector speeds up reallocation since many workers can leave agriculture without much effect on the marginal product of labor. This effect is reminiscent of Lewis (1954). Both the reduction in misallocation and the low opportunity cost of sourcing workers from agriculture result in a strong correlation between employment in nonagriculture and GDP.

Sectoral Value Added and Real Output. The model accurately predicts a number of cross correlations involving sector-specific production levels (see rows 3-6 in Table 3). Generally, the model is consistent with the empirical correlations of all variables with nonagricultural output and value added (see columns 4 and 6 in Table 3). Moreover, the model is consistent with agricultural output and value added being approximately acyclical (i.e., having close to zero correlation with GDP). The single exception is the correlation of sectoral employment with agricultural output and value added: the model predicts employment in agriculture (nonagriculture) to be strongly positively (negatively) correlated with value added and output in agriculture, while the corresponding empirical correlations are small and insignificant.

Finally, the model is consistent with the empirical observation that output in agriculture is more volatile than nonagriculture, although the differences are slightly larger than in the data.

GDP and aggregate labor supply. The benchmark model generates a negative correlation between GDP and aggregate employment. Moreover, it is consistent with the observation that aggregate labor supply is positively correlated with employment in agriculture and negatively correlated with employment in nonagriculture, although the model overpredicts the magnitude (in absolute value) of these correlations. In particular, employment in nonagriculture and aggregate employment are essentially uncorrelated in the data. However, the model implies a somewhat lower volatility of GDP than in the data and a somewhat too large volatility of employment and correlation of employment with GDP.

Finally, the model shares many of the predictions of a standard one-sector RBC model concerning consumption and investment: investment is more volatile and consumption is less volatile than output. All cross correlations in columns 1 and 2 are consistent in sign with the data, although the model underpredicts the (absolute value of the) correlation between consumption and sectoral employment in agriculture and nonagriculture. In addition, consumption is as volatile as output in the data while the model predicts a significant extent of consumption smoothing. This is a common feature in developing and emerging economies (see, e.g., Aguiar and Gopinath 2007).

[26] TFP shocks to nonagriculture are responsible for the lion's share of GDP fluctuations: a variance decomposition exercise shows that they account for 94 percent of GDP fluctuations over the 1985-2012 period.

Table 3: The table provides business-cycle statistics for China and for various various estimated models. Panel B is the benchmark model, Panel C is the unconstrained model (with $\varepsilon > \omega$) and Panel D is the model with restricted $\varepsilon = 0.5$. All series are HP-filtered.

	c	i	y^G	y^M	$\frac{P^G y^G}{P}$	$\frac{P^M y^M}{P}$	$\frac{APL^G}{APL^M}$	n^G	n^M	n
	A. China, 1985-2012							$std\,(y) = 1.7\%$		
$\frac{std(x)}{std(y)}$	0.99	3.53	0.40	1.21	1.64	1.34	2.17	1.00	1.04	0.10
$corr(x, y)$	0.70	0.65	-0.11	0.99	0.06	0.95	-0.48	-0.78	0.83	-0.23
$corr(x, n^G)$	-0.67	-0.61	0.10	-0.79	-0.01	-0.76	0.62	1.00	-0.93	0.18
$corr\left(x, n^M\right)$	0.64	0.65	-0.14	0.85	0.01	0.82	-0.65	-0.93	1.00	-0.01
	B. Benchmark							$std\,(y) = 1.3\%$		
$\frac{std(x)}{std(y)}$	0.24	2.50	2.09	1.43	1.53	1.30	2.58	3.28	1.52	0.51
$corr\,(x, y)$	0.75	0.99	-0.29	0.94	-0.15	0.96	-0.58	-0.56	0.74	-0.08
$corr\left(x, n^G\right)$	-0.31	-0.56	0.86	-0.74	0.80	-0.71	0.96	1	-0.90	0.81
$corr\left(x, n^M\right)$	0.42	0.75	-0.79	0.91	-0.70	0.89	-0.87	-0.90	1	-0.68
	C. Unconstrained Estim.							$std\,(y) = 1.3\%$		
$\frac{std(x)}{std(y)}$	0.25	2.43	2.72	1.34	2.34	1.27	1.18	2.04	1.30	0.40
$corr\,(x, y)$	0.80	0.99	0.11	0.89	0.17	0.91	-0.57	-0.29	0.62	0.36
$corr\left(x, n^G\right)$	-0.17	-0.29	0.85	-0.63	0.82	-0.60	0.38	1	-0.85	0.70
$corr\left(x, n^M\right)$	0.39	0.62	-0.59	0.89	-0.54	0.87	-0.55	-0.85	1	-0.45
	D. $\varepsilon = 0.5$							$std\,(y) = 1.2\%$		
$\frac{std(x)}{std(y)}$	0.38	2.49	0.70	1.13	2.91	1.15	2.51	1.76	1.01	0.22
$corr\,(x, y)$	0.75	0.99	0.61	0.99	0.09	0.98	-0.31	-0.54	0.71	0.32
$corr\left(x, n^G\right)$	-0.32	-0.56	-0.67	-0.49	0.39	-0.59	0.30	1	-0.89	0.53
$corr\left(x, n^M\right)$	0.47	0.74	0.74	0.67	-0.46	0.78	-0.11	-0.89	1	-0.36

5.3 Business Cycle and Economic Development

This section relates some of the main insights of the paper. We simulate the model beyond the stage of structural change reached by China in 2012. This exercise allows us to trace the path predicted by the estimated model for how the nature of the business cycles will evolve over the development process. We focus on four statistics: (i) the correlation between nonagricultural employment and the productivity gap; (ii) the correlation between agricultural and nonagricultural employment, (iii) the correlation between total employment and GDP, and (iv) the volatility of labor supply and consumption relative to GDP.

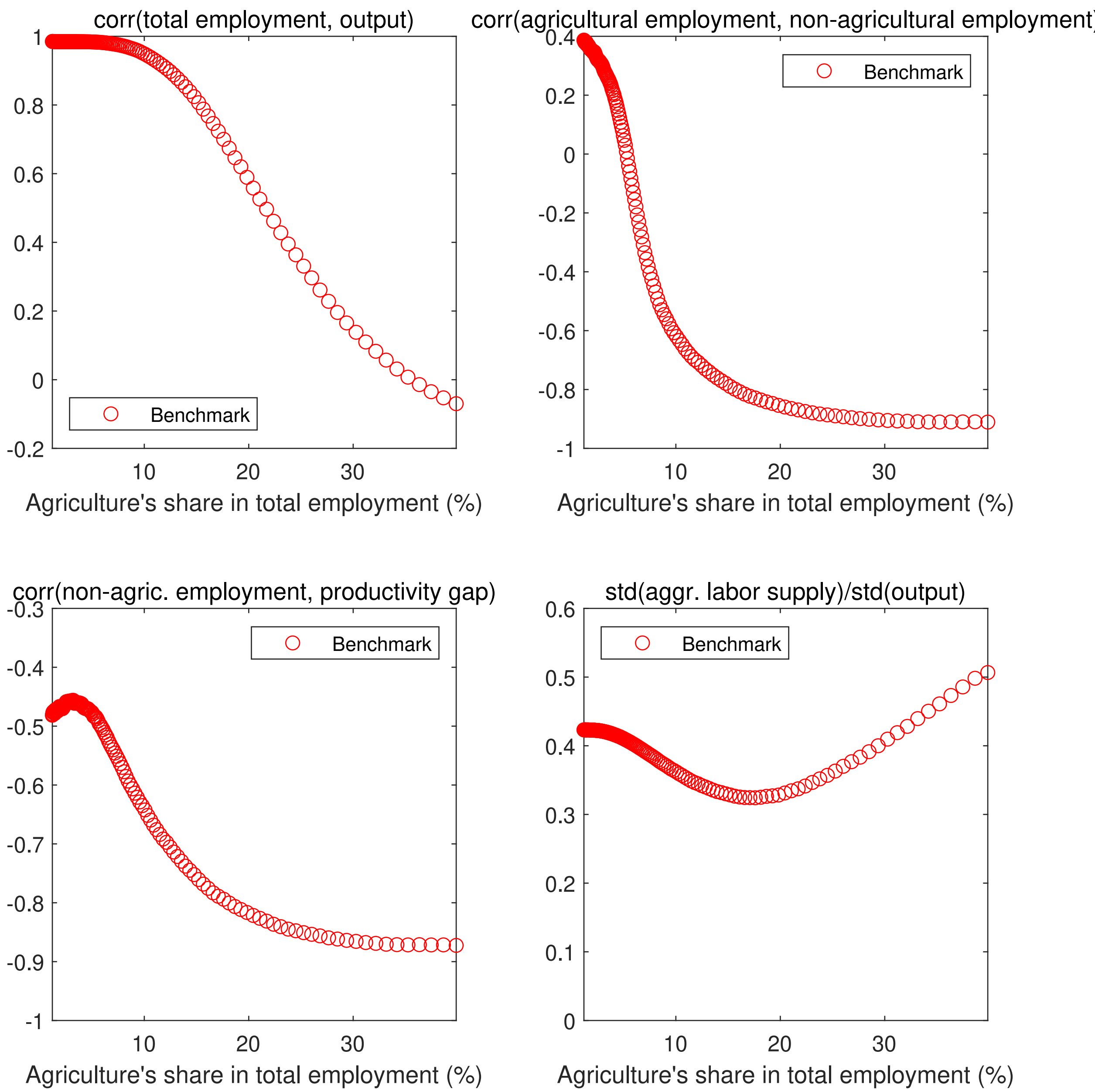
The results are shown in Figure 12. Each dot in the figure represents a statistic covering a 28-year rolling window. The first dot on the right of each figure (i.e., that corresponding to the largest agricultural share) corresponds to simulations over the period 1985-2012 which are those reported in Table 3); the second dot corresponds to 1986-2013, and so on. In each figure, the process of economic development involves a shift from left to right. A fully industrialized economy is an economy with an employment share of agriculture lower than 10%.

Panel A of Figure 12 shows that the correlation between total employment and GDP increases from -0.08 to unity as the share of agriculture falls from 40% to zero. The increase in the correlation along the process of structural change is again in line with the cross-country evidence.

Panel B in Figure 12 shows that the correlation between employment in agriculture and nonagriculture increases as the share of agriculture falls. In particular, the correlation is negative as long as the employment share in agriculture is large, while it becomes positive in the long run. The reason is twofold. First, a large agricultural sector works as a buffer: when there are sector-specific shocks, it is possible to move labor in and out of agriculture. Second, the labor reserve in the traditional sector offers a high elasticity of substitution between sector-specific employments. As this sector shrinks, the effective elasticity of substitution across sectors falls. This finding is again in line with the empirical cross-country evidence—see Panel B in Figure 5.

Panel C shows that the correlation between the size of the nonagricultural sector and the productivity gap increases as the agriculture shrinks, in line with the empirical cross-country evidence documented in Section 2. The source of this decline is the demise of the traditional sector in agriculture. As agriculture modernizes, the economy converges to a Hansen and Prescott (2002) economy with a constant productivity gap.

Panel D shows that the volatility of employment relative to GDP is approximately unchanged during development in our model. This is broadly consistent with the cross-country evidence we documented in (Panel d of Figure 5). Recall that China is an outlier in terms of having substantially lower volatility of employment than other countries at the same level of development. Finally, our model generates an approximately constant volatility of consumption relative to GDP during the process of development. Thus, the model cannot generate the decline in consumption volatility during development documented by Aguiar and Gopinath (2007).



Note: The graphs show the evolution of business cycle statistics as a function of the employment share in agriculture. Each dot shows a statistic covering a 28-year rolling window. Simulated data are HP-filtered. The upper left panel shows the correlation between employment in nonagriculture and the productivity gap. The upper right panel shows the correlation between employment in agriculture and employment in nonagriculture. The lower left panel shows the correlation between total employment and GDP. The lower right panel shows the volatility of aggregate employment relative to GDP.

5.4 Robustness Analysis

This section explores four robustness analysis exercises: (1) a low elasticity $\varepsilon = 0.5$ combined with a large food subsistence level; (2) capital adjustment costs; (3) assuming shocks to Z^S have the same persistence of as shocks to Z^{AM}; and (4) uncorrelated shocks.

5.4.1 Low Elasticity of Substitution

Our estimation identified a high elasticity of substitution ε and small subsistence component in agriculture $\bar{c}$. It is interesting to contrast the properties of our estimated model with those of a model with a low ε and a large $\bar{c}$. Section 4.1 estimates a version of the economy when ε is restricted to $\varepsilon = 0.5$. Under this constraint, the estimation yields a large subsistence level $\bar{c}$.

Consider first the response of relative sectoral employment to TFP shocks to agriculture. The lower panel of Figure 11 shows that the $\varepsilon = 0.5$ economy implies a large *decline* in agricultural employment when TFP increases in agriculture. The reason is twofold. On the one hand, the relative price of agricultural goods falls sharply when ε is low, causing an increase in nonagriculture . On the other hand, when TFP in agriculture increases, the demand for nonagricultural goods increases more, due to a stronger income effect. This implication collides with the common narrative about the Chinese business cycle discussed above. If $\varepsilon < 1$, the decline in agricultural employment should have been fast in the early 1980s when agricultural productivity increased and slow after the manufacturing boom in the years following the 2001 WTO accession. Panel (d) of Figure 3 shows that the opposite is true in the data.

Figure 11 also shows that as TFP increases in nonagriculture, employment in agriculture *increases* slightly. Thus, nonagriculture TFP shocks and nonagricultural employment are negatively correlated. This reflects two effects. On the one hand, the larger TFP in nonagriculture causes a large increase in the relative price of agriculture, which in turn induces an increase in the agricultural employment. On the other hand, as the economy becomes richer, the subsistence level in consumption becomes less relevant. In our estimated economy the former effect dominates. For this reason, TFP shocks to agriculture play a significantly more important role as drivers of business cycles in the $\varepsilon = 0.5$ economy. A variance decomposition analysis shows that agricultural TFP shocks account for about 38% of the GDP fluctuations in the $\varepsilon = 0.5$ economy whereas in the benchmark economy GDP fluctuations are almost entirely driven by TFP shocks to nonagriculture.

The business-cycle properties of the $\varepsilon = 0.5$ economy are presented in panel D of Table 3. Many of the qualitative properties are similar to those of the benchmark economy. However, an important difference is that in the $\varepsilon = 0.5$ economy has the wrong sign for all correlations with real agricultural output: in the model, agricultural output is negatively correlated with agricultural employment and positively correlated with GDP and nonagricultural employment, while the data have exact opposite correlations (see the third column of Table 3). Note that our estimated benchmark model predicts these correlations correctly. The reason for these counterfactual properties of the $\varepsilon = 0.5$ economy is that a positive TFP shock in agriculture increases output in agriculture while causing employment to flow out of agriculture because of the large income effect. For a similar reason, agricultural output is positively correlated with nonagricultural employment.

Finally, Figure 14 documents how the business-cycle properties change over time in the $\varepsilon = 0.5$ economy. As discussed above, this economy features structural change of employment from nonagriculture to agriculture. Therefore, the share of workers in agriculture never falls below 20% along the transition. It is therefore no surprise that this model does not provide a good account for the cross-country evidence on business-cycle statistics.

5.4.2 Other Robustness Exercises

We performed a number of additional robustness analysis whose results are discussed in the online appendix and documented in Table 10 and Figure 14. First, we show that adding adjustment costs to moving capital between sectors does not alter significantly the main results. The only important difference is that in the model with adjustment cost the correlation between GDP and aggregate labor supply is negative (-0.53) as in the data. Second, we consider a version of the model where TFP shocks to traditional agriculture have the same persistence as shocks to modern agriculture ($\hat{\phi}^S = \hat{\phi}^{AM} = 0.9$). We adjust the volatility of the innovations to z^S, $\sigma\left(\epsilon_t^S\right)$, so that the stationary variance of z^S remains constant. The results are basically unchanged. Finally, we consider a version of the model where the TFP shocks are orthogonal. The differences relative to the benchmark model are again small.

6 International Evidence

The elasticity of substitution ε plays an important role in our analysis. When estimating the model for China we found $\varepsilon > 1$. Using data from other countries, other studies have instead estimated $\varepsilon < 1$. For example, Herrendorf et al. (2013) shows that a model with $\varepsilon < 1$ and higher TFP growth in agriculture than in manufacturing accounts well for structural change in the US after 1950. In this section we use data from other countries to evaluate the robustness of our estimate of ε outside of China.

We proceed in two steps. First, estimate aggregate multi-sector production functions based on time-series evidence on structural change in the world's largest economies. Rather then committing to the structure of our model, we use a production function approach that is comparable to previous studies in the literature. We also consider some correlation analysis using cross-country data. Then, we review causal evidence of how TFP shocks in agriculture affect structural change, and discuss its implications for ε.

6.1 Estimating Aggregate Production Functions

Herrendorf et al. (2013) estimate a three-sector CES production function where the elasticity of substitution between agricultural goods, manufacturing goods, and services is assumed to be identical across sectors. Using a consumption value added approach, they estimate an elasticity of substitution between manufacturing, services and agriculture close to zero for the US. Following their lead, we now extend our analysis to a class of models where the nonagricultural sector comprises two sectors: a service sector and a manufacturing sector. More formally, we assume that

$$Y^M = \left[\hat{\gamma}\left(Y^{Manuf}\right)^{\frac{\epsilon_{ms}-1}{\epsilon_{ms}}} + (1-\hat{\gamma})\left(Y^{Serv}\right)^{\frac{\epsilon_{ms}-1}{\epsilon_{ms}}}\right]^{\frac{\epsilon_{ms}}{\epsilon_{ms}-1}}$$

where the superscripts $Manuf$ and $Serv$ denote manufacturing and services, respectively, and the parameter ϵ_{ms} represents the elasticity of substitution between services and manufacturing in production of the nonagricultural good. Moreover, following the approach proposed by Herrendorf et al. (2013), we construct consumption value-added data consistent with national accounts and with input-output tables. We also estimate the model using standard production value-added data from the Gröningen data (GGDC) 10-Sector Database. We estimate the model for the world's three largest countries – China, USA, and Japan.

The estimation procedure follows Herrendorf et al. (2013). More formally, we estimate the following Stone-Geary aggregate production function,

$$Y = \left[\begin{array}{c} (1-\gamma)\left(\left[\hat{\gamma}\left(Y^{Manuf}\right)^{\frac{\epsilon_{ms}-1}{\epsilon_{ms}}} + (1-\hat{\gamma})\left(Y^{Serv}+\bar{s}\right)^{\frac{\epsilon_{ms}-1}{\epsilon_{ms}}}\right]^{\frac{\epsilon_{ms}}{\epsilon_{ms}-1}}\right)^{\frac{\varepsilon-1}{\varepsilon}} \\ +\gamma\left(Y^G+\bar{c}\right)^{\frac{\varepsilon-1}{\varepsilon}} \end{array}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{19}$$

This specification follows that used in a different context by Krusell et al. (2000). In particular, we allow ε to differ from ϵ_{ms}. Moreover, as in Herrendorf et al. (2013) we allow for a subsistence level in agriculture ($\bar{c} \leq 0$) and home production in services ($\bar{s} \geq 0$). Note that the generalized model nests the specification of Herrendorf et al. (2013) as the particular case in which $\epsilon_{ms} = \varepsilon$. The estimated nonlinear equations are stated in Appendix D.

Table 4 summarizes the main results (the full estimation results are reported in Appendix Table 9). We present results for both the three-sector model and a restricted version where we do not distinguish between manufacturing and services (this is closer to the specification we use in our model).

We obtain estimates of the agriculture-nonagriculture elasticity above unity for all countries and with both consumption value added and production value added. The benchmark estimation – the three-sector model using consumption value added data – imply $\varepsilon_{US} = 2.5$, $\varepsilon_{JP} = 1.6$, and $\varepsilon_{CN} = 1.7$, and the estimates are significantly larger than unity at a 1% level for all countries. The production-based estimate of ε is higher for Japan and slightly lower for the US and China. Moreover, in all cases the estimates originating form the three-sector model are very similar to those of the two-sector model.

We estimate ϵ_{ms} to be close to zero for the US and China, and 0.8 for Japan. Thus, our findings strongly reject the null hypothesis that $\epsilon_{ms} = \varepsilon$, and suggests that the elasticity of substitution between agricultural and nonagricultural goods is substantially higher than the elasticity between manufacturing and services.[27]

[27]Note that when we impose the restriction $\epsilon_{ms} = \varepsilon$ and reestimate the model, we replicate the finding of as Herrendorf et al. (2013) that the elasticity is close to zero for the US. Similarly, when this restriction is imposed for Japan and China, we estimate the elasticity to be smaller in unity for both countries, 0.65 and 0.58, respectively.

Table 4: Estimates of elasticities under the nested CES preference specification. ε is the elasticity between agriculture and nonagriculture and ϵ_{ms} is the elasticity between manufacturing and services. Table 9 in Appendix E reports the full results.

Table 4: IFGNLS Estimates of Nested CES Elasticities

	Elasticity	Cons. value added		GGDC	
		3-sector	2-sector	3-sector	2-sector
USA	ε	2.49***	2.32***	1.36***	1.53***
		(0.28)	(0.32)	(0.13)	(0.48)
	ϵ_{ms}	0	-	0.48***	-
			-	(0.005)	-
Japan	ε	1.58***	2.24***	5.46***	6.06***
		(0.053)	(0.36)	(0.80)	(0.91)
	ϵ_{ms}	0.79***	-	1.66***	-
		(0.003)	-	(0.13)	-
China	ε	1.70***	1.34***	1.30***	1.75***
		(0.22)	(0.25)	(0.13)	(0.08)
	ϵ_{ms}	0.007	-	0.65***	-
		(0.05)	-	(0.04)	-

6.2 Correlation between Expenditure and Output (or Prices)

Cross-country data could also be used to shed light on the elasticity ε. In particular, the value of ε has implications about the correlation between relative expenditure and relative production in agriculture. To this aim, consider an aggregate CES production function like in eq. (1), which implies a standard isoelastic demand condition $P_t^G/P_t^M = \frac{\gamma}{1-\gamma}\left(Y_t^G/Y_t^M\right)^{-1/\varepsilon}$. Taking log on both sides and considering the first difference yields

$$\Delta \ln\left(\frac{Y_t^G P_t^G}{Y_t^M P_t^M}\right) = \frac{\varepsilon-1}{\varepsilon}\Delta \ln\left(\frac{Y_t^G}{Y_t^M}\right) \tag{20}$$

or, equivalently,

$$\Delta \ln\left(\frac{Y_t^G P_t^G}{Y_t^M P_t^M}\right) = (1-\varepsilon)\cdot\Delta \ln\left(\frac{P_t^G}{P_t^M}\right) \tag{21}$$

We take these two versions of the demand equation to the data. When doing so, one should bear in mind that this exercise is potentially problematic since both sides of eq. (20)-(21) are endogenous. We return to these endogeneity issues below.

When $\varepsilon > 1$, the model predicts the expenditure ratio growth will be positively correlated with the real output ratio growth (eq. (20)) and negatively correlated with the relative price growth (eq. (21)). Conversely, these correlations will be reversed if $\varepsilon < 1$.

We use value added for 10 sectors from the Gröningen Growth and Development Centre, and use the UN Analysis of Main Aggregates dataset (1970-2017) for robustness. Appendix Table 5 documents the results from estimating eq. (20) with OLS across a number of specifications. All specifications yield regression coefficients in the range 0.46-0.84, which implies an elasticity ε in the range between 2 and 6. In all cases the estimates are significantly positive at a 1 percent level. Conversely, Appendix Table

6 documents the OLS estimates implied by eq. (21). This specification yields regression coefficients in the range -0.44 to 0.78, which implies estimates of ε in the range 0.2 to 1.4. The bulk of the estimates implies $\varepsilon < 1$.

The overall evidence from a regression-based approach is mixed. In Appendix D.2, we investigate possible explanations for this discrepancy across specifications. In particular, we show that (classical) measurement error is an unlikely culprit. We assume that quantities are defined as expenditures over prices and that there is classical measurement error in both log expenditure and prices. We show that under these conditions estimating eq. (21) by OLS yields an upwards bias in the estimate of ε. In addition, if we assume that the measurement error in expenditure is smaller than that in prices (as it seems realistic because expenditure ratios are calculated from national account statistics) then, the OLS estimate ε of eq. (21)in the quantity regression (20) is biased downward. Therefore, under plausible assumptions, correcting for measurement errors would not resolve the puzzle.

Next, we investigate possible identification problems—see Appendix D.3. We assume that the comovements can be driven driven by both productivity and demand (preference) shocks. We construct a standard model with CES preferences over agricultural and nonagricultural goods and Cobb Douglas production functions (where we ignore, for simplicity, the traditional sector) subject to demand and supply shocks. Then, we formally establish that the presence of demand shocks affecting the relative demand for agricultural goods biases the estimate of ε from eq. (21) towards zero while it biases the estimate of ε from eq. (20) away from zero. We conclude that demand shocks can contribute to explain the discrepancy discussed above, although more research would be necessary to quantify the biases and allow a regression-based approach to reach a credible conclusion about whether ε is larger or smaller than unity. With this in mind, we now turn our attention to studies that consider exogenous and well identified shocks to TFP in agriculture.

6.3 Causal Effects of TFP Shocks in Agriculture

Empirical analyses of demand systems using data on prices and quantities, as in eq. (20)-(21) and (6.1), suffer from a potential omitted variable bias, since their comovements may be affected by factors other than productivity shocks that influence relative prices and output simultaneously. To address this identification problem, we turn to event studies that focus on plausibly exogenous productivity shocks.

Exogenous shocks to agriculture are especially informative about ε. In the model with $\varepsilon < 1$, a positive TFP shock to agriculture will lead to structural change from agriculture to industry. In contrast, when $\varepsilon > 1$, such shocks slow down the process of structural change. This was evident in the discussion of the impulse response functions of TFP shocks in our model, see Section 5. With this in mind we review some empirical papers studying the effect of exogenous productivity shocks in agriculture.

In a recent paper on the British Industrial Revolution, Kelly et al. (2020) find that areas that industrialized successfully were characterized at the onset of industrialization by lower labor productivity in agriculture and poorer soil than regions that did not industrialize. This led to some reversal of fortunes, where regions that used to be depressed became the most prosperous ones as the Industrial Revolution unfolded. Another prominent event that offers sharp identification is the Green Revolution. The introduction of high-yield crop varieties caused a sharp increase in agricultural TFP in developing countries, and the impact was particularly pronounced in the 1970s. Foster and Rosenzweig (2004) and Moscona (2018) exploit the fact that, due to exogenous differences in ecological and geographic characteristics, different regions were able to adopt and reap the benefits of high-yield crops to different degrees. This exogenous variation provides a valid instrument for agricultural TFP growth. Foster and Rosenzweig (2004) use time series for 240 Indian villages and document that during the period 1970-2000 the growth of rural industry was systematically slower in areas where crop yields grew faster. They conclude that the evidence is consistent with a model in which industrialization and technological development in agriculture are substitutes.

While these pattern is suggestive of a high elasticity of substitution between agriculture and nonagriculture, regional evidence could arguably be affected by trade and specialization within a country. However, Moscona (2018) finds the pattern to hold true not only across regions in India, but also across countries. He shows that countries that because of exogenous ecological characteristics were better placed to benefit from new crop-specific technologies witnessed a larger increase in agricultural TFP together with an expansion of employment in agriculture. However, this also caused a slower growth of manufacturing and non-farm labor as well a slower process of urbanization. His conclusions again suggest substitutability rather than complementarity between agriculture and industrial technology. Not all studies reach this conclusion. Based on a different identification strategy exploiting variation in the timing of adoption of high-yield variety crops, Gollin et al. (2021) find that the early adoption of high-yield variety crops tends to increase GDP per capita in a panel of 81 countries. Moreover, the adoption of these high yield varieties is associated with negative employment effects in agriculture. Turning to case studies, Bustos et al. (2016) find that the introduction of high-yield maize in Brazil resulted in a reduction in industrial employment.[28] Finally, Jayachandran (2006) estimates the response of agricultural wages and labor supply to positive TFP shocks in agriculture. He uses rainfall as an instrument for TFP shocks and shows that higher crop yield is strongly associated with higher wages and higher labor supply in agriculture across districts in India over the 1956-1987 period.

In conclusion, the evidence from causal studies estimating the effects of technological progress in agriculture on structural change is not uniform. However, a number of studies reach conclusions consistent with the view that the output of agriculture and nonagriculture are substitutes.[29]

[28]However, they also find that the introduction of genetically modified soy, which does not require tilling of land and hence was labor saving, tended to reduce employment in agriculture.

[29]We also note that the elasticity of substitution between agriculture and nonagriculture is potentially different at the household preferences (consumption) level versus at the aggregate country level (production). There are many reasons for that. For example, if goods could be stored or traded across countries, the relative production would differ from the relative consumption and the effective production elasticity would be larger than the consumption elasticity.

7 Conclusion

Business cycle fluctuations in countries undergoing structural transformation differ systematically from business cycles in mature economies. We document that countries with large declining agricultural sectors – including China – have aggregate employment fluctuations that are smooth and acyclical, while these countries experience volatile and procyclical reallocation of labor between agriculture and nonagriculture. One might expect that structural change accelerates in recessions, as the declining sectors shrink in those times. Instead, the decline of agriculture accelerates during booms.

We provide a unified theoretical framework for studying business cycle during structural change consistent with those facts. The central aspect of the theory is the modernization of agriculture that occurs during the structural change: agriculture is becoming increasingly capital intensive and less labor intensive as a large traditional sector is crowded out. With a large traditional sector the expansion of manufacturing draws workers from traditional agriculture, triggering modernization in agriculture and sustaining aggregate productivity. This process is driven by capital accumulation and differential productivity growth between agriculture and nonagriculture. At business cycle frequencies, positive TFP shocks in nonagriculture accelerate this process while TFP shocks in agriculture slowdown structural change in line with the evidence from the Green Revolution discussed in the paper.

We estimate the model using data for China and show that its quantitative predictions are consistent both with China's structural transformation and China's business cycles. Moreover, the model is consistent with the changing business cycle properties as a country evolves from a poor economy with a large agricultural sector to a modern industrialized economy with negligible agricultural employment.

Our business cycle analysis only focuses on productivity shocks. Future research will extend it to a broader set of disturbances including demand shocks. Another limitation that we plan to address in future work is the closed-economy environment. In spite of these and other shortcomings, we believe the theory casts new light on the relationship between business cycles and economic development.

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Online Appendix for the paper
"Business Cycle during Structural Change:
Arthur Lewis' Theory from a Neoclassical Perspective"

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A Appendix: Data Description in Section 2

In this appendix, we describe how we constructed the data to document the evidence on the structural change and business cycle facts across countries. In all cases, we exclude countries. We exclude the following city states: Bahrein, Hong Kong, Macao, Qatar, Singapore. Agricultural sector's shares of employment and GDP are negligible in these countries.

When computing the K/Y ratio, we exclude countries that have K/Y ratio higher than 100. When calculating the productivity gap, we exclude countries that have productivity gap higher than 20.

The data for aggregate GDP, capital stocks, investment, and consumption are from the World Development Indicators. The data for value added in agriculture and capital stocks in agriculture is from the FAO. The data for sectoral employment comes from the International Labor Organization (ILO). The real consumption is constructed by dividing households consumption by CPI index. The real output is constructed by dividing the GDP in current price by GDP deflator.

The data set is constructed as follows. First, we use data from the labor force surveys, households surveys, official statistics, and population censuses. We exclude data from firm surveys. Second, we exclude data that are not representative of the whole country. In particular, we exclude data from some countries which report data that only cover the urban population. Third, if multiple sources exist for the same country and these data cover overlapping time periods, we merge (by chaining) the different sources provided that data in the overlapping time periods are small. However, if the differences are large across different sources, we only retain the most recent data source, provided that the sample period is at least 15 years. If the most recent data cover less than 15 years, we retain the less recent data series (provided the sample covers at least 15 years).

Fourth, if multiple sources exist for the same country and these data do not cover overlapping time periods, then we do not merge the data. Instead we retain only the most recent data, provided that the sample period is at least 15 years. Again, if the most recent data covers less than 15 years, we use less recent data, provided that the data cover at least 15 years. The country is dropped if there are no data series longer than 15 years.

The sample of countries plotted in Figure 3 ranges from 63 to 66 across panels a–c. We further exclude countries with relative consumption volatility higher than 3 in panel d. The detailed description of which countries appear in each panel and for which sample period is provided in Appendix A.3.

B Appendix: Formal Analysis in Section 3

This appendix contains proofs and analytical derivations.

B.1 Proof of Proposition 1

We take FOCs of the program 8 using standard methods. After rearranging terms, the equalization of the marginal product of capital in modern agriculture and nonagriculture yields:

$$\frac{1-\kappa}{\kappa}=\frac{1-\beta}{1-\alpha}\frac{\gamma}{1-\gamma}\left(\frac{y^G}{y^M}\right)^{\frac{\varepsilon-1}{\varepsilon}}\upsilon. \tag{22}$$

The equalization of the marginal product of labor in modern agriculture and nonagriculture, and in traditional agriculture and nonagriculture yield, respectively,

$$\nu^{AM} = \frac{1}{1-\tau}\frac{1-\alpha}{1-\beta}\frac{\beta}{\alpha}\frac{1-\kappa}{\kappa}\nu^M, \tag{23}$$

$$\nu^S = \frac{1}{\beta}\frac{1-\upsilon}{\upsilon}\nu^{AM}. \tag{24}$$

Combining (22), (23), and (24) with the resource constraint on labor, and solving for ν^M, yields

$$\nu^M=\left(1+\frac{1}{1-\tau}\frac{1-\kappa}{\kappa}\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\left(1+\frac{1}{\beta}\frac{1-\upsilon}{\upsilon}\right)\right)^{-1}. \tag{25}$$

These four equations, together with the definitions of y^G and y^M provided in the text, implicitly define the unique set of equilibrium policy functions $\kappa=\upsilon(\chi,\boldsymbol{Z})$, $\nu^M(\kappa(\chi,\boldsymbol{Z}),\upsilon(\chi,\boldsymbol{Z}))$, $\nu^{AM}(\kappa(\chi,\boldsymbol{Z}),\upsilon(\chi,\boldsymbol{Z}))$, and $\nu^S(\kappa(\chi,\boldsymbol{Z}),\upsilon(\chi,\boldsymbol{Z}))$.

B.2 Derivation of eq. 10

Proof. The FOC (22) that equates the marginal product of capital can be rewritten as

$$\frac{1-\kappa}{\kappa}=\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\left(\frac{Y^G}{Y^M}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{Y^{AM}}{Y^G}\right)^{\frac{\omega-1}{\omega}}.$$

Taking logarithms and letting $\omega \to \varepsilon$ yields

$$\ln(1-\kappa) - \ln\kappa = \ln\left(\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right) + \frac{\varepsilon-1}{\varepsilon}\ln\left(\frac{y^{AM}}{y^{M}}\right)$$

Substituting in the expressions for y^{AM} and y^{G}, and differentiating with respect to $\ln\chi$ yields

$$\left(\frac{\varepsilon-1}{\varepsilon}(\beta-\alpha) - \frac{1}{\varepsilon}\frac{1}{1-\kappa}\right)\frac{\partial\ln\kappa}{\partial\ln\chi} = -\frac{\varepsilon-1}{\varepsilon}(\beta-\alpha)\times\left(1-\frac{\partial\ln\nu^{M}}{\partial\ln\chi}\right) \tag{26}$$

Next, consider (25). Differentiating with respect to $\ln\chi$ yields

$$\begin{aligned}\frac{\partial\ln\left(\nu^{M}\right)}{\partial\ln\chi} &= \frac{\nu^{AM} + (1+(1-\kappa)(\varepsilon-1)(1-\beta))\nu^{S}}{(1+\nu^{S}(1-\beta)(\omega-1))}\frac{1}{1-\kappa}\frac{\partial\ln\kappa}{\partial\ln\chi} \\ &\quad + \frac{\nu^{S}(\omega-1)(1-\beta)}{1+\nu^{S}(1-\beta)(\varepsilon-1)}\end{aligned} \tag{27}$$

Plugging in (27) into (26) and letting $\omega \to \varepsilon$ yields

$$\lim_{\omega\to\varepsilon}\frac{\partial\ln\kappa}{\partial\ln\chi} = \frac{(\varepsilon-1)(\beta-\alpha)(1-\kappa)}{1+(\varepsilon-1)\left((\beta-\alpha)\left(\kappa-\nu^{M}\right)+\nu^{S}(1-\beta)\right)} > 0. \tag{28}$$

■

B.3 Proof of Lemma 2

Proof. Define $z_A \equiv \left(Z^{AM}\right)^{\beta}/\left(Z^{M}\right)^{\alpha}$ and $z_S \equiv Z^{S}/\left(Z^{M}\right)^{\alpha}$. The first order conditions of the static planning problem can be expressed as

$$\frac{\nu^{S}}{\nu^{AM}} = \left(\frac{1}{\beta}\right)^{\varepsilon}\left(\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\Xi\right)^{-(1-\beta)(\varepsilon-1)}\left(\frac{z_S}{z_A}\right)^{\varepsilon-1}, \tag{29}$$

$$\frac{\nu^{AM}}{\nu^{M}} = \frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{1-\kappa}{\kappa}, \tag{30}$$

$$\left(\frac{\nu^{AM}}{\nu^{M}} + \frac{\nu^{S}}{\nu^{M}}\right)\kappa\chi = \left(\frac{1}{\nu^{M}}-1\right)\kappa\chi = \Xi - \kappa\chi, \tag{31}$$

$$\left(\frac{1-\kappa}{\kappa}\right)(\Xi)^{(\beta-\alpha)(\varepsilon-1)} = \left(\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^{\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{\beta(\varepsilon-1)}(z_A)^{(\varepsilon-1)}, \tag{32}$$

where $\Xi \equiv \frac{\kappa\chi}{\nu^{M}} = \left(\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\right)^{-1}\frac{(1-\kappa)\chi}{\nu^{AM}}$. Consider, first, the case where $\beta = \alpha$. Then, eq. (32) establishes that κ is decreasing in z_A. Next, suppose $\beta \neq \alpha$. Then we can rewrite eq. (32) as

$$\Xi = \left(\left(\frac{1-\kappa}{\kappa}\right)^{-1}\left(\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^{\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{\beta(\varepsilon-1)}(z_A)^{(\varepsilon-1)}\right)^{\frac{1}{(\beta-\alpha)(\varepsilon-1)}}. \tag{33}$$

Substituting (29)-(30) into (31) to get rid of the ratios $\frac{\nu^S}{\nu^{AM}}$ and $\frac{\nu^{AM}}{\nu^M}$ yields

$$\frac{\Xi}{\kappa\chi}-1=\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{1-\kappa}{\kappa}+\left(\frac{1}{\alpha}\frac{\gamma}{1-\gamma}\right)^{\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{(\varepsilon-1)(\beta-\alpha)}(\Xi)^{-(\varepsilon-1)(1-\alpha)}(z_S)^{\varepsilon-1}$$

Substituting in Ξ from eq. (33), and simplifying terms, yields

$$\begin{aligned}
&\frac{1}{\chi}\left(\frac{\kappa}{1-\kappa}\right)^{\frac{1}{(\beta-\alpha)(\varepsilon-1)}}\left(\left(\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^{\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{\beta(\varepsilon-1)}\right)^{\frac{1}{(\beta-\alpha)(\varepsilon-1)}}(z_A)^{\frac{1}{\beta-\alpha}}\\
=\;&\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}-\frac{\beta-\alpha}{\alpha(1-\beta)}\kappa+\left(\frac{1}{\alpha}\frac{\gamma}{1-\gamma}\right)^{\varepsilon}\left(\frac{\gamma}{1-\gamma}\frac{\beta}{\alpha}\right)^{-\frac{(1-\alpha)}{(\beta-\alpha)}\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{-\frac{(1-\alpha)}{(\beta-\alpha)}(\beta(\varepsilon-1)-\varepsilon)+(\varepsilon-1)(\beta-\alpha)}\\
&\times(1-\kappa)^{\frac{(1-\alpha)}{(\beta-\alpha)}}(\kappa)^{-\frac{1-\beta}{\beta-\alpha}}(z_A)^{-(\varepsilon-1)\frac{(1-\alpha)}{(\beta-\alpha)}}(z_S)^{\varepsilon-1}
\end{aligned}$$

Consider two separate cases. First, if $\beta>\alpha$, then, the LHS is increasing in z_A and in κ, while the RHS is decreasing in z_A and in κ. Standard differentiation implies then that κ is decreasing in z_A. Second, if $\beta<\alpha$, then, the LHS is decreasing in z_A and in κ, while the RHS is increasing in z_A and in κ. It follows again that that κ is decreasing in z_A. In both cases (as well as in the case where $\beta=\alpha$), eq. (30) implies then that ν^M/ν^{AM} also decreases in z_A. ■

B.4 Analysis of the Dynamic Equilibrium

The dynamic equilibrium can be characterized by solving the Hamiltonian (12). The following proposition summarizes the results equilibrium:

Proposition 3 *The dynamic competitive equilibrium is characterized by the following system of ordinary differential equations:*

$$\begin{aligned}
\frac{\dot{c}_t}{c_t} &= \left(\eta\left(\kappa\left(\chi_t,\boldsymbol{Z}_t\right),\upsilon\left(\chi_t,\boldsymbol{Z}_t\right)\right)\right)^{\frac{1}{\varepsilon}}(1-\gamma)(1-\alpha)\times \qquad (34)\\
&\quad\left(\frac{\kappa\left(\chi_t,\boldsymbol{Z}_t\right)\chi_t}{Z_t^M\nu^M\left(\kappa\left(\chi_t,\boldsymbol{Z}_t\right),\upsilon\left(\chi_t,\boldsymbol{Z}_t\right)\right)}\right)^{-\alpha}-\delta-\rho\\
\dot{\chi}_t &= \eta\left(\kappa\left(\chi_t,\boldsymbol{Z}_t\right),\upsilon\left(\chi_t,\boldsymbol{Z}_t\right)\right)\times\left(\chi_t\kappa\left(\chi_t,\boldsymbol{Z}_t\right)\right)^{1-\alpha}\times \qquad (35)\\
&\quad\left(Z_t^M\nu^M\left(\kappa\left(\chi_t,\boldsymbol{Z}_t\right),\upsilon\left(\chi_t,\boldsymbol{Z}_t\right)\right)\right)^{\alpha}-(\delta+n)\chi_t-c_t\\
\frac{Z_t^M}{Z_t^M} &= g^M,\ \frac{Z_t^{AM}}{Z_t^{AM}}=\frac{g^G}{\beta},\ \frac{Z_t^S}{Z_t^S}=g^G
\end{aligned}$$

where $\eta(\kappa,\upsilon)$ is given by (14), $\nu^M(\kappa,\upsilon)$ satisfies (25), and $\kappa(\chi_t,\boldsymbol{Z}_t)$ and $\upsilon(\chi_t,\boldsymbol{Z}_t)$ are the static equilibrium policy functions. The solution is subject to a vector of initial conditions $(\chi_0,\boldsymbol{Z}_0)=(\bar{\chi}_0,\bar{\boldsymbol{Z}}_0)$ and a transversality condition.

Eq. (34) is a standard Euler Equation for consumption. For constant TFPs, the growth rate of consumption is decreasing in χ because the aggregate production function exhibit decreasing returns to capital. Eq. (35) is a resource constraint.

It is useful to rewrite the equilibrium conditions in Proposition 3 in terms of an autonomous system of differential equations. To this aim, we differentiate with respect to time the set of static equilibrium (22), (23), (24), and (25). After rearranging terms, we obtain:

$$\frac{\dot{\kappa}_t}{\kappa_t} = (1-\kappa_t)\frac{\left(\begin{array}{c}\left(\alpha g^M - g^G + (\beta-\alpha)\frac{\dot{\chi}_t}{\chi_t}\right) + \\ \left(\frac{1}{\omega-1} - \frac{(\beta-\alpha)\left(1-\nu^M(\kappa_t,\upsilon_t)\right)}{1-\upsilon_t(1-\beta)}\right)\frac{\dot{\upsilon}_t}{\upsilon_t}\end{array}\right)}{\frac{1}{\varepsilon-1} + (\beta-\alpha)\left(\kappa_t - \nu^M(\kappa_t,\upsilon_t)\right)}, \tag{36}$$

$$\frac{\dot{\upsilon}_t}{\upsilon_t} = \frac{(1-\upsilon_t)(1-\beta)\left(\frac{\dot{\chi}_t}{\chi_t} - \frac{\dot{\kappa}_t}{\kappa_t}\frac{\kappa_t-\nu^M(\kappa_t,\upsilon_t)}{1-\kappa_t}\right)}{\frac{1}{\omega-1} + \frac{(1-\upsilon_t)(1-\beta)(1-\nu^M(\kappa_t,\upsilon_t))}{1-\upsilon_t(1-\beta)}}. \tag{37}$$

This dynamic system is defined up to a pair of initial conditions: $\kappa_0 = \kappa(\chi_0, \boldsymbol{Z}_0)$ and $\upsilon_0 = \upsilon(\chi_0, \boldsymbol{Z}_0)$ consistent with the static equilibrium conditions at time zero.

Corollary 1 *The dynamic competitive equilibrium is fully characterized by the solution to the autonomous system of Ordinary Differential Equations (34)-(35)-(36)-(37) and the exogenous law of motion $\dot{Z}_t^M/Z_t^M = g^M$, after setting $\kappa(\chi_t, \boldsymbol{Z}_t) = \kappa_t$ and $\upsilon(\chi_t, \boldsymbol{Z}_t) = \upsilon_t$ for $t > 0$, with initial conditions $\kappa(\chi_0, \boldsymbol{Z}_0) = \kappa\left(\bar{\chi}_0, \bar{\boldsymbol{Z}}_0\right) \equiv \kappa_0$ and $(\chi_0, \boldsymbol{Z}_0) = \upsilon\left(\bar{\chi}_0, \bar{\boldsymbol{Z}}_0\right) \equiv \upsilon_0$.*

Eq. (36)-(37) allow us to eliminate Z_t^{AM} and Z_t^S from the dynamic system, while only retaining their initial levels and their growth rates. In other words, $\kappa_0 = \kappa\left(\bar{\chi}_0, \bar{\boldsymbol{Z}}_0\right)$ and $\upsilon_0 = \upsilon\left(\bar{\chi}_0, \bar{\boldsymbol{Z}}_0\right)$ are sufficient statistics. If κ_0 and υ_0 are set at the static equilibrium level at time zero, eq. (36)-(37) guarantee that κ_t and υ_t will also be consistent with the static equilibrium in all future periods.

B.5 Proof of Proposition 2

Proof. We start by evaluating eq. (34)-(35) under the ABGP conditions. Note that (14) implies that $\eta(1,1) = (1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}$. Thus,

$$\begin{aligned}\frac{\dot{c}_t}{c_t} &= g_M = (1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}(1-\alpha)\left(\frac{\chi}{Z^M}\right)^{-\alpha} - \delta - \rho, \\ \frac{\dot{\chi}_t}{\chi_t} &= g_M = (1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}} \times \left(\frac{\chi}{Z^M}\right)^{-\alpha} - (\delta+n) - \frac{c}{\chi}.\end{aligned}$$

Solving for c/χ and χ/Z^M yields the expressions in (15) and (16). Therefore, (34)-(35) hold true under the ABGP conditions. It is straightforward to see that under the ABGP conditions (in particular, when $\kappa = \upsilon = 1$) (36)-(37) yields $\frac{\dot{\kappa}}{\kappa} = \frac{\dot{\upsilon}}{\upsilon} = 0$. Likewise, (22) holds true when $\kappa = \upsilon = 1$.

Next, consider the asymptotic growth rates of the sectoral capital. Taking logarithms and differentiating with respect to time the definitions of k^M and k^{AM} yields $\dot{k}^M/k^M = \dot{\kappa}/\kappa + \dot{\chi}/\chi = g^M$ and $\dot{k}^{AM}/k^{AM} = -(1-\kappa)^{-1} \times \dot{\kappa}/\kappa + \dot{\chi}/\chi = g^M - (\varepsilon-1)\left(\beta g^M - g^G\right)$.

Next, consider the asymptotic growth rates of the sectoral employments of labor. First, observe that eq. (25) yields $\nu^M = 1$ at the ABGP conditions $\kappa = \upsilon = 1$. Second, note that $\nu^M = 1$ implies that $\dot{N}^M/N^M = \dot{N}/N = n$. In order to establish the growth rate of N^{AM}, observe that taking logarithms on both side of eq. (23), differentiating with respect to time, and using the ABGP conditions and eq. (36) yields

$$\frac{\dot{N}^{AM}}{N^{AM}} = -\frac{1}{1-\kappa}\frac{\dot{\kappa}}{\kappa} + \frac{\dot{N}^{AM}}{N^{AM}} = n - (\varepsilon - 1)\left(\beta g^M - g^G\right).$$

Finally, to establish the growth rate of N^S, observe that taking logarithms on both side of eq. (24), differentiating with respect to time, and using the ABGP conditions and eq. (37) yields

$$\frac{\dot{N}^S}{N^S} = -\frac{1}{1-\upsilon}\frac{\dot{\upsilon}}{\upsilon} + \frac{\dot{N}^{AM}}{N^{AM}} = \frac{\dot{N}^{AM}}{N^{AM}} - (\omega - 1)(1-\beta)\, g^M.$$

To establish convergence, we linearize the dynamic system in a neighborhood of the ABGP. The system has three predetermined variables (χ, κ, υ) and one jump variable (c). Therefore, we must prove that the linear system has three negative eigenvalues and one positive eigenvalue. The rest of the proof is devoted to establish that this is the case.

Let $\tilde{\chi} = \frac{\chi}{Z^M}$ and $\tilde{c} = \frac{c}{Z^M}$, implying that $\frac{d\tilde{\chi}/dt}{\tilde{\chi}} = \frac{\dot{\chi}}{\chi} - g^M$. Then, we can write the dynamic system (34)-(35)-(36)-(37). We can rewrite the system as

$$\begin{aligned}
\frac{d\tilde{c}/dt}{\tilde{c}} &= \eta(\kappa_t, \upsilon_t)^{\frac{1}{\varepsilon}}(1-\gamma)(1-\alpha)\left(\frac{\kappa_t \tilde{\chi}_t}{\nu^M(\kappa_t, \upsilon_t)}\right)^{-\alpha} - \delta - \rho - g^M \\
\frac{d\tilde{\chi}/dt}{\tilde{\chi}} &= \eta(\kappa_t, \upsilon_t)^{1-\alpha}\left(\frac{\kappa_t \tilde{\chi}_t}{\nu^M(\kappa_t, \upsilon_t)}\right)^{-\alpha}\kappa_t - \delta - \tilde{c}_t/\tilde{\chi}_t - n - g^M \\
\frac{\dot{\kappa}}{\kappa} &= (1-\kappa)\frac{\left(\begin{array}{c}\left(\alpha g^M - g^G + (\beta-\alpha)\left(\frac{d\hat{\chi}/dt}{\hat{\chi}} + g^M\right)\right) \\ + \left(\frac{1}{\omega-1} - \frac{(\beta-\alpha)\left(1-\nu^M\right)}{1-\upsilon(1-\beta)}\right)\frac{\dot{\upsilon}}{\upsilon}\end{array}\right)}{\frac{1}{\varepsilon-1} + (\beta-\alpha)\left(\kappa - \nu^M\right)} \\
\frac{\dot{\upsilon}}{\upsilon} &= (1-\upsilon)\frac{(1-\beta)\left(\frac{\dot{\chi}_t}{\chi_t} - \frac{\dot{\kappa}_t}{\kappa_t}\frac{\kappa_t - \nu^M(\kappa_t,\upsilon_t)}{1-\kappa_t}\right)}{\frac{1}{\omega-1} + \frac{(1-\upsilon)(1-\beta)(1-\nu^M)}{1-\upsilon(1-\beta)}}
\end{aligned}$$

where we use eq. (25) implying that

$$\frac{1-\nu^M}{\nu^M} = \frac{1-\kappa}{\kappa}\frac{1-\alpha}{1-\beta}\left(\frac{\beta}{\alpha} + \frac{1}{\alpha}\frac{1-\upsilon}{\upsilon}\right).$$

The transversality condition (TVC) becomes

$$\lim_{t\to\infty} \xi e^{-(\rho-n)t} K = 0$$

Substitute the condition that

$$\lim_{t\to\infty} \frac{\dot{K}}{K} = n + g^M$$

The TVC becomes

$$\lim_{t\to\infty}\frac{\dot{\xi}}{\xi}+g^M+n<\rho-n$$

Because

$$-\frac{\dot{c}}{c}=(1-\theta)(1-\sigma)\frac{\dot{h}}{h}\frac{h}{1-h}+\frac{\dot{\xi}}{\xi}+n$$

Then we have

$$\lim_{t\to\infty}\frac{\dot{c}}{c}=-\lim_{t\to\infty}\frac{\dot{\xi}}{\xi}-n$$

where we use $\lim_{t\to\infty}\frac{\dot{h}}{h}=0$. Plug $\lim_{t\to\infty}\frac{\dot{\xi}}{\xi}=-g^M-n$ into the TVC to get

$$-g^M-n+g^M+n<\rho-n$$

In the end, the TVC can be rewritten as

$$\rho-n>0$$

Letting $\Psi=(\tilde{c},\tilde{\chi},\kappa,\upsilon)'$, we can write the system of differential equations as

$$A\dot{\Psi}=f(\Psi)$$

where

$$A=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&a&1&b\\0&c&d&1\end{bmatrix}$$

and, after

$$\begin{aligned}
a &= -\frac{(1-\kappa)(\beta-\alpha)}{\frac{1}{\varepsilon-1}+(\beta-\alpha)(\kappa-\nu^M)}\\
b &= -(1-\kappa)\frac{\left(\frac{1}{\omega-1}-\frac{(\beta-\alpha)\left(1-\nu^M\right)}{1-\upsilon(1-\beta)}\right)}{\frac{1}{\varepsilon-1}+(\beta-\alpha)(\kappa-\nu^M)}\\
c &= -\frac{(1-\upsilon)(\omega-1)(1-\beta)}{\left(1+\frac{1-\upsilon}{\upsilon}\frac{1-\kappa}{\kappa}\frac{1-\alpha}{\alpha}(\omega-1)\nu^M\right)}\\
d &= -\frac{(1-\upsilon)(\omega-1)(1-\beta)\left(1-\frac{1-\alpha}{(1-\beta)\alpha}\frac{\nu^M}{\kappa}\left(\beta+\frac{1-\upsilon}{\upsilon}\right)\right)}{1+(\omega-1)\frac{1-\upsilon}{\upsilon}\frac{1-\kappa}{\kappa}\frac{1-\alpha}{\alpha}\nu^M}
\end{aligned}$$

$$f=\begin{bmatrix}\eta(\kappa_t,\upsilon_t)^{\frac{1}{\varepsilon}}(1-\gamma)(1-\alpha)\left(\frac{\kappa_t\tilde{\chi}_t}{\nu^M(\kappa_t,\upsilon_t)}\right)^{-\alpha}-\delta-\rho-g^M\\ \eta(\kappa_t,\upsilon_t)\left(\frac{\kappa_t\tilde{\chi}_t}{\nu^M(\kappa_t,\upsilon_t)}\right)^{-\alpha}\kappa_t-\delta-\tilde{c}_t/\tilde{\chi}_t-n-g^M\\ (1-\kappa)\frac{\beta g^M-g^G}{\frac{1}{\varepsilon-1}+(\beta-\alpha)(\kappa-\nu^M)}\\ \frac{(1-\upsilon)(\omega-1)(1-\beta)g^M}{1+\frac{1-\upsilon}{\upsilon}\frac{1-\kappa}{\kappa}\frac{1-\alpha}{\alpha}\nu^M(\omega-1)}\end{bmatrix}$$

where we use the

$$\begin{aligned}
\frac{1-\nu^M}{\nu^M} &= \frac{1-\kappa}{\kappa}\frac{1-\alpha}{(1-\beta)}\left(\frac{\beta}{\alpha}+\frac{1}{\alpha}\frac{1-\upsilon}{\upsilon}\right) \\
\frac{1-\nu^M}{1-\upsilon(1-\beta)} &= \nu^M\frac{1-\kappa}{\kappa}\frac{1-\alpha}{(1-\beta)}\frac{1}{\alpha\upsilon} \\
\frac{\kappa-\nu^M}{1-\kappa} &= -1+\frac{\nu^M}{\kappa}\frac{1-\alpha}{(1-\beta)\alpha}\left(\beta+\frac{1-\upsilon}{\upsilon}\right)
\end{aligned}$$

The inverse of matrix A is given by

$$A^{-1}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{1}{bd-1}(a-bc) & -\frac{1}{bd-1} & \frac{b}{bd-1} \\ 0 & \frac{1}{bd-1}(c-ad) & \frac{d}{bd-1} & -\frac{1}{bd-1} \end{bmatrix}$$

Along the approximate balanced growth path,

$$a^*=0, b^*=0, c^*=0, d^*=0$$

Now we compute

$$J(\Psi)=\begin{pmatrix} J_1(\Psi) \\ J_2(\Psi) \\ J_3(\Psi) \\ J_4(\Psi) \end{pmatrix}=A^{-1}(\Psi)\, f(\Psi)$$

where

$$\begin{aligned}
J_1(\Psi) &= \eta^{\frac{1}{\varepsilon}}(1-\gamma)(1-\alpha)\left(\frac{\kappa\tilde{\chi}}{\nu^M(\kappa,\upsilon)}\right)^{-\alpha}-\delta-\rho-g^M \\
J_2(\Psi) &= \eta\left(\frac{\kappa\tilde{\chi}}{\nu^M(\kappa,\upsilon)}\right)^{-\alpha}\kappa-\delta-\tilde{c}(\tilde{\chi})^{-1}-n-g^M \\
J_3(\Psi) &= \frac{1}{bd-1}(a-bc)\left(\left(\frac{\kappa\tilde{\chi}}{\nu^M(\kappa,\upsilon)}\right)^{-\alpha}\kappa-\delta-\tilde{c}/\tilde{\chi}-n-g^M\right) \\
&\quad -\frac{1}{bd-1}(1-\kappa)\frac{\beta g^M-g^G}{\frac{1}{\varepsilon-1}+(\beta-\alpha)(\kappa-\nu^M)} \\
&\quad +\frac{b}{bd-1}\frac{(1-\upsilon)(\omega-1)(1-\beta)g^M}{1+\frac{1-\upsilon}{\upsilon}\frac{1-\kappa}{\kappa}\frac{1-\alpha}{\alpha}(\omega-1)\nu^M} \\
J_4(\Psi) &= \frac{1}{bd-1}(c-ad)\left(\left(\frac{\kappa\tilde{\chi}}{\nu^M(\kappa,\upsilon)}\right)^{-\alpha}\kappa-\delta-\tilde{c}/\tilde{\chi}-n-g^M\right) \\
&\quad +\frac{d}{bd-1}(1-\kappa)\frac{\beta g^M-g^G}{\frac{1}{\varepsilon-1}+(\beta-\alpha)(\kappa-\nu^M)} \\
&\quad -\frac{1}{bd-1}\frac{(1-\upsilon)(\omega-1)(1-\beta)g^M}{1+\frac{1-\upsilon}{\upsilon}\frac{1-\kappa}{\kappa}\frac{1-\alpha}{\alpha}(\omega-1)\nu^M}
\end{aligned}$$

From (25) it follows that

$$\begin{aligned}
\dot{\nu}^M &= \nu^M \left(\frac{\dot{\kappa}}{\kappa} \frac{1-\nu^M}{1-\kappa} + \frac{\dot{\upsilon}}{\upsilon} \frac{1-\nu^M}{1-\upsilon(1-\beta)} \right) \\
&= \nu^M \left(\frac{\dot{\kappa}}{\kappa} \frac{\nu^M}{\kappa} \frac{1-\alpha}{(1-\beta)} \left(\frac{\beta}{\alpha} + \frac{1}{\alpha} \frac{1-\upsilon}{\upsilon} \right) + \frac{\dot{\upsilon}}{\upsilon} \frac{1-\nu^M}{1-\upsilon(1-\beta)} \right)
\end{aligned}$$

Hence

$$\begin{aligned}
\frac{\partial \nu^M}{\partial \kappa} &= \left(\frac{\nu^M}{\kappa} \right)^2 \frac{1-\alpha}{(1-\beta)} \left(\frac{\beta}{\alpha} + \frac{1}{\alpha} \frac{1-\upsilon}{\upsilon} \right) \\
\frac{\partial \nu^M}{\partial \upsilon} &= \frac{\nu^M}{\upsilon} \frac{1-\nu^M}{1-\upsilon(1-\beta)}.
\end{aligned}$$

Computing the Jacobian evaluated at the balanced growth path $(\tilde{c}, \tilde{\chi}, \kappa, \upsilon)'$, we obtain

$$J = \begin{bmatrix} 0 & J_{12}^* & J_{13}^* & 0 \\ J_{21}^* & J_{22}^* & J_{23}^* & 0 \\ 0 & 0 & J_{33}^* & 0 \\ 0 & 0 & J_{43}^* & J_{44}^* \end{bmatrix}$$

with determinant given by $-J_{12}^* J_{21}^* J_{33}^* J_{44}^*$ and four eigenvalues given by

$$\begin{bmatrix} \frac{1}{2} J_{22}^* + \frac{1}{2} \sqrt{(J_{22}^*)^2 + 4 J_{12}^* J_{21}^*} \\ \frac{1}{2} J_{22}^* - \frac{1}{2} \sqrt{(J_{22}^*)^2 + 4 J_{12}^* J_{21}^*} \\ J_{33}^* \\ J_{44}^* \end{bmatrix},$$

where

$$\begin{aligned}
J_{12}^* &= -\alpha (1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}} (1-\alpha) (\tilde{\chi}^*)^{\alpha-1} < 0 \\
J_{21}^* &= -(\tilde{\chi}^*)^{-1} < 0 \\
J_{22}^* &= (\tilde{\chi}^*)^{-1} (\rho - n) > 0 \\
J_{33}^* &= -(\varepsilon - 1) \left(\beta g^M - g^G \right) < 0 \\
J_{44}^* &= -(\omega - 1)(1-\beta) g^M < 0
\end{aligned}$$

Thus, three eigenvalues are negative, while one is positive ($\frac{1}{2} J_{22}^* + \frac{1}{2} \sqrt{(J_{22}^*)^2 + 4 J_{12}^* J_{21}^*} > 0$), establishing the result. ■

B.6 The Lewis Model

In this section we provide the details of the analysis in Section 3.3. We abstract from technical progress and set $Z^M = Z^{AM} = Z^S = 1$. Moreover, we set $\tau = 0$. Endogenous capital accumulation is then the only source of transition. We continue to assume that $\varepsilon > 1$ and $\beta > \alpha$.

Stage 1 (Early Lewis). Consider an economy in which capital is very scarce. When $\chi < \underline{\chi}$, then, $\nu^{AM} = 0$, $\nu^{M} > 0$, $\nu^{S} > 0$, and $\kappa = 1$.[30] Intuitively, because capital is scarce, it is optimal to use it only in nonagriculture, where it is an essential factor, to take advantage of the high relative price of the nonagricultural good. Over time, employment grows in nonagriculture and falls in agriculture.[31]

The average labor productivity is higher in nonagriculture than in agriculture, reflecting the nonagriculture uses capital. More formally, the productivity gap is given by the inverse ratio of the labor-income shares in the two sectors, which equals $1/\alpha$.

Consider, next, the evolution of the aggregate capital-output ratio and factor prices in the Early Lewis stage. If the agricultural and nonagricultural goods were perfect substitutes, both the wage and the interest rate would stay constant as capital accumulates. However, for $\varepsilon < \infty$ capital accumulation triggers an increase in the relative price of agricultural goods and real wage growth. Wage growth in turn causes capital deepening in the nonagricultural sector and a declining interest rate.

As capital accumulation progresses, the relative price of the agricultural good increases. Once capital is sufficiently abundant, (i.e., as $\chi \geq \underline{\chi}$), the relative price of agriculture is so high that it becomes optimal to put some capital in the modern agricultural sector. At this point the modernization process of agriculture starts and the economy enters the Advanced Lewis stage.

Stage 2 (Advanced Lewis). In this stage the equalization of factor returns across the two sectors implies that they have a constant capital-labor ratios. These are given by

$$k^{AM} \equiv \frac{(1-\kappa)\chi}{\nu^{AM}} = \left(\frac{1-\zeta}{\beta\zeta}\right)^{\frac{1}{1-\beta}}, \ k^{M} \equiv \frac{\kappa\chi}{\nu} = \frac{\beta(1-\alpha)}{\alpha(1-\beta)} k^{AM}. \tag{39}$$

The share of capital that goes to nonagriculture declines over the process of development in Stage 2:

$$\kappa = \frac{K^M}{L^M}\frac{L^M}{L}\frac{1}{\chi} = \frac{1 + \left(\frac{(1-\zeta)}{\beta\zeta}\right)^{\frac{1}{1-\beta}} \frac{\beta}{1-\beta}\frac{1}{\chi}}{1 + \frac{\alpha}{1-\alpha}\left(\frac{1+\Xi}{\Xi}\right)} \tag{40}$$

The optimal allocation of labor in manufacturing and modern agriculture yields

$$\nu^{M} = \frac{\chi + k^{AM}\frac{\beta}{1-\beta}}{k^{AM}\frac{\beta}{1-\beta}\left(\frac{1+\Xi}{\Xi}\right) + k^{M}}, \ \nu^{AM} = \frac{\beta}{1-\beta}\frac{1+\Xi}{\Xi}\nu^{M} - \frac{\beta}{1-\beta}. \tag{41}$$

These expressions shows that employment in both manufacturing and modern agriculture increase as χ

[30] In particular, $\underline{\chi} = \frac{\beta(1-\alpha)}{\alpha(1-\beta)}\left(\frac{(1-\zeta)}{\beta\zeta}\right)^{\frac{1}{1-\beta}}\frac{\Xi}{1+\Xi}$, where $\Xi \equiv \left(\frac{\alpha(1-\gamma)}{\gamma(1-\zeta)^{\frac{\varepsilon-1}{\varepsilon}}}\right)^{\varepsilon}\left(\frac{\beta(1-\alpha)}{\alpha(1-\beta)}\left(\frac{(1-\zeta)}{\beta\zeta}\right)^{\frac{1}{1-\beta}}\right)^{(\varepsilon-1)(1-\alpha)}$.

[31] The key equilibrium condition is the equalization of the marginal product of labor in nonagriculture and traditional agriculture. Using the implicit function theorem, we can show that ν^M is an increasing function of χ. More formally,

$$\frac{\nu^M}{1-\nu^M} = \left(\frac{\alpha(1-\gamma)}{\gamma(1-\zeta)^{\frac{\varepsilon-1}{\varepsilon}}}\right)^{\varepsilon}\left(\frac{\chi}{\nu^M}\right)^{(\varepsilon-1)(1-\alpha)}, \tag{38}$$

where the LHS is increasing in ν^M and the RHS is increasing in χ and decreasing in ν^M. Thus, standard differentiation implies that $\partial\nu^M/\partial\chi > 0$.

grows. Since the sectoral capital-labor ratios are constant, this also implies that capital and output in these sectors are increasing at the expense of a falling production of the traditional agriculture. Since factor prices are constant while the aggregate capital intensity in the economy is increasing, then the aggregate share of GDP accruing to capital grows while the labor share falls.

An interesting observation is that throughout this stage the expenditure share on agriculture and nonagriculture remain constant, even though $\varepsilon \neq 1$. To understand why, consider an economy without a Lewis sector. In this case, when $\varepsilon > 1$ and $\beta > \alpha$, capital accumulation would imply that the capital-intensive sector (in our case, nonagriculture) would grow faster over time. Although this implies an increase in the relative price of the agricultural product, the expenditure share on nonagricultural goods would increase over time. However, reallocation within agriculture with the decline of the Lewis sector offsets this force by increasing labor productivity in agriculture.

More formally, we can show that $\frac{P^M Y^M}{P^G Y^G} = \frac{1-\gamma}{\gamma}\left(\frac{Y^M}{Y^G}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \Psi$, where $\Psi \equiv \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{\alpha}{1-\zeta}\right)^{\varepsilon-1}\left(k^M\right)^{(1-\alpha)(\varepsilon-1)}$. This implies that the productivity gap between agriculture and nonagriculture is shrinking, since

$$\frac{\frac{P^M Y^M}{L^M}}{\frac{P^G Y^G}{L^G}} = \Psi \frac{1-\nu^M}{\nu^M},$$

and, recall, ν^M is increasing in the Advanced Lewis stage. In particular, the productivity gap (which is the inverse of the ration between the labor income share in the two sectors) declines from $1/\alpha$ to β/α in this stage, where, recall, β is the labor income share in modern agriculture. Finally, in the Advanced Lewis stage, the capital-output ratio in agriculture increases relative to the capital-output ratio in nonagriculture. More formally,

$$\frac{\frac{K^G}{P^G Y^G}}{\frac{K^M}{P^M Y^M}} = \Psi \frac{\alpha}{1-\alpha}\left(\frac{1+\Xi}{\Xi} - \frac{\frac{\alpha}{1-\alpha}\left(\frac{1+\Xi}{\Xi}\right)+1}{\frac{1}{k^M}\chi + \frac{\alpha}{1-\alpha}}\right),$$

which is increasing in χ.

Stage 3 (Neoclassical). As the process of capital accumulation proceeds, the labor reserve in traditional agriculture becomes eventually exhausted. This happens when

$$\bar{\chi} = \frac{\beta}{\beta+\Xi}\left(\frac{(1-\zeta)}{\beta\zeta}\right)^{\frac{1}{1-\beta}}\left(1+\frac{(1-\alpha)\,\Xi}{\alpha\,(1-\beta)}\right) > \underline{\chi}.$$

For any $\chi > \bar{\chi}$, $\nu^S = 0$ and $\upsilon = 1$. Henceforth, the economy exhibit standard properties of neoclassical models. In particular, if $\varepsilon > 1$ and $\beta > \alpha$, the nonagriculture sector grows in relative size, capital share (i.e., κ increases) and expenditure share. The productivity gap remains constant at β/α and the relative (agriculture vs. nonagriculture) capital-output ratio is also constant. During this stage, the interest rate falls and the real wage increases as capital accumulates.

Proposition 4 *Suppose $\varepsilon > 1$, $\beta > \alpha$ and $\omega \to \infty$. Then, as χ grows, economic development progresses through three stages:*

1. ***Early Lewis:*** *If* $\chi \leq \underline{\chi}$*, then,* $\nu^{AM} = \upsilon = 0$*,* $\kappa = 1$*. Moreover,* ν^M *is increasing and* ν^S *is decreasing in* χ*. The interest rate is decreasing and the wage rate is increasing in* χ*. The (average labor) productivity gap is constant and equal to* $1/\alpha$*.*

2. ***Advanced Lewis:*** *If* $\chi \in [\underline{\chi}, \bar{\chi}]$ *then,* ν^M *and* ν^{AM} *are increasing linearly in* χ *while* ν^S *is falling linearly in* χ *(cf. eq. (41)). Therefore,* υ *is increasing in* χ*. Moreover,* κ *is decreasing in* χ *(cf. eq. (40)). The capital-labor ratio in nonagriculture and modern agriculture is constant (cf. eq. (39)), but the relative nonagriculture-to-agriculture capital-output ratio is falling in* χ*. The interest rate and the wage rate are constant, implying that the aggregate labor income share is falling. The (average labor) productivity gap is monotonically decreasing.*

3. ***Neoclassical:*** *If* $\chi \geq \bar{\chi}$*, then,* $\nu^S = 0$ *and* $\upsilon = 1$*.* ν^M *is increasing and* ν^{AM} *is decreasing in* χ*. Moreover,* κ *is increasing in* χ*. The capital-labor ratio is increasing in* χ *in both nonagriculture and modern agriculture, but the relative nonagriculture-to-agriculture capital-output ratio is constant. The interest rate is decreasing in* χ *and the wage rate is increasing in* χ*, while the aggregate labor income share is falling. The (average labor) productivity gap is constant. As* χ *becomes arbitrarily large,* $\kappa \to 1$*,* $\nu^M \to 1$ *and the expenditure share of agriculture tends to zero.*

C Appendix: Details on Estimating the Cyclical TFP Process

This appendix describes how we estimate the stochastic process in equation (18). The moment conditions rely on variances and covariances of current and up to two lags of the cyclical TFP components, including $var(z_t^j)$, $cov(z_t^j, z_{t+1}^j)$, $cov(z_t^j, z_{t+2}^j)$, $var(\Delta z_t^j)$, and $cov(\Delta z_t^j), \Delta z_{t+1}^j))$, where $j \in M, AM, S$. The exact conditions are reported in equations (42)–(46).

$$var\left(\tilde{z}_t^j\right) = \frac{\sigma_{\varepsilon,j}^2}{1-\rho_j^2} + \sigma_{\zeta,j}^2 \tag{42}$$

$$cov\left(\tilde{z}_t^j, \tilde{z}_{t-1}^j\right) = \rho_j \frac{\sigma_{\varepsilon,j}^2}{1-\rho_j^2} \tag{43}$$

$$cov\left(\tilde{z}_t^j, \tilde{z}_{t-2}^j\right) = \rho_j^2 \frac{\sigma_{\varepsilon,j}^2}{1-\rho_j^2} \tag{44}$$

$$var\left(\Delta\tilde{z}_t^j\right) = \frac{2}{1+\rho_j}\sigma_{\varepsilon,j}^2 + 2\sigma_{\zeta,j}^2 \tag{45}$$

$$cov\left(\Delta\tilde{z}_t^j, \Delta\tilde{z}_{t+1}^j\right) = \frac{\rho_j\left(1-\rho_j\right)}{1+\rho_j}\sigma_{\varepsilon,j}^2 - \sigma_{\zeta,j}^2 \tag{46}$$

It is straightforward to verify that the moments (42)–(46) identify the model.

The off-diagonal elements of the matrix A, i.e., $cov\left(\varepsilon_t^j, \varepsilon_t^i\right)$ are then computed according to

$$cov\left(\varepsilon_t^j, \varepsilon_t^i\right) = cov\left(\tilde{z}_t^j - \hat{\rho}_j \tilde{z}_{t-1}^j, \tilde{z}_t^i - \hat{\rho}_i \tilde{z}_{t-1}^i\right), \tag{47}$$

where the measurement errors are assumed to independent across sectors.

D Appendix: Details of the Analysis of Section 6

D.1 CES estimation with cross-sectional data

We now estimate the elasticities implied by a CES model based on eq. (20)-(21) and cross-country data.

We use the 10-sector data from the Gröningen Growth and Development Centre as our benchmark data set, henceforth GGDC. These data provide long time series on sectoral output for 41 countries. We also use the UN Analysis of Main Aggregates dataset which covers more countries (220 countries) albeit over a shorter time period (1970-2017), henceforth UN AMA. Using value-added sectoral output and real output data (in 2005 prices) from GGDC, we can derive the implied price deflator for each sector. We then aggregate the sectoral price indices and real quantities into agriculture, manufacturing, and nonagriculture following the cyclical expansion procedure used in Herrendorf et al. (2013). We define manufacturing sector as the union of "Mining", "Manufacturing", "Utilities", "Construction". Similarly, we aggregate all sectors except agriculture sector and government sector into the nonagriculture sector. Following Comin et al. (2015), our benchmark specification focuses on ratios of agriculture to manufacturing. We show that all our findings are robust to using a broader data set (UN data) and a broader definitions of nonagriculture.

It is appropriate to estimate the model in first differences since one cannot reject the hypothesis that the logarithm of the empirical relative value added-, price-, and output ratios have unit roots. For robustness we also estimate the models in levels, allowing country-specific fixed effects.

We start by estimating eq. (20). Table 5 documents the results. Specification (1) is based on calculating average ratios over non-overlapping 10-year periods and taking the first difference across these observations (the results are robust to instead taking simple 10-year differences of annual data). Specification (2), which we consider our benchmark specification, adds time fixed effects to the regression. The estimated coefficient of interest is significantly positive (0.838 and 0.746). According to eq. (20), a coefficient of 0.746 implies a high elasticity; $\varepsilon \approx 4$. This finding is robust to different data and specifications. Specification (3) adds log of aggregate real consumption as a control. Specification (4) estimates the model in levels, allowing country and time fixed effects. Motivated by our focus on China, specification (5) estimates the model on Asian countries. Specification (6) estimates the model using a higher frequency (annual). Finally, specification (7) considers ratios of agriculture to nonagriculture (as opposed to manufacturing).[32] The coefficient of interest is significantly positive in all specifications and imply an elasticity ε between 2 and 6. These results are robust to using UN AMA data.

We now estimate eq. (21). Table 6 documents the results. The specifications (1) and (2) yield positive coefficients which are all significant at a 10% level. According to eq. (21), the benchmark estimate in column (2) implies a low elasticity; $\varepsilon \approx 0.6$. This finding is robust to adding aggregate

[32]The coefficient in column (7) of Table 5 is somewhat lower than the other estimates. However, when estimating this specification with UN AMA data, the coefficient is higher, 0.862.

Table 5: The table shows regressions based on equation 20. Specification (1) is based on differences over averages for 10-year periods for all countries in the GGDC data. Specification (2) adds time fixed effects as additional control. Specification (3) adds log real consumption from PWT as additional control. Specification (4) is based on 10-year average levels. Specification (5) is based on GGDC Asia sample, excluding HKG and SGP. (6) uses 1-year difference instead, and specification (7) estimates the model based on ratios of agriculture to nonagriculture.

EXPENDITURE SHARE OF AGRICULTURE, regressed on QUANTITY RATIO							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Data	GGDC	GGDC	GGDC	GGDC	GGDC Asia	GGDC	GGDC
VARIABLES			Agriculture/Manufacturing				Agri./Nonagr.
	10yr FD	10yr FD	10yr FD	10yr level	10yr FD	1yr FD	10yr FD
Δlogrel_quantity	0.838***	0.746***	0.664***		0.685***	0.681***	0.459***
	(0.0893)	(0.100)	(0.141)		(0.0625)	(0.0591)	(0.119)
Δlogconsumption			-0.204				
			(0.142)				
logrel_quantity				0.787***			
				(0.0928)			
Year Fixed Effect	N	Y	Y	Y	Y	Y	Y
Observations	139	139	139	179	33	1,923	139
R-squared	0.684	0.756	0.763	0.830	0.961	0.361	0.541

Standard errors (clustered at country level) in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 6: The table shows regressions based on equation 21. Specification (1) is based on differences over averages for 10-year periods for all countries in the GGDC data. Specification (2) adds time fixed effects as additional control. Specification (3) adds log real consumption from PWT as additional control. Specification (4) is based on 10-year average levels. Specification (5) is based on GGDC Asia sample, excluding HKG and SGP. (6) uses 1-year difference instead, and specification (7) estimates the model based on ratios of agriculture to nonagriculture.

EXPENDITURE SHARE OF AGRICULTURE, regressed on PRICE RATIO							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Data	GGDC	GGDC	GGDC	GGDC	GGDC Asia	GGDC	GGDC
VARIABLES			Agriculture/Manufacturing				Agri./Nonagr.
	10yr FD	10yr FD	10yr FD	10yr level	10yr FD	1yr FD	10yr FD
Δlogrel_price	0.539**	0.356*	0.501**		-0.445	0.775***	0.518***
	(0.233)	(0.203)	(0.200)		(0.285)	(0.0497)	(0.103)
Δlogconsumption			-0.983***				
			(0.125)				
logrel_price				0.451***			
				(0.112)			
Year Fixed Effect	N	Y	Y	Y	Y	Y	Y
Observations	139	139	139	179	33	1,923	139
R-squared	0.099	0.381	0.648	0.671	0.820	0.548	0.591

Standard errors (clustered at country level) in parentheses. *** p<0.01, ** p<0.05, * p<0.1

consumption as a control (3), to estimating the model in levels (4), at a higher frequency (6), and using ratios of agriculture to nonagriculture (7). These regressions suggest that ε is between 0.2 and 0.6. However, when considering only Asian countries, the estimated coefficient implies an elasticity above unity, $\varepsilon \approx 1.4$.

In conclusion, the simple regressions based on expenditure, prices, and quantities, summarized in Tables 5 and 6, give mixed evidence on ε. These inconclusive results could be due to the fact that both sides of eq. (20)-(21) are endogenous. In Appendix D.3 we present a structural model with two goods where relative prices, quantities, and expenditures are endogenous and where the economy is subject to both supply shocks and demand shocks, captured by TFP shocks and preference shocks. We show that demand shocks and supply shocks have opposite implications for the sign of the correlations between prices and quantities. This problem applies to the estimation of both eq. (20) and (21).

Finally, note that (classical) measurement error is not a likely cause of the differences across specifications. To see this, note that in the presence of classical measurement error, potential biases should be smaller at lower frequencies, as the signal to noise ratio is larger. Consider first Table 5. The estimated coefficients are virtually unchanged when going from differences over 10-year periods to annual differences (specification (6) versus (2)) in Table 5). This suggests that potential biases due to measurement error must be negligible. Consider, next, Table 6. The coefficient of interest in Table 6 is higher at an annual frequencies (see specification (6) relative to (2)). We show below that measurement error in this regression gives rise to an attenuation bias, which should bias the estimated coefficient toward zero. The fact that this coefficient is larger at higher frequencies suggests that classical measurement error is not plausible explanation for the differences in results across specifications.

D.2 Classical measurement error in the price-quantity regression

This section examines how the estimates of ε from regressions in (20)-(21) would be influenced by classical measurement error.

Assume that there is classical measurement error in both log expenditure and prices, and that quantities are defined as expenditures over prices, i.e.,

$$\begin{aligned}
\ln \widehat{\frac{P_t^G Y_t^G}{P_t^M Y_t^M}} &= \ln \frac{P_t^G Y_t^G}{P_t^M Y_t^M} + \eta_e \\
\ln \widehat{\frac{P_t^G}{P_t^M}} &= \ln \frac{P_t^G}{P_t^M} + \eta_p \\
\ln \widehat{\frac{Y_t^G}{Y_t^M}} &\equiv \ln \widehat{\frac{P_t^G Y_t^G}{P_t^M Y_t^M}} - \ln \widehat{\frac{P_t^G}{P_t^M}} = \ln \frac{Y_t^G}{Y_t^M} + \eta_e - \eta_p
\end{aligned}$$

Note first that $\hat{\beta}_p$ in the price regression is biased toward zero (relative to the true coefficient) through an attenuation bias;

$$
\begin{aligned}
\ln \widehat{\frac{P_t^G Y_t^G}{P_t^M Y_t^M}} &= \beta_{0p} + \hat{\beta}_p \ln \widehat{\frac{P_t^G}{P_t^M}} + \delta_t \\
&\Rightarrow \hat{\beta}_p = \beta_p \cdot \frac{var\left(\frac{P_t^G}{P_t^M}\right)}{var\left(\frac{P_t^G}{P_t^M}\right) + var\left(\eta_p\right)} = \begin{cases} < \beta_p & \text{if } \beta_p > 0 \\ \geq \beta_p & \text{if } \beta_p \leq 0 \end{cases},
\end{aligned}
$$

where β_p is the true coefficient. It follows that since $\beta_p = 1 - \varepsilon$ and $\hat{\beta}_p > 0$, the estimated $\hat{\varepsilon}$ in the price regression is biased upward.

Second, note that $\hat{\beta}_y$ in the quantity regression is also biased toward zero (relative to the true coefficient) provided that measurement error in expenditures is small relative to measurement error in prices. This is again driven by through an attenuation bias;

$$
\begin{aligned}
\ln \widehat{\frac{P_t^G Y_t^G}{P_t^M Y_t^M}} &= \beta_0 + \hat{\beta}_y \ln \widehat{\frac{Y_t^G}{Y_t^M}} + \delta_t \\
\Rightarrow \quad \hat{\beta}_y &= \frac{cov\left(\frac{P_t^G Y_t^G}{P_t^M Y_t^M} + \eta_e, \frac{Y_t^G}{Y_t^M} + \eta_e - \eta_p\right)}{var\left(\frac{Y_t^G}{Y_t^M} + \eta_e - \eta_p\right)} \\
&= \beta_y \frac{var\left(\frac{Y_t^G}{Y_t^M}\right)}{var\left(\frac{Y_t^G}{Y_t^M}\right) + var\left(\eta_e\right) + var\left(\eta_p\right)} + \frac{var\left(\eta_e\right)}{var\left(\frac{Y_t^G}{Y_t^M}\right) + var\left(\eta_e\right) + var\left(\eta_p\right)}.
\end{aligned}
$$

Assume that $var\left(\eta_e\right)$ is small. This seems reasonable given that expenditure ratios are calculated from national accounts. It follows that if $\beta_p > 0$, then the attenuation bias will dominate and $\hat{\beta}_y < \beta_y$. Since $\beta_y = \left(\varepsilon - 1\right)/\varepsilon$ and the estimated coefficient $\hat{\beta}_y >> 0$, the estimated $\hat{\varepsilon}$ in the quantity regression must be biased downward.

D.3 Estimation bias in the presence of demand shocks

This appendix argues that in the presence of demand shocks affecting the relative demand for agricultural goods, the estimate of ε from eq. (21) is biased toward zero while that from eq. (20) is biased away from zero.

Consider a dynamic production economy where a representative household has preferences over the goods Y_g and Y_m given by eq. (1). There is no capital and no storage, so equilibrium consumption equals production of each good.

The goods are traded at prices P_g and P_m. Utility maximization implies the demand equation

$$
\frac{Y_g}{Y_m} = \frac{\gamma}{1-\gamma}\left(\frac{P_g}{P_m}\right)^{-\varepsilon}. \tag{48}
$$

A continuum of firms, owned by the household, produces Y_g and Y_m via the function

$$Y_i = A_i L_i^{\varphi}, \tag{49}$$

where $\varphi \in [0, 1)$ and $i \in \{g, m\}$. The household has one unit of labor that is split between sectors. Firm optimization implies that the marginal product of labor is equalized across sectors, i.e., $P_g \varphi A_g (L_g)^{\varphi-1} = P_m \varphi A_m (L_m)^{\varphi-1}$.

Combine this equation with eq. (48)-(49) to substitute out the labor ratio L_g/L_m and the price ratio P_g/P_m. Rearranging terms yields an expression for Y_g/Y_m in terms of $\gamma/(1-\gamma)$ and A_g/A_m,

$$\ln\left(\frac{Y_g}{Y_m}\right) = \frac{\varphi}{\varphi + \varepsilon(1-\varphi)} \ln\left(\frac{\gamma}{1-\gamma}\right) + \frac{\varepsilon}{\varphi + \varepsilon(1-\varphi)} \ln\left(\frac{A_g}{A_m}\right). \tag{50}$$

Combining eq. (48) and (50) yields an expression for P_g/P_m in terms of $\gamma/(1-\gamma)$ and A_g/A_m,

$$\ln\left(\frac{P_g}{P_m}\right) = \frac{1-\varphi}{\varphi + \varepsilon(1-\varphi)} \ln\left(\frac{\gamma}{1-\gamma}\right) - \frac{1}{\varphi + \varepsilon(1-\varphi)} \ln\left(\frac{A_g}{A_m}\right). \tag{51}$$

The economy is subject to supply and demand shocks, interpreted as shocks to growth in relative TFP and relative preference weights. For notational convenience, define the variables $p_t = \ln(P_{g,t}/P_{m,t}) - \ln(P_{g,t-1}/P_{m,t-1})$, $y_t = \ln(Y_{g,t}/Y_{m,t}) - \ln(Y_{g,t-1}/Y_{m,t-1})$, $\xi_t = \ln[\gamma_t/(1-\gamma_t)] - \ln[\gamma_{t-1}/(1-\gamma_{t-1})]$, and $a_t = \ln(A_{g,t}/A_{m,t}) - \ln(A_{g,t-1}/A_{m,t-1})$. The growth in relative expenditure, $x_t \equiv \ln[P_{gt}Y_{gt}/(P_{mt}Y_{mt})] - \ln[P_{g,t-1}Y_{g,t-1}/(P_{m,t-1}Y_{m,t-1})]$, is then $x_t = p_t + y_t$. For simplicity, assume that $var(\xi_t)$ and $var(a_t)$ are constant and that $corr(\xi, a) = 0$.

Consider now simulating time-series data from this model and running OLS regressions on eq. (20)-(21). The following proposition derives the implied coefficients from these regressions.

Proposition 5 *The regression coefficient on the relative price growth p_t in the expenditure-price regression $x_t = \beta_0 + \beta_p p_t + \eta_t$ is given by*

$$\beta_p = 1 - \varepsilon + \left(\varepsilon + \frac{\varphi}{1-\varphi}\right) \frac{(1-\varphi)^2 var(\xi)}{(1-\varphi)^2 var(\xi) + var(a)} \geq 1 - \varepsilon \tag{52}$$

The regression coefficient on the relative output growth y_t in the expenditure-output regression $x_t = \beta_0 + \beta_y y_t + \delta_t$ is given by

$$\beta_y = \frac{\varepsilon - 1}{\varepsilon} + \frac{\varphi + \varepsilon(1-\varphi)}{\varphi\varepsilon} \frac{\varphi^2 \cdot var(\xi)}{\varphi^2 \cdot var(\xi) + \varepsilon^2 \cdot var(a)} \geq \frac{\varepsilon - 1}{\varepsilon} \tag{53}$$

Proof. The regression coefficients are defined as $\beta_p = cov(p+y, p)/var(p)$ and $\beta_y = cov(p+y, y)/var(y)$. The expressions in (52)-(53) follow from the equilibrium relationships (50)-(51) and the assumption $corr(\xi, a) = 0$. The inequalities in eq. (52)-(53) follow from $\varphi \in [0, 1)$ so the last term in each equation is non-negative. ■

Proposition 5 shows that when there are no demand shocks ($var(\xi) = 0$) then β_p and β_y are unbiased estimates of $1 - \varepsilon$ and $(\varepsilon - 1)/\varepsilon$, respectively. However, when $var(\xi) > 0$, then $\hat{\varepsilon} \equiv 1 - \beta_p < \varepsilon$

and $\tilde{\varepsilon} \equiv 1/(1-\beta_y) > \varepsilon$. It follows that the estimate from eq. (21) yields a downward bias in ε and that from eq. (20) yields an upward bias in ε.

D.4 Estimated equations in Section 6.1

The production function (19) in Section 6.1 gives rise to the following equilibrium conditions, based on first-order conditions,

$$\begin{aligned}
\frac{p^{Serv}Y^{Serv}}{p^{Serv}Y^{Serv}+p^{Manuf}Y^{Manuf}} &= \frac{(1-\hat{\gamma})\left(p^{Serv}\right)^{1-\varepsilon_{ms}}}{\hat{\gamma}\left(p^{Manuf}\right)^{1-\varepsilon_{ms}}+(1-\hat{\gamma})\left(p^{Serv}\right)^{1-\varepsilon_{ms}}} \\
&\quad \times \frac{p^{Serv}Y^{Serv}+p^{Manuf}Y^{Manuf}+p^{Serv}\bar{s}}{p^{Serv}Y^{Serv}+p^{Manuf}Y^{Manuf}} - \frac{p^{Serv}\bar{s}}{p^{Serv}Y^{Serv}+p^{Manuf}Y^{Manuf}} \\
\frac{p^{G}Y^{G}}{p^{G}Y^{G}+p^{M}Y^{M}} &= \frac{\gamma\left(p^{G}\right)^{1-\varepsilon}}{\gamma\left(p^{G}\right)^{1-\varepsilon}+(1-\gamma)\left(p^{M}\right)^{1-\varepsilon}} \\
&\quad \times \frac{p^{G}Y^{G}+p^{M}Y^{M}+p^{G}\bar{c}+p^{S}\bar{\bar{s}}}{p^{G}Y^{G}+p^{M}Y^{M}} - \frac{p^{G}\bar{c}}{p^{G}Y^{G}+p^{M}Y^{M}}
\end{aligned}$$

The nonlinear least square regressions in Section 6.1 are based on these conditions.

D.5 Calibrating ε using time-series data for the U.S.

Combining eq. (22) and (25) with the demand relationship $\Delta \ln \left(Y_t^G/Y_t^M\right) = -\varepsilon \cdot \Delta \ln \left(P_t^G/P_t^M\right)$ yields

$$\varepsilon = 1 - \left[\Delta \ln \left(\frac{1-\nu^M}{\nu^M}\right) - \Delta \ln \left(\upsilon + \frac{1}{\beta}(1-\upsilon)\right)\right] / \Delta \ln \left(\frac{P_t^G}{P_t^M}\right).$$

Alvarez-Cuadrado and Poschke (2010) report that the employment share of U.S. agriculture – $1-\nu^M$ in our model – went from 73% in 1800 to 16% in 1945. Over the same time period the price ratio of agricultural goods to nonagricultural goods increased by 50%. Our model is consistent with these changes for $\varepsilon \approx 7.5$. This calculation assumes the factor shares estimated for China, and any $\upsilon \geq 0$. A shorter time span, for example 1800-1920, yields an even larger estimate for ε.

E Appendix: Tables and Figures

Table 7: Panel (a) reports the results of a set of regressions whose dependent variable is the logarithm of the K-Y ratio. Panel (b) reports the results of a set of regressions whose dependent variable is the logarithm of the productivity gap. Columns (1) and (2) are pooled regressions with time effects. Columns (3) and (4) are within regressions with country fixed effects. Robust standard errors clustered at country level are reported in parentheses. *** p¡0.01, ** p¡0.05, * p¡0.1

	(a) Relative K-Y ratio in agriculture					
	(1)	(2)	(3)	(4)	(5)	(6)
log(agr. empl. share)	-0.296***	-0.297***	-0.262***	-0.249**	-0.281*	-0.219
	(0.0186)	(0.0241)	(0.0964)	(0.105)	(0.150)	(0.223)
Sample	FULL	NO AFRICA	FULL	NO AFRICA	FULL	NO AFRICA
Time dummy	YES	YES	NO	NO	YES	YES
Country FE	NO	NO	YES	YES	YES	YES
Observations	292	206	292	206	292	206
R-squared	0.638	0.586	0.047	0.052	0.047	0.052
	(b) Productivity Gap					
	(1)	(2)	(3)	(4)	(5)	(6)
log(agr. empl. share)	0.370***	0.348***	0.300***	0.191**	0.901***	0.965***
	(0.0165)	(0.0176)	(0.0979)	(0.0863)	(0.139)	(0.163)
Sample	FULL	NO AFRICA	FULL	NO AFRICA	FULL	NO AFRICA
Time dummy	YES	YES	NO	NO	YES	YES
Country FE	NO	NO	YES	YES	YES	YES
Observations	290	208	290	208	290	208
R-squared	0.797	0.771	0.066	0.036	0.261	0.281

Supplemetary Material for the paper "Business Cycle during Structural Change: Arthur Lewis' Theory from a Neoclassical Perspective"

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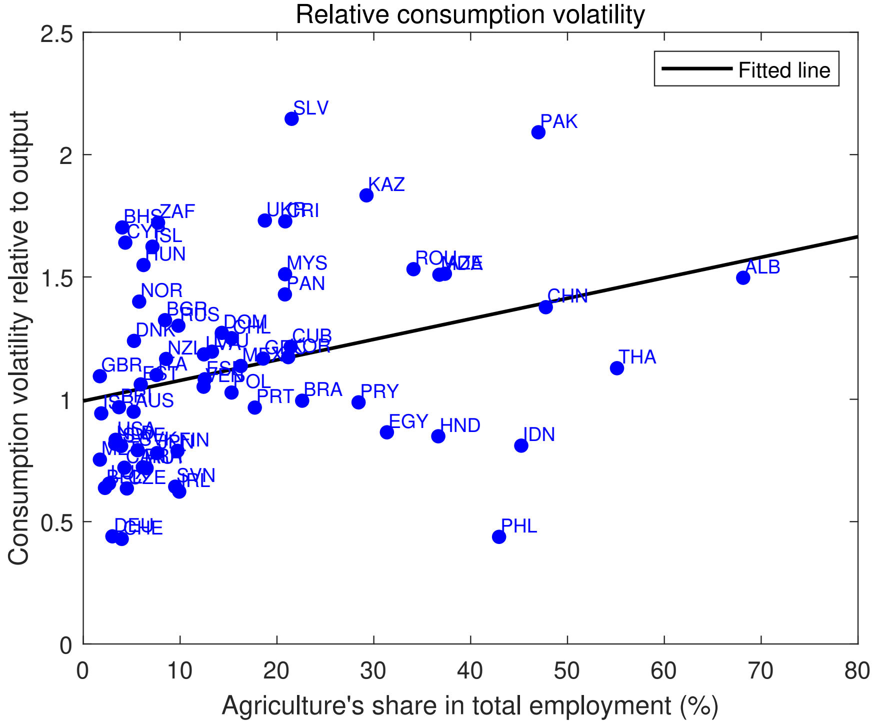
Table 8: Summary Statistics for China data and Model: First-differenced DATA AND MODEL.

	c	i	y^G	y^M	$\frac{P^G y^G}{P}$	$\frac{P^M y^M}{P}$	$\frac{APL^G}{APL^M}$	n^G	n^M	n
	A. China Data, 1985-2012									$std\,(y) = 2.4\%$
$\frac{std(x)}{std(y)}$	1.27	3.34	0.50	1.18	1.82	1.31	2.43	1.60	1.02	0.30
$corr(x, y)$	0.57	0.63	0.05	0.99	0.12	0.93	-0.30	-0.54	0.71	-0.25
$corr(x, n^G)$	-0.75	-0.43	-0.12	-0.46	-0.13	-0.45	0.56	1.00	-0.55	0.68
$corr\left(x, n^M\right)$	0.39	0.56	-0.11	0.76	-0.04	0.75	-0.53	-0.55	1.00	0.078
	B. Benchmark									$std\,(y) = 2.2\%$
$\frac{std(x)}{std(y)}$	0.37	2.42	2.06	1.47	1.50	1.34	2.51	3.20	1.54	0.52
$corr\,(x, y)$	0.65	0.97	-0.30	0.94	-0.15	0.96	-0.56	-0.55	0.75	-0.16
$corr\left(x, n^G\right)$	-0.23	-0.56	0.86	-0.71	0.80	-0.69	0.96	1.00	-0.89	0.80
$corr\left(x, n^M\right)$	0.43	0.74	-0.79	0.91	-0.70	0.89	-0.85	-0.89	1.00	-0.72
	C. Unconstrained Estim.									$std\,(y) = 2.2\%$
$\frac{std(x)}{std(y)}$	0.38	2.35	2.67	1.45	2.28	1.36	1.16	2.04	1.40	0.42
$corr\,(x, y)$	0.71	0.98	0.06	0.89	0.13	0.90	-0.55	-0.33	0.65	0.16
$corr\left(x, n^G\right)$	-0.20	-0.32	0.85	-0.63	0.82	-0.61	0.42	1.00	-0.84	0.70
$corr\left(x, n^M\right)$	0.50	0.61	-0.60	0.91	-0.55	0.89	-0.54	-0.84	1.00	-0.58
	D. $\varepsilon = 0.5$									$std\,(y) = 2.1\%$
$\frac{std(x)}{std(y)}$	1.01	2.27	0.68	1.29	2.67	1.30	2.24	1.63	1.01	0.24
$corr\,(x, y)$	0.67	0.97	0.30	0.98	0.05	0.97	-0.26	-0.59	0.76	-0.14
$corr\left(x, n^G\right)$	-0.36	-0.60	-0.48	-0.54	0.40	-0.62	0.26	1.00	-0.89	0.60
$corr\left(x, n^M\right)$	0.54	0.76	0.46	0.73	-0.45	0.82	-0.06	-0.89	1.00	-0.57

Table 9: Estimation results for the models in Section 6.1. "3-sector" refers to agriculture, manufacturing, and services. "2-sector" refers to agriculture and nonagriculture. Consumption value added data for USA are from Herrendorf et al. (2013). Consumption value added data for China and Japan are constructed following the methodology in Herrendorf et al. (2013) using official input-output tables from NBS and Japan's Ministry of Internal Affairs and Communications, respectively. Production value added data are from GGDC.

Param.	USA				Japan				China			
	Cons. value added		GGDC		Cons. value added		GGDC		Cons. value added		GGDC	
	3-sector	2-sector	3-sector	2-sector	3-sector	2-sector	3-sector	2-sector	3-sector	2-sector	3-sector	2-sector
ε	2.49***	2.32***	1.36***	1.53***	1.58***	2.24***	5.46***	6.06***	1.70***	1.34***	1.30***	1.75***
	(0.28)	(0.32)	(0.13)	(0.48)	(0.05)	(0.36)	(0.80)	(0.91)	(0.22)	(0.25)	(0.13)	(0.08)
ϵ_{ms}	0	-	0.48***	-	0.79***	-	1.66***	-	0.007	-	0.65***	-
			(0.005)		(0.003)		(0.13)		(0.05)		(0.04)	
$\bar{c}$	-160.6***	-160.6***	-215.0***	-224.0***	-669.6***	-828.2***	-58334***	-59631***	-9.0***	-9.2***	-654.2***	-614.9***
	(3.8)	(3.8)	(6.0)	(5.7)	(43.1)	(46.1)	(5417)	(5417)	(0.4)	(0.4)	(41.6)	(41.4)
$\bar{s}$	4299.4***	-	12897***	-	3234.0***	-	36056***	-	0.01	-	139.8***	-
	(444.0)		(431.8)		(92.9)		(6269)				(31.1)	
γ	0.99***	0.99***	0.997***	0.994***	0.97***	0.97***	0.994***	0.995***	0.89***	0.92***	0.90***	0.90***
	(0.0005)	(0.005)	(0.0004)	(0.0005)	(0.03)	(0.005)	(0.003)	(0.003)	(0.02)	(0.02)	(0.005)	(0.006)
$\hat{\gamma}$	0.15***	-	0.20***	-	0.21***	-	0.44***	-	0.63***	-	0.61***	-
	(0.004)		(0.003)		(0.002)		(0.004)		(0.008)		(0.004)	
Sample	1947-2010		1947-2010		1955-2017		1953-2010		1985-2012		1980-2010	
AIC	-909.3	-525.0	-840.0	-506.0	-596.6	-259.6	-663.8	-392.2	-270.2	-9.2	-302.1	-158.8
RMSE_G	0.004	0.004	0.003	0.003	0.022	0.022	0.008	0.008	0.015	0.015	0.018	0.017
RMSE_M	0.011	-	0.020	-	0.016	-	0.021	-	0.016	-	0.025	-
RMSE_S	0.012	-	0.018	-	0.037	-	0.025	-	0.020	-	0.016	-

Figure 13: Cross-country evidence on consumption volatility



Note: The figure shows the volatility of total consumption relative to the volatility of GDP in a sample of 64 countries. We use a smoothing parameter 6.25 for the HP filter Ravn and Uhlig (2002). The x-axis denotes the mean agriculture's share in total employment over the sample period for each country. The solid regression line stands for unweighted OLS regression.

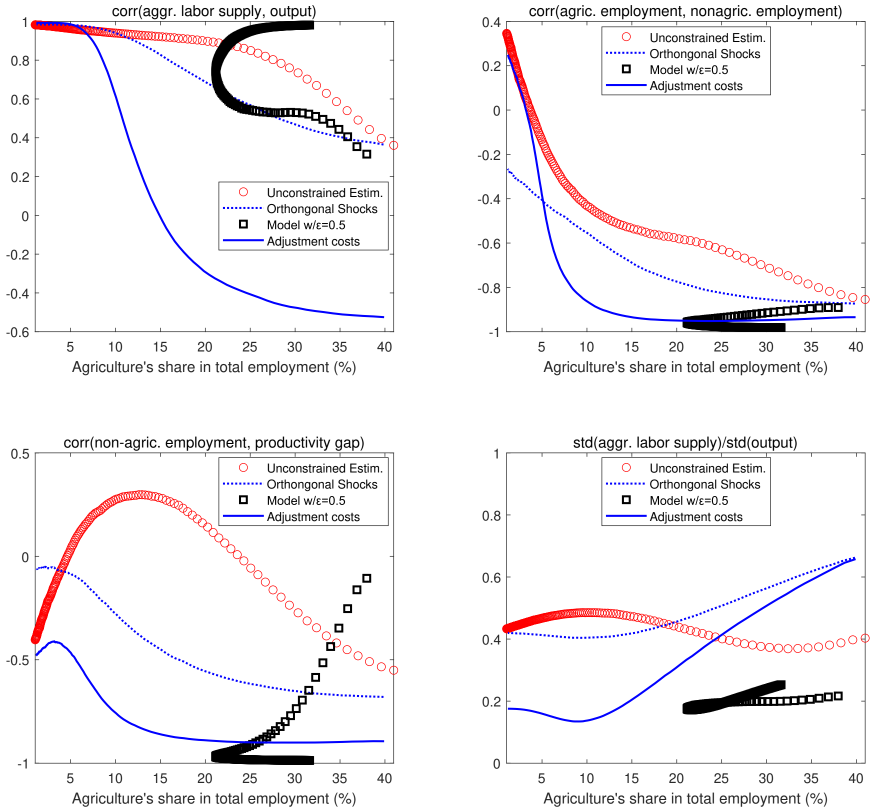
A Additional Material

A.1 Additional Robusness Analysis

A.1.1 Capital Adjustment Costs

In the benchmark model, capital in each sector is set one period in advance, and after one period reallocation of capital between sectors can occur without cost. In this appendix, we investigate the effect of introducing additional costs of reallocating capital between agriculture and nonagriculture. Capital adjustment costs are commonly assumed in the quantitative DSGE literature (cf. Christiano et al. (2005); Smets and Wouters (2007)), including papers studying business cycles in models with multiple sectors (see e.g. Horvath (2000); Bouakez et al. (2009); Iacoviello and Neri (2010)).

Following Bouakez et al. (2009) and Iacoviello and Neri (2010), we consider a canonical model where it is costly to change the investment rate. Recall that for each sector j, the law of motion of capital is given by $K_{t+1}^{j} = (1-\delta)\, K_t^j + I_t^j$, where $j \in \{M, G\}$ and I_t^j is the effective investment in sector j. The



Note: The figure reports the result of the robustness analysis discussed in Section 5.4.

investment cost is reflected in an aggregate resource constraint for investment goods,

$$\left[\gamma\left(X_t^G\right)^{\frac{\varepsilon-1}{\varepsilon}}+(1-\gamma)\left(X_t^M\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} = \Psi_t^G\left(\frac{I_t^G}{K_t^G},K_t^G\right) + \Psi_t^M\left(\frac{I_t^M}{K_t^M},K_t^M\right)+I_t^G+I_t^M,$$

where the terms Ψ_t^{AG} and Ψ_t^M capture the adjustment cost. Recall also that X_t^i is the quantity of good $i \in \{M,G\}$ allocated to investment. We assume that the adjustment cost function Ψ has a standard quadratic form,

$$\Psi_t^j\left(\frac{I_t^j}{K_t^j},K_t^j\right)=\frac{\xi}{2}\left(\frac{I_t^j}{K_t^j}-\delta-g_t^j\right)^2 K_t^j,$$

where the parameter g_t^j is the growth rate of capital K_t^j in period t in the deterministic structural transition and ξ is a nonnegative adjustment cost parameter. It follows that as long as the capital stock K_t^j grows at the same rate as in the deterministic transition, g_t^j, the adjustment costs are zero. However, when the investment rate deviates from this level then quadratic costs are incurred.

We set the adjustment cost parameter to $\xi = 2.5$, which is slightly lower than the annual equivalent to the value of ξ estimated by Iacoviello and Neri (2010) based on quarterly data for the US ($\xi = 11$).[33] Since the deterministic model of structural change is not affected by the capital adjustment cost, we keep all other parameters the same as in the benchmark economy.

Because adjustment costs make investments more sluggish, consumption must respond more to TFP shocks, which in turn increases its volatility. Adjustment costs also affect sectoral employment because sluggish capital makes less advantageous moving labor across sectors in the short run.

Panel C of Table 10 shows the results. The main effect of introducing adjustment costs is that the cyclical behavior of aggregate labor supply and consumption is more in line with the data: n becomes negatively correlated with GDP and positively correlated with nonagricultural labor supply, as it is for China (see panel A of Table 3). Moreover, consumption becomes substantially more volatile, more negatively correlated with agricultural labor supply, and get a more reasonable correlation with GDP. However, aggregate investment turns too smooth.

Table 10: Robustness analysis, benchmark model versus alternative model. All statistics refer to HP-filtered simulated data for 1985-2012.

	c	i	y^G	y^M	$\frac{P^G y^G}{P}$	$\frac{P^M y^M}{P}$	$\frac{APL^G}{APL^M}$	n^G	n^M	n
	A. China Data, 1985-2012									$std\,(y) = 1.7\%$
$\frac{std(x)}{std(y)}$	0.99	3.53	0.40	1.21	1.64	1.34	2.17	1.00	1.04	0.10
$corr(x,y)$	0.70	0.65	-0.11	0.99	0.06	0.95	-0.48	-0.78	0.83	-0.23
$corr(x,n^G)$	-0.67	-0.61	0.10	-0.79	-0.01	-0.76	0.62	1.00	-0.93	0.18
$corr\left(x,n^M\right)$	0.64	0.65	-0.14	0.85	0.01	0.82	-0.65	-0.93	1.00	-0.01
	B. Capital adjustment cost									$std\,(y) = 1.2\%$
$\frac{std(x)}{std(y)}$	0.69	1.54	2.45	1.52	1.76	1.38	2.85	3.96	1.56	0.67
$corr\,(x,y)$	1.00	1.00	-0.41	0.93	-0.30	0.95	-0.63	-0.65	0.69	-0.53
$corr\left(x,n^G\right)$	-0.61	-0.65	0.90	-0.81	0.86	-0.79	0.98	1.00	-0.93	0.95
$corr\left(x,n^M\right)$	0.65	0.69	-0.90	0.89	-0.84	0.86	-0.89	-0.93	1.00	-0.96
	C. Orthogonal shocks									$std\,(y) = 1.2\%$
$\frac{std(x)}{std(y)}$	0.26	2.49	3.59	1.46	2.71	1.31	2.14	3.54	1.52	0.64
$corr\,(x,y)$	0.77	0.99	-0.05	0.84	0.04	0.89	-0.12	-0.17	0.47	0.33
$corr\left(x,n^G\right)$	-0.01	-0.16	0.90	-0.58	0.88	-0.51	0.83	1	-0.88	0.83
$corr\left(x,n^M\right)$	0.20	0.46	-0.82	0.85	-0.78	0.79	-0.67	-0.88	1.00	-0.62

A.1.2 More Persistent Shocks in the Traditional Sector

In the benchmark model the persistence of shocks to z^S is significantly lower than that of shocks to z^{AM} ($\hat{\phi}^S = 0.42$ versus $\hat{\phi}^{AM} = 0.90$). We argued above that the transitory nature of shocks to traditional sector z^S was the cause behind the falling volatility of aggregate employment in the initial phase of the transition (see the lower right panel of Figure 12).

[33] Recall that in an annual model it is impossible to change the capital stock more often than annually, so our benchmark economy already embeds some frictions since sectoral capital is set one period in advance.

Since neither traditional nor modern agricultural production are directly observed, measurement error in agricultural capital and employment will show up as movements in the two TFP levels. This will affect the estimated TFP processes for z^S and z^M. For this reason, we consider a sensitivity analysis where TFP shocks to traditional agriculture have the same persistence as shocks to modern agriculture ($\hat{\phi}^S = \hat{\phi}^{AM} = 0.9$). Moreover, we adjust the volatility of the innovations to z^S, $\sigma\left(\epsilon_t^S\right)$, so that the stationary variance of z^S is kept constant. The results are shown in panel C of Table 10. The main effect of a higher persistence of shocks to z^S is that the aggregate volatility of labor supply n falls and the correlation between n and GDP increase somewhat. Moreover, the relative volatility of employment is predicted to increase monotonically during structural change (see panel D of appendix Figure 14). Lowering the standard deviation of innovations to z^S, $\sigma\left(\epsilon_t^S\right)$, has similar effects.

A.2 Details on employment data for China

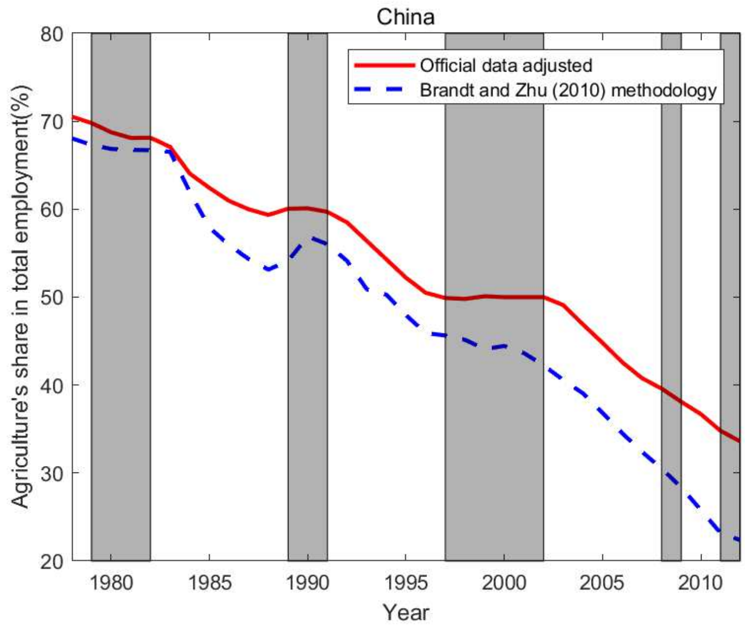
We rely on official Chinese data on aggregate and sectoral employment, applying two standard adjustments. The official series has a structural break in 1990. After 1990, the data are based on the labour force survey and the population census. Before 1990 the official data comes from the annual administrative data on registered workers from the Administration for Industry and Commerce. To make the series consistent, we apply the correction proposed by Holz (2006), which revises the official employment data before 1990 using the 1982 and 1990 population census. See appendix 13 in Holz (2006) for details.

The official NBS series for sector-specific employment relies on a survey where working status is based on the reference period, which is usually one week before the survey starts. Brandt and Zhu (2010) argue that these data exaggerate the extent of agricultural employment. They document that NBS most likely includes non-agricultural workers in rural private enterprises and rural individual enterprises (i.e., firms employing less than eight employees) in their measure of agricultural workers. We follow Brandt and Zhu (2010) and construct an adjusted series for agricultural employment given by total rural employment minus rural employment in private enterprises and individual enterprises minus employment in Township and Village Enterprises (TVEs). We are interested in both structural change and cyclical fluctuations in labor supply, including agricultural labor. Figure 15 plots the revised Brandt and Zhu (2010) series we use against the official data. The figure shows that the data of Brandt and Zhu (2010) has a steeper downward trend in agricultural employment. However, the cyclical fluctuations in agricultural employment share are similar in the two data series.

A.3 Details of Samples used in Figure 3

For panels a, b, and d in Figure 5 (aggregate employment, GDP, agricultural employment, and nonagricultural employment), the sample comprises 66 countries. The sample time periods for each country are the following,

Figure 15: Agricultural employment in China



Note: he figure plots NBS data on agricultural employment against data data calculated using the methodology of Brandt and Zhu (2010). Shaded areas are times with GDP growth rate below 9.7%.

Countrycode	Start	End	Countrycode	Start	End
ALB	1994	2014	ITA	1977	2015
AUS	1970	2015	JAM	1992	2015
AUT	1983	2015	JPN	1970	2015
AZE	1983	2015	KOR	1970	2015
BEL	1983	2015	LTU	1998	2015
BGR	2000	2015	LUX	1983	2015
BHS	1989	2011	LVA	1996	2015
BRA	1981	2014	MDA	1999	2015
BRB	1981	2015	MEX	1995	2015
CAN	1970	2012	MLT	2000	2015
CHE	1991	2015	MMR	1978	1994
CHL	1975	2015	MYS	1980	2015
CHN	1985	2012	NLD	1987	2015
CRI	1980	2013	NOR	1972	2015
CUB	1995	2014	NZL	1986	2015
CYP	1999	2015	PAK	1973	2008
CZE	1993	2015	PAN	1982	2015
DEU	1983	2015	PHL	1971	2015
DNK	1972	2015	POL	1999	2015
DOM	1996	2015	PRI	1970	2011
EGY	1989	2015	PRT	1974	2015
ESP	1970	2015	PRY	1997	2015
EST	1989	2015	ROU	1997	2015
FIN	1970	2015	RUS	1997	2015
FRA	1970	2015	SLV	1994	2015
GBR	1983	2015	SVK	1994	2015
GRC	1983	2015	SVN	1993	2015
HND	1990	2015	SWE	1970	2015
HUN	1992	2015	THA	1971	2015

For panel c in Figure 5 (productivity gap versus nonagricultural employment), the sample comprises 63 countries. The sample time periods for each country are the following

Countrycode	Start	End	Countrycode	Start	End
ALB	1994	2014	ITA	1990	2015
AUS	1990	2015	JAM	1993	2015
AUT	1983	2015	JPN	1970	2015
AZE	1990	2015	KOR	1970	2015
BEL	1995	2015	LTU	1998	2015
BGR	2000	2015	LUX	1995	2015
BHS	1989	2011	LVA	1996	2015
BRA	1981	2014	MDA	1999	2015
BRB	1990	2014	MEX	1995	2015
CHE	1991	2015	MLT	2000	2015
CHL	1975	2015	MYS	1980	2015
CHN	1985	2012	NLD	1987	2015
CRI	1980	2013	NOR	1972	2015
CUB	1995	2014	NZL	1986	2014
CYP	1999	2015	PAK	1973	2008
CZE	1993	2015	PAN	1982	2015
DEU	1991	2015	PHL	1971	2015
DNK	1972	2015	POL	1999	2015
DOM	1996	2015	PRI	1971	2011
EGY	1989	2015	PRT	1995	2015
ESP	1995	2015	PRY	1997	2015
EST	1995	2015	ROU	1997	2015
FIN	1975	2015	RUS	1997	2015
FRA	1970	2015	SLV	1994	2015
GBR	1990	2015	SVK	1995	2015
GRC	1995	2015	SVN	1995	2015
HND	1990	2015	SWE	1980	2015
HUN	1995	2015	THA	1971	2015
IDN	1987	2015	TTO	1984	2010
IRL	1995	2015	USA	1970	2015
ISL	1997	2015	VEN	1975	2013
			ZAF	2000	2015

For panel d in Figure 5 (relative consumption to output volatility versus nonagricultural employment), the sample comprises 64 countries. The sample time periods for each country are the following,

Countrycode	Start	End	Countrycode	Start	End
BGR	2000	2015	LUX	1983	2015
BHS	1989	2011	LVA	1996	2015
BRA	1981	2014	MDA	1999	2015
CAN	1970	2012	MEX	1995	2015
CHE	1991	2015	MLT	2000	2015
CHL	1975	2015	MYS	1980	2015
CHN	1990	2012	NLD	1987	2015
CRI	1980	2013	NOR	1972	2015
CUB	1995	2014	NZL	1986	2015
CYP	1999	2015	PAK	1986	2008
CZE	1993	2015	PAN	1982	2015
DEU	1983	2015	PHL	1971	2015
DNK	1972	2015	POL	1999	2015
DOM	1996	2015	PRI	1972	2011
EGY	1989	2015	PRT	1974	2015
ESP	1970	2015	PRY	1997	2015
EST	1995	2015	ROU	1997	2015
FIN	1970	2015	RUS	1997	2015
FRA	1970	2015	SLV	1994	2015
GBR	1983	2015	SVK	1994	2015
GRC	1983	2015	SVN	1995	2015
HND	1990	2015	SWE	1970	2015
HUN	1992	2015	THA	1971	2015
IDN	1985	2015	UKR	2001	2015
IRL	1983	2015	USA	1970	2015
ISL	1991	2015	VEN	1975	2013
ISR	1995	2015	ZAF	2000	2015

A.4 Discrete Time Model

In this section we provide a complete description of the discrete time model with endogenous labor supply estimated in Section 4. Our baseline discrete time model adds the following model features to the continuous-time model: (1) endogenous labor supply (2) land as a factor of production in modern-agriculture sector (3) TFP shocks (4) capital stocks in each sector are predetermined.

Time t is discrete, indexed by $0, 1, 2, ...$ Given the initial capital stock in each sector, i.e., $\bar{K}_0 \kappa_0$ and $\bar{K}_0 (1 - \kappa_0)$, and initial TFP levels, $Z_0^i, i = AM, M, S$, the representative household maximizes expected utility:

$$\max E_0 \sum_{t=0}^{\infty} \mu^t \left(\theta \log c_t + (1-\theta) \log (1 - h_t)\right)$$

subject to the budget constraint

$$N_t c_t + K_{t+1} = W_t N_t + (R_t - \delta) K_t + Tr_t$$

where $K_t = K_t^M + K_t^{AM}, N_t = N_t^M + N_t^{AM} + N_t^S$. W_t denotes the after-tax equilibrium wage. $Tr_t = \tau W_t^M h_t N_t^M$ denotes the lump-sum transfer from the government to the representative household. Note that μ denotes the discount factor.

The production side is identical to the model in the text, except that the production of modern agriculture has been modified to include land

$$Y_t^{AM} = \left(K_t^{AM}\right)^{1-\beta-\beta_T} \left(Z_t^{AM} H_t^{AM}\right)^{\beta}$$

where the land income share is denoted by $\beta_T \geq 0$. We assume $\beta + \beta_T < 1$.

As explained in Section 3.2, we can exploit the equivalence between the competitive equilibrium and the distorted social planner problem, and write the Lagrangian as:

$$L = E_0 \sum_{t=0}^{\infty} \mu^t \left\{ \begin{array}{c} \theta \log c_t + (1-\theta) \log (1 - h_t) + \\ \xi_t \left[\begin{array}{c} Y_t + (1-\delta) K_t - c_t N_t - K_{t+1} \\ -\tau W_t^M H_t^M + Tr_t \end{array} \right] \end{array} \right\}$$

where we use the notation $\chi_t, \kappa_t, \nu_t^M, \nu_t^{AM}, \nu_t^S, \upsilon_t$ introduced in the main text, but we modify the notations of η_t and introduce $\tilde{\eta}_t$

$$\eta_t \equiv \left[\gamma \left(\frac{Y_t^G}{Y_t^M}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\tilde{\eta}_t \equiv \left[\gamma + (1-\gamma) \left(\frac{Y_t^M}{Y_t^G}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Therefore, by definition $Y_t = Y_t^M \eta_t$ and $Y_t = Y_t^G \tilde{\eta}_t$. Recall that $H_t^i \equiv h_t N_t^i$.

The FOC with respect to ν_t^{AM} and ν_t^S are as in the continuous time problem.

$$\begin{aligned}
\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}\upsilon_t\beta\frac{1}{\nu_t^A} &= (1-\tau)(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\alpha\frac{1}{\nu_t^M}\\
\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}(1-\upsilon_t)\frac{1}{\nu_t^S} &= (1-\tau)(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\alpha\frac{1}{\nu_t^M}
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
\frac{\nu_t^{AM}}{\nu_t^M} &= \frac{\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}\upsilon_t\beta}{(1-\tau)(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\alpha}\\
\frac{\nu_t^S}{\nu_t^M} &= \frac{\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}(1-\upsilon_t)}{(1-\tau)(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\alpha}
\end{aligned}$$

sum up together to have the expenditure ratio agr./non-agr. as

$$\frac{\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}}{(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}} = \frac{1-\nu_t^M}{\nu_t^M}\frac{(1-\tau)\alpha}{(1-\upsilon_t)+\upsilon_t\beta}$$

and express ν_t^M as

$$\nu_t^M = \left(1+\frac{\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}}{(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}}\frac{(1-\upsilon_t)+\upsilon_t\beta}{(1-\tau)\alpha}\right)^{-1}$$

Those with respect to c_t and h_t yield, respectively:

$$\theta\frac{1}{c_t} = \xi_t N_t, \tag{54}$$

$$\frac{1-\theta}{1-h_t} = \xi_t Y_t^{\frac{1}{\varepsilon}}\left[\begin{array}{c}\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}\left(\upsilon_t\frac{\beta}{h_t}+(1-\upsilon_t)\frac{1}{h_t}\right)\\ +(1-\tau)(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\frac{\alpha}{h_t}\end{array}\right]. \tag{55}$$

Substituting the FOCs with respect to ν_t^{AM} and ν_t^S into (55) yields

$$\frac{1-\theta}{1-h_t} = \xi_t Y_t^{\frac{1}{\varepsilon}}(1-\tau)\alpha(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\frac{1}{h_t\nu_t^M}. \tag{56}$$

Combining (56) with (54) yields

$$\frac{1-\theta}{\theta}\frac{c_t}{1-h_t} = (1-\tau)Y_t^{\frac{1}{\varepsilon}}\alpha(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\frac{1}{\nu_t^M h_t N_t}.$$

The FOC w.r.t. κ_{t+1} and K_{t+1} yield, respectively (after combining the two equations and rearranging terms):

$$\xi_t = E_t\left[\mu\xi_{t+1}\left(Y_{t+1}^{\frac{1}{\varepsilon}}\gamma\left(Y_{t+1}^{G}\right)^{-\frac{1}{\varepsilon}}\left(\upsilon_{t+1}\right)^{-\frac{1}{\omega-1}}\left(1-\beta-\beta_T\right)\frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}}+1-\delta\right)\right] \tag{57}$$

$$\xi_t = \mu E_t\left[\xi_{t+1}\left(Y_{t+1}^{\frac{1}{\varepsilon}}\frac{1}{K_{t+1}}\frac{1}{\kappa_{t+1}}\left(1-\gamma\right)\left(Y_{t+1}^{M}\right)^{1-\frac{1}{\varepsilon}}\left(1-\alpha\right)+1-\delta\right)\right]. \tag{58}$$

Substitute $Y_t = \eta_t Y_t^M$ and $Y_t = Y_t^G\tilde{\eta}_t$ Substituting in this into (57) and (58) respectively to eliminate Y yields

$$\xi_t = \mu E_t\left[\xi_{t+1}\left(\gamma\tilde{\eta}_{t+1}^{\frac{1}{\varepsilon}}\left(\upsilon_{t+1}\right)^{-\frac{1}{\omega-1}}\left(1-\beta-\beta_T\right)\frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}}+1-\delta\right)\right] \tag{59}$$

$$\xi_t = \mu E_t\left[\xi_{t+1}\left(\eta_{t+1}^{\frac{1}{\varepsilon}}\frac{Y_{t+1}^{M}}{K_{t+1}^{M}}\left(1-\gamma\right)\left(1-\alpha\right)+1-\delta\right)\right]. \tag{60}$$

Eq. (54), 59), and (60) yields the standard Euler equations for consumption:

$$1 = \frac{\mu}{1+n}E_t\left\{\frac{c_t}{c_{t+1}}\left[\begin{array}{c}\gamma\tilde{\eta}_{t+1}^{\frac{1}{\varepsilon}}\left(\upsilon_{t+1}\right)^{-\frac{1}{\omega-1}}\left(1-\beta-\beta_T\right)\left(\left(1-\kappa_{t+1}\right)\chi_{t+1}\right)^{-\beta-\beta_T}\\ \times\left(Z_{t+1}^{AM}h_{t+1}\nu_{t+1}^{AM}\right)^{\beta}N_{t+1}^{-\beta_T}+1-\delta\end{array}\right]\right\}$$

$$1 = \frac{\mu}{1+n}E_t\left\{\frac{c_t}{c_{t+1}}\left[1+\eta_{t+1}^{\frac{1}{\varepsilon}}\left(1-\gamma\right)\left(1-\alpha\right)\left(\frac{Z_{t+1}^{M}h_{t+1}\nu_{t+1}^{M}}{\kappa_{t+1}\chi_{t+1}}\right)^{\alpha}-\delta\right]\right\}$$

We can simplify the intertemporal condition:

$$\frac{1-\theta}{\theta}\frac{c_t}{1-h_t} = \left(1-\tau\right)\eta_t^{\frac{1}{\varepsilon}}\alpha\left(1-\gamma\right)\left(\frac{Z_t^M h_t\nu_t^M}{\kappa_t\chi_t}\right)^{\alpha-1}$$

The resource constraint becomes

$$\chi_{t+1}\left(1+n\right) = \eta_t\left(\kappa_t\right)^{1-\alpha}\left(Z_t^M\nu_t^M h_t\right)^{\alpha}\chi_t^{1-\alpha}+\left(1-\delta\right)\chi_t - c_t$$

A.5 Algorithm to Solve the Rational Expectation Equilibrium

We can rewrite the model in discrete time recursively. Denote the state space as $\Theta_t \equiv \left(\hat{\chi}_t, \kappa_t, z_t^M, z_t^{AM}, z_t^S, t\right)$. The Bellman Equation (during the structural change transition) is given by

$$V\left(\Theta_t\right) = \max_{\hat{c}_t, h_t, \kappa_{t+1}, \hat{\chi}_{t+1}, \upsilon_t, \nu_t^M}\left\{u\left(\hat{c}_t, h_t\right)+\mu E_t V'\left(\Theta_{t+1}\right)\right\}$$

where use the following notations to detrend the variables

$$\hat{\chi}_t = \frac{\chi_t}{\Lambda_t^M}, \hat{c}_t = \frac{c_t}{\Lambda_t^M}, z_t^i = \frac{Z_t^i}{\Lambda_t^i}, i = AM, M, S$$

and

$$\Lambda_t^i = Z_0^i \Pi_{k=1}^t \left(1 + g_k^i\right), i = AM, M, S$$

We solve the Bellman Equation using its detrended first order conditions and budget constraint.

In each period, we have 6 unknown policy function to solve

$$\hat{c}_t(\Theta_t), h_t(\Theta_t), \kappa_{t+1}(\Theta_t), \hat{\chi}_{t+1}(\Theta_t), \upsilon_t(\Theta_t), \nu_t^M(\Theta_t).$$

We can reduce the number of unknown policy function further by expressing $\hat{c}_t$ and $\hat{\chi}_{t+1}$ in terms of other state variables and decision variables.

$$\begin{aligned}
\hat{c}_t &= (1-\tau)\,\eta_t^{\frac{1}{\varepsilon}}(1-\gamma)\,\alpha\left(\kappa_t\hat{\chi}_t\right)^{1-\alpha}\left(z_t^M h_t \nu_t^M\right)^{\alpha-1}(1-h_t)\frac{\theta}{1-\theta} \\
\hat{\chi}_{t+1} &= \frac{\eta_t\left(\kappa_t\right)^{1-\alpha}\left(z_t^M\nu_t^M h_t\right)^{\alpha}\left(\hat{\chi}_t\right)^{1-\alpha} + (1-\delta)\,\hat{\chi}_t - \hat{c}_t}{(1+n)\left(1+g_{t+1}^M\right)}
\end{aligned}$$

where

$$\begin{aligned}
\eta_t &= \left[\gamma\left(\frac{Y_t^G}{Y_t^M}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
\frac{Y_t^G}{Y_t^M} &= \frac{\left(z_t^{AM}\Lambda_t^{AM}h_t\frac{\nu_t^{AM}}{\nu_t^M}\nu_t^M N_t\right)^{\beta}\left((1-\kappa_t)\,\hat{\chi}_t\Lambda_t^M N_t\right)^{1-\beta-\beta_T}}{\left(z_t^M\Lambda_t^{AM}h_t\nu_t^M N_t\right)^{\alpha}\left(\kappa_t\hat{\chi}_t\Lambda_t^M N_t\right)^{1-\alpha}}\left(\upsilon_t\right)^{-\frac{\omega}{\omega-1}} \\
\frac{\nu_t^{AM}}{\nu_t^M} &= \frac{1-\nu_t^M}{\nu_t^M}\frac{(1-\tau)\,\alpha}{(1-\upsilon_t)+\upsilon_t\beta}\frac{\upsilon_t\beta}{(1-\tau)\,\alpha}
\end{aligned}$$

Therefore, we are left with four unknowns $h_t(\Theta_t), \kappa_{t+1}(\Theta_t), \nu_t^M(\Theta_t), \upsilon_t(\Theta_t)$. We need to solve four nonlinear equations as well.

The first two are the detrended the Euler Equations

$$\hat{c}_t^{-1} = \frac{\mu}{(1+n)\left(1+g_{t+1}^M\right)}E_t\left\{\hat{c}_{t+1}^{-1}\times\left[\eta_{t+1}^{\frac{1}{\varepsilon}}\left(\frac{z_{t+1}^M\nu_{t+1}^M h_{t+1}}{\kappa_{t+1}\hat{\chi}_{t+1}}\right)^{\alpha}(1-\gamma)(1-\alpha)+(1-\delta)\right]\right\} \quad (61)$$

$$\hat{c}_t^{-1} = \frac{\mu}{(1+n)\left(1+g_{t+1}^M\right)}E_t\left[\hat{c}_{t+1}^{-1}\left(\gamma\tilde{\eta}_{t+1}^{\frac{1}{\varepsilon}}\left(\upsilon_{t+1}\right)^{-\frac{1}{\omega-1}}(1-\beta-\beta_T)\frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}}+1-\delta\right)\right] \quad (62)$$

where

$$\frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}} = \left((1-\kappa_{t+1})\,\hat{\chi}_{t+1}\Lambda_{t+1}^M\right)^{-\beta-\beta_T}\left(z_{t+1}^{AM}\Lambda_{t+1}^{AM}\lambda_{t+1}^{AM}h_{t+1}\right)^{\alpha_A}N_{t+1}^{-\beta_T}$$

and

$$\frac{Y_{t+1}^G}{Y_{t+1}^M} = \frac{\left(z_{t+1}^{AM}\Lambda_{t+1}^{AM}h_{t+1}\nu_{t+1}^{AM}N_{t+1}\right)^{\beta}\left((1-\kappa_{t+1})\,\hat{\chi}_{t+1}\Lambda_{t+1}^M N_{t+1}\right)^{1-\beta-\beta_T}}{\left(z_{t+1}^M\Lambda_{t+1}^{AM}h_{t+1}\nu_{t+1}^M N_{t+1}\right)^{\alpha}\left(\kappa_{t+1}\hat{\chi}_{t+1}\Lambda_{t+1}^M N_{t+1}\right)^{1-\alpha}}\left(\upsilon_{t+1}\right)^{-\frac{\omega}{\omega-1}}$$

The other two are the equations governing υ_t and υ_t^M.

$$\upsilon_t = \frac{\left(Y_t^{AM}\right)^{\frac{\omega-1}{\omega}}}{\left(Y_t^{AM}\right)^{\frac{\omega-1}{\omega}} + \left(Y_t^{S}\right)^{\frac{\omega-1}{\omega}}} \tag{63}$$

$$\nu_t^M = \left(1 + \frac{\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}}{(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}} \frac{(1-\upsilon_t)+\upsilon_t\beta}{(1-\tau)\,\alpha}\right)^{-1} \tag{64}$$

where

$$\begin{aligned}
Y_t^{AM} &= (1-\kappa_t)^{1-\beta-\beta_T}\left(\hat{\chi}_t\Lambda_t^M\right)^{1-\beta-\beta_T}\left(\frac{\nu_t^{AM}}{\nu_t^M}\right)^{\beta}\left(z_t^{AM}\Lambda_t^{AM}\nu_t^M\right)^{\beta} h_t^{\beta} N_t^{-\beta_T} \\
Y_t^S &= z_t^S\nu_t^M\frac{\nu_t^S}{\nu_t^M}h_t\Lambda_t^S N_t \\
\frac{\nu_t^{AM}}{\nu_t^M} &= \frac{1-\nu_t^M}{\nu_t^M}\frac{(1-\tau)\,\alpha}{(1-\upsilon_t)+\upsilon_t\beta}\frac{\upsilon_t\beta}{(1-\tau)\,\alpha} \\
\frac{\nu_t^S}{\nu_t^M} &= \frac{1-\nu_t^M}{\nu_t^M}\frac{(1-\tau)\,\alpha}{(1-\upsilon_t)+\upsilon_t\beta}\frac{(1-\upsilon_t)}{(1-\tau)\,\alpha}
\end{aligned}$$

Summary 1 *We solve the Bellman Equation backwards from the last period, which corresponds to the long-run approximate balanced growth path with (approximately) only one sector, that is, the non-agr. sector.*

1. *We choose to number of transition period to be a large number ($T = 250$). In practice, we can increases the number until the beginning transition period we are interested in are no longer affected by the choice of T. We can also check whether the economy will converge to the long-run ABGP within the period T.*

2. *We discretize the state space. In the deterministic case, we choose the state space for $\hat{\chi}$ as $[0.5\hat{\chi}_0, 1.5\hat{\chi}^*]$ and choose the state space for κ as $[0.5, 1]$. We discretize both $\hat{\chi}$ and κ using 250 equally spaced grid points; In the stochastic case, we choose the state space for $\hat{\chi}$ as $[0.9\hat{\chi}_t^*, 1.1\hat{\chi}_t^*]$ and choose state space for κ as $[\kappa_t^* - 0.025, \kappa_t^* + 0.025]$. We discretize both $\hat{\chi}$ and κ using 75 equally spaced grid points, where $\hat{\chi}_t^*$ and κ_t^* are the realized path in the deterministic model. We further discretize the joint process for the three types of shocks using 27 grid points using Tauchen's method Tauchen (1986).*

3. *We solve the transitional path backwards.*

   (a) *In the last period, the economy is almost identical to a one-sector RBC model. Therefore, we set*

$$\begin{aligned}
\kappa_{T+1}(\Theta_T) &= 1, \upsilon_T(\Theta_T) = 1 \\
\nu_T^M(\Theta_T) &= 1, \nu_T^{AM}(\Theta_T) = 0, \nu_T^S(\Theta_T) = 0
\end{aligned}$$

We solve the Bellman Equation using value function iteration, with linear interpolation between grid points. We can solve for the rest of the policy functions

$$c_T(\Theta_T), h_T(\Theta_T), \hat{\chi}_{T+1}(\Theta_T)$$

(b) *From period* $t = T - 1$ *to* 1, *we solve the nonlinear system of eq. (61), (62), (63), and (64) for the policy functions* $\kappa_{t+1}(\Theta_t), \upsilon_t(\Theta_t), \nu_t^M(\Theta_t), h_t(\Theta_t)$. *In each period, we first express* $\hat{c}_t(\Theta_t), \hat{\chi}_{t+1}(\Theta_t)$ *in terms of other state variables and decision variables*

A.6 Measuring sector-specific TFP levels: theory

We observe $\{Y, Y^M, K^M, N^M, Y^G, K^G, N^G\}$ but we do not have direct observations of allocations of labor and output across the two agricultural technologies (we also presume that we have already estimated all relevant parameters). We describe the procedure to estimate the three-shock process

To measure the sector-specific TFP levels we impose three equilibrium conditions: (i) marginal return to capital are equated across manufacturing and modern agriculture, and (ii) marginal return to labor is equated across the two agricultural technologies; and (iii) hours per worker h is equalized across sectors. In addition we assume that assume that h is constant over time so that employment is a sufficient statistic for measuring labor input.

Recall the definitions of sector-specific outputs;

$$\begin{aligned} Y^G &= \left[\left(Y^{AM}\right)^{\frac{\omega-1}{\omega}} + \left(Y^S\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \\ Y^M &= \left(K^M\right)^{1-\alpha_M} \times \left(Z^M H^M\right)^{\alpha_M} \\ Y^{AM} &= \left(K^{AM}\right)^{\beta_{AM}} \times \left(Z^{AM} H^{AM}\right)^{\alpha_{AM}} \\ Y^S &= Z^S H^S \end{aligned}$$

We express aggregate output using current prices,[34] $Y = P^G Y^G + P^M Y^M$.

[34] The advantage of focusing on aggregate output in terms of current prices instead of specifying the production function is that while a subsistence level in agricultural consumption will be subsumed in the relative prices, it will not affect the equations determining how TFP levels are measured.

The marginal products of sector-specific capital and labor are

$$\begin{aligned}
\frac{\partial Y}{\partial K_M} &= P^M \frac{\partial Y_M}{\partial K_M} = P^M \left(1-\alpha_M\right) \left(\frac{K^M}{H^M}\right)^{-\alpha_M} \times \left(Z^M\right)^{\alpha_M} \\
\frac{\partial Y}{\partial K_G} &= P^G \frac{\partial Y_G}{\partial Y_{AM}} \frac{\partial Y_{AM}}{\partial K_G} = P^G \left(\frac{Y^{AM}}{Y}\right)^{-\frac{1}{\omega}} \beta_{AM} \left(K^G\right)^{\beta_{AM}-1} \times \left(Z^{AM} H^{AM}\right)^{\alpha_{AM}} \\
\frac{\partial Y}{\partial N_S} &= P^G \frac{\partial Y_G}{\partial Y_S} \frac{\partial Y_S}{\partial N_S} = P^G \left(\frac{Y^S}{Y^G}\right)^{-\frac{1}{\omega}} Z^S h \\
\frac{\partial Y}{\partial N_{AM}} &= P^G \frac{\partial Y_G}{\partial Y_{AM}} \frac{\partial Y_{AM}}{\partial N_{AM}} = P^G \left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}} \alpha_{AM} \left(K^G\right)^{\beta_{AM}} \times \left(H^{AM}\right)^{\alpha_{AM}-1} \left(Z^{AM}\right)^{\alpha_{AM}} h
\end{aligned}$$

We now proceed to measuring the TFP levels in five steps:

1. The marginal product of capital is the same in manufacturing and modern agriculture,

$$\begin{aligned}
P^M \left(1-\alpha_M\right) \left(K^M\right)^{-\alpha_M} \times \left(Z^M H^M\right)^{\alpha_M} &= \left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}} P^G \beta_{AM} \left(K^G\right)^{\beta_{AM}-1} \times \left(Z^{AM} H^{AM}\right)^{\alpha_{AM}} \\
&\Rightarrow
\end{aligned}$$

$$\left(\frac{Y^{AM}}{Y^G}\right)^{1-\frac{1}{\omega}} = \frac{K^G}{K^M} \frac{P^M Y^M}{P^G Y^G} \frac{1-\alpha_M}{\beta_{AM}} \tag{65}$$

The variables on the right-hand side of eq. (65) are observable. This identifies the ratio Y^{AM}/Y^G.

2. Use the agricultural production function (3) to derive a relationship between the ratios Y^S/Y^G and Y^{AM}/Y^G,

$$1 = \left(\frac{Y^{AM}}{Y^G}\right)^{\frac{\omega-1}{\omega}} + \left(\frac{Y^S}{Y^G}\right)^{\frac{\omega-1}{\omega}} \tag{66}$$

Given the imputed ratio Y^{AM}/Y^G, this equation identifies the ratio Y^S/Y^G.

3. The marginal product of labor is equated across the two agricultural sectors. This implies

$$\begin{aligned}
P^G \left(\frac{Y^S}{Y^G}\right)^{-\frac{1}{\omega}} Z^S h &= P^G \left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}} \alpha_{AM} \left(K^G\right)^{\beta_{AM}} \times \left(H^{AM}\right)^{\alpha_{AM}-1} \left(Z^{AM}\right)^{\alpha_{AM}} h \\
&\Rightarrow
\end{aligned}$$

$$\frac{H^{AM}}{H^S} = \alpha_{AM} \left(\frac{Y^{AM}}{Y^G} \frac{Y^G}{Y^S}\right)^{1-\frac{1}{\omega}} \tag{67}$$

4. Use the accounting identity $N^G = N^{AM} + N^S$ to identify N^{AM} and N^S;

$$\frac{H^{AM}}{H^S} = \frac{N^{AM}}{N^S} = \frac{N^{AM}}{N^G - N^{AM}} \Rightarrow$$

$$
\begin{aligned}
N^{AM} &= \frac{N^G}{1+\left(\frac{H^{AM}}{H^S}\right)^{-1}} && (68) \\
N^S &= N^G - N^{AM} && (69)
\end{aligned}
$$

5. Normalize hours per worker to $h = 1$ for all periods (so aggregate hours equals employment). Eq. (65)-(69) then allows us to identify Z^M, Z^S, and Z^{AM};

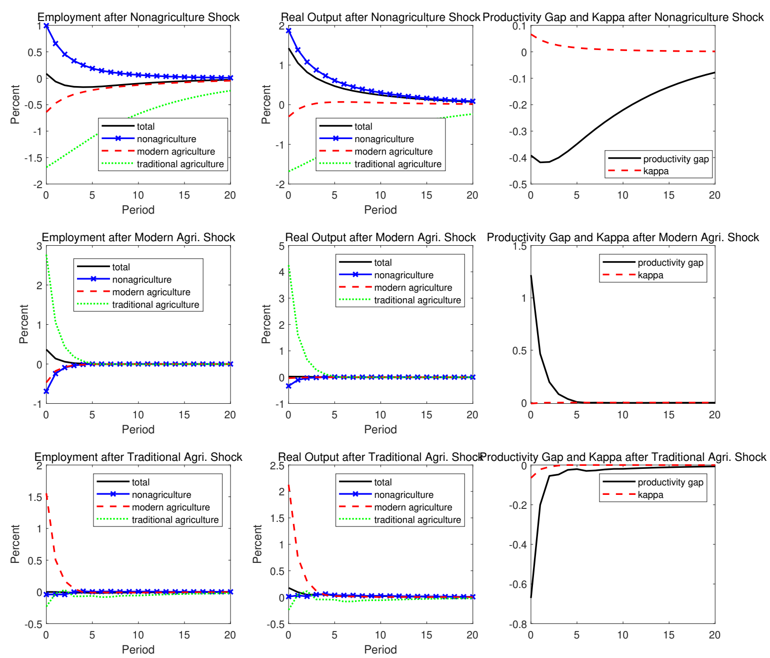
$$
\begin{aligned}
\ln\left(Z^S\right) &= \ln\left(Y^S\right) - \ln\left(N^S\right) && (70) \\
\ln\left(Z^{AM}\right) &= \frac{1}{\alpha_{AM}} \ln\left(Y^{AM}\right) - \frac{\beta_{AM}}{\alpha_{AM}} \ln\left(K^{AM}\right) - \ln\left(N^{AM}\right) && (71)
\end{aligned}
$$

and finally TFP in manufacturing;

$$
\ln\left(Z^M\right) = \frac{1}{\alpha_M} \ln\left(Y^M\right) - \frac{1-\alpha_M}{\alpha_M} \ln\left(K^M\right) - \ln\left(N^M\right) \quad (72)
$$

A.7 Additional Figures

Figure 16: Impulse Responses in year 2000



Note: The graphs show impulse response as percentage deviation from the deterministic path in year 2000. The three top panels show responses to a one-standard deviation change in nonagricultural TFP (Z^M). The three middle panels show responses to modern agricultural TFP (Z^{AM}). The three bottom panels show responses to traditional agricultural TFP (Z^S).