Should monetary policy care about redistribution? Optimal fiscal and monetary policy with heterogeneous agents

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Introduction

- Monetary policy generates redistributive effects through various channels. (from Friedman 1968; Bewley, 1980; Kehoe Levine Woodford 1992; Erosa and Ventura 2002; Bilbiie 2008; Algan Ragot 2010; Kaplan, Moll, and Violante 2018; Auclert 2019, among many others).
- Research question: Should monetary policy care about redistribution or only focus on monetary objectives?
- Our strategy:
  - Derive optimal fiscal and monetary policy in a HANK model (het-agent economy with aggregate shocks and nominal rigidities).
  - Fiscal policy: distorting labor and capital taxes, lump-sum transfer and one-period riskless public debt.
  - Monetary policy: New-Keynesian setup.

Results' preview

- **Theoretical results.**

  Irrelevance result: No redistributive role for monetary policy when capital and labor taxes are available with commitment (both timeless and time-0 perspective).

- **Quantitative results.**

  1. Monetary policy (timeless perspective; McCallum and Nelson 2000; Woodford, 2003): Even with incomplete fiscal tools (no optimal capital tax), inflation has little role to play for redistribution.
  2. Monetary policy (time-0 perspective): Inflation is an imperfect substitute to missing capital tax.
  3. Fiscal policy (both perspectives): Public debt is more countercyclical and capital tax much less volatile than in the standard complete market setup.

How? Truncating individual histories

Computing Ramsey policies in HANK is challenging. We do so thanks to a methodological contribution (Le Grand and Ragot, 2021).

- We use the Lagrangian approach of Marcet and Marimon (2019).
- We consider a "truncated representation" of idiosyncratic histories.

Allows for simple and accurate quantitative investigation. See Le Grand and Ragot (2022), for results.

Literature (Selected)

- Monetary policy in het-agent economies with nominal frictions: Bilbiie, 2008; McKay, Nakamura, and Steinsson (2016), Gornemann, Kuester, and Nakajima (2017), Kaplan, Moll, and Violante (2018), Nuño and Moll (2018), Auclert (2019).
- Interactions between monetary and fiscal policies (no idiosyncratic risk): Chari and Kehoe (1999), Aiyagari, Marcet, Sargent, and Seppälä (2002), Bhandari, Evans, Golosov, and Sargent (2017). Similar equivalence results in **Correia, Nicolini, and Teles (2008)** and Correia, Farhi, Nicolini, and Teles (2013), Bilbiie, Monacelli, Perotti (2020).
- Computing optimal Policy HA : Nuno and Thomas (2020), Dyrda and Pedroni (2021), Bhandari, Evans, Golosov, and Sargent (2021), McKay and Wolf (2022).
- Closest papers: **Bhandari, Evans, Golosov, and Sargent (2021), Acharya, Challe, and Dogra (2020), Nuno and Thomas (2020)**. Slightly different conclusions (more on this later).

Outline of the presentation

1. The environment
2. Optimal fiscal-monetary policy
3. Numerical simulations: Timeless
4. Numerical simulations: Time-0 problem

1 - The environment

- Aggregate state Z_t, affects TFP or public consumption. First-order Markov process.
- Unit mass of agents facing uninsurable productivity risk.
- Productivity levels $y \in \mathcal{Y}$: constant discrete first-order Markov process with transition matrix $\Pi_{y\tilde{y}}$. Constant share S_y.
- Credit constraint $a_t \geq -\bar{a}$
- GHH utility function over consumption and labor supply (works with more general utility functions):

$$U(c,l) = u\left(c - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}\right).$$

Production

Standard NK production sector with capital.

- Rotemberg cost: $\frac{\kappa}{2}\left(\frac{p_t(j)}{p_{t-1}(j)}-1\right)^2 Y_t$ + production subsidy
- Phillips curve:

$$\Pi_t(\Pi_t-1)=\frac{\varepsilon-1}{\kappa}(\zeta_t-1)+\beta\mathbb{E}_t\Pi_{t+1}(\Pi_{t+1}-1)\frac{Y_{t+1}}{Y_t}\frac{M_{t+1}}{M_t}$$

with marginal cost

$$\zeta_t=\frac{1}{Z_t}\left(\frac{\tilde{r}_t^K+\delta}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha}.$$

Asset markets

- Risk-neutral fund collects capital and public debt and issues claims to agents with rate $\tilde{r}_t$ (and borrowing limits). (as Gornemann et al. 2012)

  $\rightarrow$ no actual portfolio choice by agents.

- Funds hold two assets:
  - Capital stock with pre-tax tax net rate $\tilde{r}_t^K$.
  - Public debt with nominal pre-tax gross rate $\tilde{R}_{t-1}^{B,N}$ that is predetermined.

Government

Has to finance exogenous G_t and transfers T_t, with fiscal tools:

- Distorting taxes on capital and labor (τ_t^K and τ_t^L).
- Public debt issuance B_t.
- Firms' profits are fully taxed.

Affine tax system : Not so bad (Dyrda and Perdoni, 2021, Heathcote and Tsujiyama, 2021). Governmental budget constraint

$$G_t + \frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} B_{t-1} + T_t \leq \tau_t^L \tilde{w}_t L_t + \tau_t^K \left(\tilde{r}_t^K K_{t-1} + \left(\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1 \right) B_{t-1} \right) + \left(1 - \zeta_t - \frac{\kappa}{2}\pi_t^2\right) Y_t + B_t.$$

2 - Optimal fiscal-monetary policy

- Aggregate welfare criterion: $\sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U(c_t^i, l_t^i) \ell(di)$, with weights $\omega_t^i = \omega_t(a_0, y^{i,t})$.
- Consistently, pricing kernel $M_t = \int_i \omega_t^i U_c(c_t^i, l_t^i) \ell(di)$.

Ramsey program: Find $\tau_t^L, \tau_t^K, T_t, B_t, \tilde{R}_t^{B,N}, \pi_t$ for $t \geq 0$ that maximize aggregate welfare among competitive equilibria. I.e. maximize aggregate welfare subject to:

- individual and governmental budget constraints,
- individual Euler equations,
- Phillips curve, fund conditions
- market clearing conditions,
- factor price relationships. See program

The Lagrangian approach

- Rich framework for normative questions.

→ Rely on Lagrangian approach of Marcet and Marimon (2019), adjusted in LeGrand and Ragot (2021)

- Enables the derivation of first-order conditions. Useful for:
  - Interpretation and understanding of mechanisms.
  - Simulations of solutions.
- Two Lagrange multipliers of interest:
  - λ_t^i: on agents' Euler equation between t and $t+1$.
  - μ_t: on governmental budget constraint.

See here

Nominal economy with all fiscal tools

An irrelevance result

With capital, labor tax and public debt, the planner implements $\Pi = 1$ and there is no further role for monetary policy.

Intuition. There are sufficient independent instruments to set the mark-up wedge of firms to $\zeta_t = 1$ (as in the real case). Inflation variations are then only costly and $\Pi_t = 1$. Valid for both timeless and time-0 perspectives. Spirit of Correia, Nicolini, and Teles (2008)

Very general:

- General utility function
- Portfolio choice (as many taxes as assets)
- Different shocks (cost-push shocks...)
- Steady-state may not exist

Question. What happens with missing fiscal instruments?

2 - Optimal fiscal-monetary policy

Steps

1. Consider a steady state (Bewley model) with a realistic fiscal system.
2. Assume that the actual fiscal system is optimal.

   ⇒ From allocations to Pareto weights. Invert Optimal approach Heathcote and Tsujiyama, 2021.
3. Analyze the optimal dynamics with aggregate shocks (Ramsey allocation) around the steady state.

   ⇒ From Pareto weights to allocations in the dynamics.
4. Simulate the dynamics with a timeless or a time-0 perspective.

To do so, we use a **truncated representation**. Only the idiosyncratic history over the last N periods matters (N is the truncation length). here

3 - Numerical simulations: Timeless

	Preference and technology (quarterly)	
β	Discount factor	0.99
α	Capital share	0.36
δ	Depreciation rate	0.025
$\bar{a}$	Credit limit	0
χ	Scaling param. labor supply	0.068
φ	Frisch elasticity labor supply	0.5
	Shock process	
ρ_z	Autocorrelation TFP	0.95
σ_z	Standard deviation TFP shock	0.31%
ρ_y	Autocorrelation idio. income	0.99
σ_y	Standard dev. idio. income	14%
n_y	Numb. idio. states, from Rouwenhorst	5
	Tax system	
τ^K	Capital tax	36%
τ^L	Labor tax	28%
T	Transfer over GDP	8%
B/Y	Public debt over yearly GDP	60%
G/Y	Public spending over yearly GDP	12.4%
	Monetary parameters	
κ	Price adjustment cost	100
ε	Elasticity of sub.	6

Distribution

	Data		Model
Wealth statistics	PSID, 06	SCF, 07	
Q1	-0.9	-0.2	0.0
Q2	0.8	1.2	0.1
Q3	4.4	4.6	3.5
Q4	13.0	11.9	15.1
Q5	82.7	82.5	81.3
Top 5%	36.5	36.4	37.8
Top 1%	30.9	33.5	10.7
Gini	0.77	0.78	0.77

Truncated model calibration

Truncation length. $N = 5 \;\rightarrow\; 5^5 = 3125$ truncated histories.

Simulations

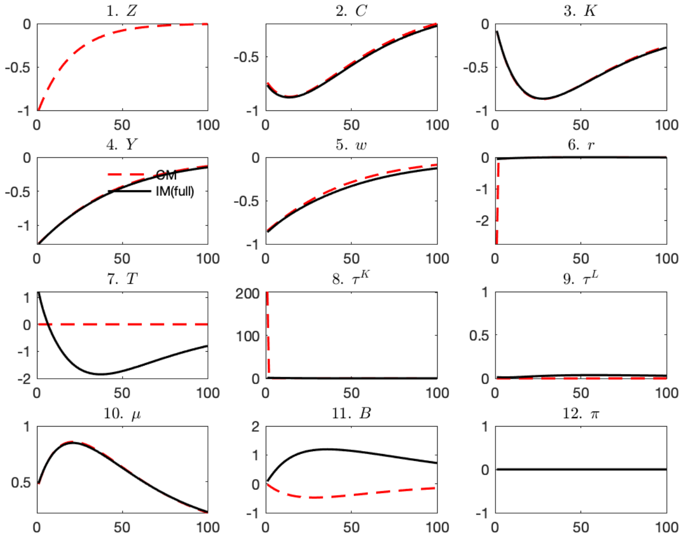
Dynamics: Timeless perspective

Steady-state + planner cannot renege on past commitments (Woodford 1999; McCallum and Nelson, 2000): closest to a rule.

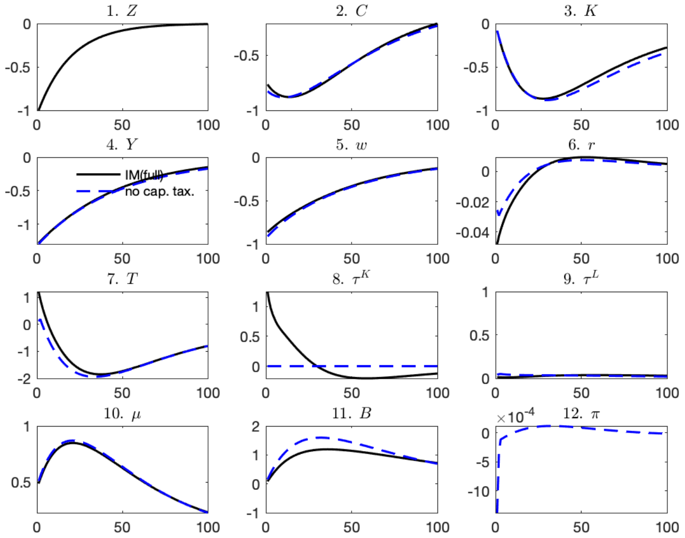
We compare several cases:

1. Complete market with a representative agent.
2. Incomplete market with full set of fiscal instruments. Our setup with monetary irrelevance.
3. Incomplete market with missing capital tax. Possible role for monetary policy.

Full set incomplete markets vs. complete markets



Missing capital tax vs. full set (incomplete markets)



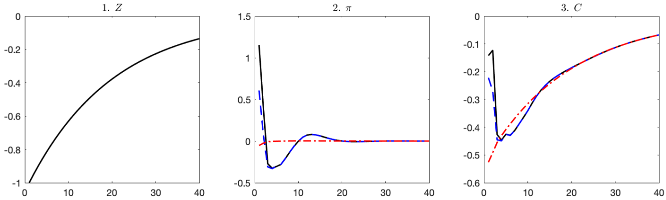
Second-order moments

		CM	Full	No cap.tax
C	Mean	0.754	0.754	0.754
	Std	0.026	0.027	0.027
K	Mean	11.056	11.054	11.054
	Std	0.027	0.027	0.029
Y	Mean	1.176	1.176	1.176
	Std	0.026	0.027	0.027
τ^K	Mean	0.000	0.360	0.360
	Std	0.885	0.014	0.000
τ^L	Mean	0.000	0.280	0.280
	Std	0.000	0.002	0.001
B	Mean	-10.933	2.843	2.843
	Std	0.015	0.046	0.054
T	Mean	0.000	0.094	0.094
	Std	0.000	0.061	0.064
π	Mean	0.000	0.000	0.000
	Std	0.000	0.000	0.001
$corr(\tau^K, Y)$		-0.208	-0.487	0.000
$corr(\tau^L, Y)$		0.927	-0.637	-0.925
$corr(B, Y)$		-0.835	-0.759	-0.829
$corr(Y, Y_{-1})$		0.978	0.978	0.978
$corr(B, B_{-1})$		0.999	0.999	0.999

4 - Numerical simulations: Time-0 problem

Model and other results

- Model without capital, to compare other papers (no fund)
- We set initial Lagrange multipliers to 0, when the shock hits (as Bhandari, et al. 2020; Asharya, Challe, and Dogra, 2020; Nuno and Thomas, 2020): Planner reoptimizes.
- **Our equivalence result holds in all periods** (also for a time-0 problem): To have inflation dynamics, we must have τ^K not optimal.



Inflation used as a missing capital tax (Dyrda and Pedroni, 2018).

Robustness checks

- Validation of the truncation method:

  Comparison with other methods (Reiter and Boppart et al.) Simulations

- Modification of the pricing kernel / of parameters for nominal rigidities.
- Public spending shock.
- Fixed labor tax vs. fixed labor and capital taxes.

Other results

Conclusion

- **Take-away**
  - **Irrelevance result** for the monetary policy when the full set of fiscal tools is available. Confirmed in simulations.
  - **Incomplete markets matter.** Public debt is actual debt and capital tax volatility is reduced by two orders of magnitude.
  - **Importance of public debt.** Even in absence of capital tax, shock is mostly smoothed out by public debt.
  - **Monetary policy.** In absence of capital tax, monetary policy plays a role only for a **time-0 problem**.
- **Work in Progress**
  - Optimal Monetary Policy with sub-optimal fiscal policy
  - Separable Utility Function
  - Utilitarian Welfare Function

Ramsey program formulation Go back

$$\max_{(T_t,w_t,r_t,\tilde{w}_t,\tilde{r}_t^K,\tilde{R}_t^{B,N},\tau_t^K,\tau_t^L,B_t,K_t,L_t,\Pi_t,(a_t^i,c_t^i,l_t^i)_i)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i \omega_t^i U(c_t^i,l_t^i)\ell(di)\right],$$

$$G_t + B_{t-1} + r_t(B_{t-1}+K_{t-1}) + w_tL_t + T_t = B_t + (1-\frac{\kappa}{2}\pi_t^2)Y_t - \delta K_{t-1},$$

for all i: $a_t^i + c_t^i = (1+r_t)a_{t-1}^i + w_t y_t^i l_t^i,$

$$a_t^i \geq -\bar{a}, (a_t^i+\bar{a})\nu_t^i = 0$$

$$U_c(c_t^i,l_t^i) = \beta\mathbb{E}_t\left[U_c(c_{t+1}^i,l_{t+1}^i)(1+r_{t+1})\right] + \nu_t^i,$$

$$l_t^{i,1/\varphi} = \chi w_t y_t^i,$$

$$\Pi_t(\Pi_t-1) = \frac{\varepsilon-1}{\kappa}(\zeta_t-1) + \beta\mathbb{E}_t\Pi_{t+1}(\Pi_{t+1}-1)\frac{Y_{t+1}}{Y_t}\frac{M_{t+1}}{M_t},$$

$$K_t + B_t = \int_i a_t^i\ell(di),\ L_t = \int_i y_t^i l_t^i \ell(di),$$

$$r_t = (1-\tau_t^K)\frac{\tilde{r}_t^K K_{t-1} + (\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t}-1)B_{t-1}}{K_{t-1}+B_{t-1}},$$

$$\mathbb{E}_t[\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}}] = \mathbb{E}_t[1+\tilde{r}_{t+1}^K].$$

First-order conditions

Go back

Key concept: social valuation of liquidity ψ_t^i for agent i.

$$\psi_t^i \equiv \omega_t^i U_c(c_t^i, l_t^i) - U_{cc}(c_t^i, l_t^i)\left(\lambda_t^i - (1+r_t)\lambda_{t-1}^i\right)$$
$$-\left((\gamma_t - \gamma_{t-1})\,\Pi_t\,(\Pi_t - 1) - \frac{\varepsilon - 1}{\kappa}\gamma_t\,(\zeta_t - 1)\right) Y_t \omega_t^i U_{cc}(c_t^i, l_t^i).$$

$\rightarrow$ Benefit, from planner's perspective, of transferring an extra unit of consumption to agent i.

Related concept: net social valuation.

$$\hat{\psi}_t^i = \psi_t^i - \mu_t.$$

Reformulation

Lagrangian simplification:

$$J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U_t^i \ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \lambda_t^i \omega_t^i \left(U_{c,t}^i - \nu_t^i - \beta \mathbb{E}_t \left[(1 + r_{t+1}) U_{c,t+1}^i \right] \right) \ell(di)$$

$$- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_t \left(\Pi_t \left(\Pi_t - 1 \right) Y_t M_t - \frac{\varepsilon - 1}{\kappa} \left(\zeta_t - 1 \right) Y_t M_t - \beta \mathbb{E}_t \left[\Pi_{t+1} \left(\Pi_{t+1} - 1 \right) Y_{t+1} M_{t+1} \right] \right).$$

We get

$$J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U_t^i \ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\omega_t^i \lambda_t^i - (1 + r_t) \lambda_{t-1}^i \omega_{t-1}^i \right) U_{c,t}^i \ell(di).$$

Optimal fiscal policy Go back

- Euler equation for T_t:
$$\int_i \hat{\psi}_t^i \ell(di) = 0$$
- capital tax:
$$\underbrace{\int_i \hat{\psi}_t^i a_{t-1}^i \ell(di)}_{=\text{redistributive effects}} = \underbrace{-\int_i \lambda_{t-1}^i U_c(c_t^i, l_t^i)\ell(di)}_{=\text{savings distortions}}.$$
- Labor wage:
$$\underbrace{\int_i \hat{\psi}_t^i y_t^i l_t^i \ell(di)}_{=\text{redistributive effects}} = \underbrace{\varphi \mu_t \left(L_t - (1-\alpha)\frac{Y_t}{w_t}\right)}_{=\text{labor supply distortions}}.$$

FOCs truncated model Go back

$$\hat{\psi}_{y^N} = \omega_{y^N}\xi_{y^N}U_c(c_{y^N}, l_{y^N}) - \left(\lambda_{y^N} - \tilde{\lambda}_{y^N}(1+r)\right)\xi_{y^N}U_{cc}(c_{y^N}, l_{y^N}) - \mu$$

and

$$\hat{\psi}_{y^N} = \beta\mathbb{E}(1+r)\hat{\psi}_{\tilde{y}^N} \text{ if } \nu_{y^N} = 0 \text{ and } \lambda_{y^N} = 0 \text{ otherwise,}$$

$$\sum_{y^N\in\mathcal{Y}^N} S_{y^N}\hat{\psi}_{y^N}y_{y^N}l_{t,y^N} = \varphi\mu(L - (1-\alpha)\frac{Y}{w}),$$

$$\sum_{y^N\in\mathcal{Y}^N} S_{y^N}\hat{\psi}_{y^N}\tilde{a}_{y^N} = -\sum_{y^N\in\mathcal{Y}^N} S_{t,y^N}\tilde{\lambda}_{y^N}\xi_{y^N}U_c(c_{y^N}, l_{y^N}),$$

$$\mu = \beta\mathbb{E}\left[\mu(1+\tilde{r}^K)\right], \text{and } \sum_{y^N\in\mathcal{Y}^N} S_{y^N}\hat{\psi}_{y^N} = 0.$$

Dynamic truncated model (all fiscal tools) Go back

$$a_{t,y^N} + c_{t,y^N} = w_t y_0^N l_{t,y^N} + (1+r_t)\tilde{a}_{t,y^N} + T_t,$$

$$\xi_{y^N} U_c(c_{t,y^N}, l_{t,y^N}) = \beta(1+r_{t+1})\mathbb{E}\xi_{\tilde{y}^N} U_c(c_{t+1,\tilde{y}^N}, l_{t+1,\tilde{y}^N}) + \nu_{y^N}, \text{ and } l_{t,y^N}^{1/\varphi} = \chi w_t y_0^N,$$

$$\tilde{a}_{t,y^N} = \sum_{\hat{y}^N \in \mathcal{Y}^N} \frac{S_{\hat{y}^N}}{S_{y^N}} \Pi_{\hat{y}^N, y^N} a_{t-1,\hat{y}^N},$$

$$\hat{\psi}_{t,y^N} = \omega_{y^N} \xi_{y^N} U_c(c_{t,y^N}, l_{t,y^N}) - \left(\lambda_{t,y^N} - \tilde{\lambda}_{t,y^N}(1+r_t)\right) \xi_{y^N} U_{cc}(c_{t,y^N}, l_{t,y^N}) - \mu_t,$$

$$G_t + B_{t-1} + r_t\left(B_{t-1} + K_{t-1}\right) + w_t L_t + T_t = B_t + Y_t - \delta K_{t-1},$$

$$\hat{\psi}_{t,y^N} = \beta\mathbb{E}(1+r_{t+1})\hat{\psi}_{t+1,\tilde{y}^N} \text{ if } \nu_{t,y^N} = 0 \text{ and } \lambda_{t,y^N} = 0 \text{ otherwise},$$

$$\sum_{y^N \in \mathcal{Y}^N} S_{y^N} \hat{\psi}_{t,y^N} y_{y^N} l_{t,y^N} = \varphi\mu_t(L_t - (1-\alpha)\frac{Y_t}{w_t}),$$

$$\sum_{y^N \in \mathcal{Y}^N} S_{y^N} \hat{\psi}_{t,y^N} \tilde{a}_{t,y^N} = -\sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \tilde{\lambda}_{t,y^N} \xi_{y^N} U_c(c_{t,y^N}, l_{t,y^N}),$$

$$\mu_t = \beta\mathbb{E}\left[\mu_{t+1}(1+\tilde{r}_{t+1}^K)\right], \text{ and } \sum_{y^N \in \mathcal{Y}^N} S_{y^N} \hat{\psi}_{t,y^N} = 0,$$

$$K_t = \sum_{y^N \in \mathcal{Y}^N} S_{y^N} a_{y_t^N}, L_t = \sum_{y^N \in \mathcal{Y}^N} S_{y^N} y_0^N l_{t,y^N}, Y_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha}.$$

Method comparison

Go back: Aggregation | Go back: Robustness

Model without fiscal policy.

		Reiter	BKM	Trunc
GDP Y	Mean	1.397	1.397	1.397
	Std	0.017	0.017	0.016
Capital K	Mean	18.34	18.34	18.34
	Std	0.015	0.015	0.015
Aggregate consumption C	Mean	0.939	0.939	0.939
	Std	0.014	0.014	0.013
Aggregate labor L	Mean	0.328	0.328	0.328
	Std	0.006	0.006	0.005
$corr(C, Y)$		0.954	0.958	0.951
$corr(Y, Y_{-1})$		0.973	0.974	0.971

Table: Method Comparison

Time-0 model

Go back time-0 Go back robustness

To compare with the literature:

- model without capital (as Acharya et al., 2020; Bhandari et al., 2020),
- model with time-varying idiosyncratic risk (as Acharya et al., 2020),
- model with nominal debt (no fund), (to compare with Nuno and Thomas, 2020).

Economy	κ	$\bar{a}$	ϕ^{TV}	$\Delta\pi$ (%)
(0)	30	5	0	1.19
(i)	60	5	0	0.79
(ii)	30	2	0	0.78
(iii)	30	5	3	1.25

Table: Comparative statics about the inflation on impact.

Algorithm Go back

1. Choose the steady-state fiscal system
2. Solve the Bewley model
3. Choose a N, construct the Aggregated model

   Compute the ξs (Simple linear algebra)
4. Construct the FOC of the planner on the Truncated model

   Compute the ωs (Simple linear algebra)
5. Write the dynamic truncated model, with fixed ξs (Dynare, Julia/JuMP)
6. Simulate the truncated model (exogenous instruments) compare with other methods, increase N if necessary
7. Simulate the truncated model with endogenous instruments

Step 1: Go back

For any Bewley model (with a realistic fiscal system $\tau^K, \tau^L, T, G, \Pi = 1$).

- Steady-state distribution $\Lambda(a, y_k)$ and policy rules $g_a(a, y_k)$, $g_c(a, y_k)$ for $a \geq -\bar{a}$ and $k = 1 \ldots Y$.
- For $k, k' = 1 \ldots Y$. Iterating $g_c(a, y_{k'})$ on $\Lambda(a, y_k)$, find the distribution $\Lambda(a, \{y_k, y_{k'}\})$
- Consider a **truncated history** of length N $y^N = (y_{-N+1}, \ldots, y_0) \in \mathcal{Y}^N$, (there are Y^N histories).

  $\Rightarrow$ We can find the distribution $\widetilde{\Lambda}(a, y^N)$
- We can then compute for histories $y^N \in \mathcal{Y}^N$ (a_{y^N} is the end-of-period wealth):

$$S_{y^N},\ c_{y^N},\ l_{y^N},\ a_{y^N},\ \nu_{y^N},\ u'_{c,y^N}.$$

Step 1: Aggregation

- Law of motion across histories:

$$\Pi_{\hat{y}^N y^N} = 1_{y^N \succeq \hat{y}^N} \Pi_{\hat{y}_0^N y_0^N}.$$

- Check that

$$S_{y^N} \equiv \sum_{\hat{y}^N \in \mathcal{Y}^N} \Pi_{\hat{y}^N, y^N} S_{\hat{y}^N}.$$

- Beginning-of-period wealth $\tilde{a}_{y^N}$ is:

$$\tilde{a}_{y^N} \equiv \sum_{\hat{y}^N \in \mathcal{Y}^N} \frac{S_{\hat{y}^N}}{S_{y^N}} \Pi_{\hat{y}^N, y^N} a_{\hat{y}^N}.$$

- Budget constraint:

$$a_{y^N} + c_{y^N} = w y_0^N l_{y^N} + (1+r)\tilde{a}_{y^N} + T, \text{ for } y^N \in \mathcal{Y}^N.$$

Step 1: Aggregation

Euler equations $u'_{c,y^N} \neq U_c(c_{y^N}, l_{y^N})$, Jensen inequality due to distribution within truncated histories.

We can find ξ_{y^N} such that (related to Werning, 2015):

$$\xi_{y^N} U_c(c_{y^N}, l_{y^N}) = \beta(1+r)\mathbb{E}_{\tilde{y}^N} \xi_{\tilde{y}^N} U_c(c_{\tilde{y}^N}, l_{\tilde{y}^N}) + \nu_{y^N}.$$

ξ_{y^N} in closed-form from steady-state allocation.

Step 1: Aggregation

Aggregated model, for $y^N \in \mathcal{Y}^N$. Finite state space

$$\begin{aligned}
a_{y^N} + c_{y^N} &= w y_0^N l_{y^N} + (1+r)\tilde{a}_{y^N} + T, \\
\xi_{y^N} U_c(c_{y^N}, l_{y^N}) &= \beta(1+r)\mathbb{E}_{\tilde{y}^N} \xi_{\tilde{y}^N} U_c(c_{\tilde{y}^N}, l_{\tilde{y}}) + \nu_{y^N}, \\
l_{y^N}^{1/\varphi} &= \chi w y_0^N, \\
\tilde{a}_{y^N} &= \sum_{\hat{y}^N \in \mathcal{Y}^N} \frac{S_{\hat{y}^N}}{S_{y^N}} \Pi_{\hat{y}^N, y^N} a_{\hat{y}^N},
\end{aligned}$$

and

$$K = \sum_{y^N \in \mathcal{Y}^N} S_{y^N} a_{y^N}, L = \sum_{y^N \in \mathcal{Y}^N} S_{y^N} y_0^N l_{y^N}.$$

Remark: exact aggregation, no simplifying assumption. Go back

Step 2: Identification of Pareto weights

Compute optimal policy on aggregated model. Finite state space objective:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{y^N \in \mathcal{Y}^N} S_{y^N} \omega_{y^N} \xi_{y^N} U(c_{t,y^N}, l_{t,y^N}).$$

Assumption. The planner doesn't consider within-history heterogeneity: ξ_{y^N} are constant. FOCs

Step 2: Identification of Pareto weights

We can find ω_{y^N} for $y^N \in \mathcal{Y}^N$, such that the planner's FOCs are fulfilled for the truncated model given a fiscal system. We choose the weights minimizing:

$$\min_{(\omega_{y^N})} \left\| (\omega_{y^N})_{y^N} - \mathbf{1}_{y^N} \right\|_2$$

subject to planner's FOCs fulfilled with actual fiscal system.

$\rightarrow$ can be computed with basic linear algebra (Linear-Quadratic problem)

Step 2: Consistent truncated model of Pareto weights Go back

$$a_{y^N} + c_{y^N} = w y_0^N l_{y^N} + (1+r)\tilde{a}_{y^N} + T,$$

$$\xi_{y^N} U_c(c_{y^N}, l_{y^N}) = \beta(1+r)\mathbb{E}\xi_{\tilde{y}^N} U_c(c_{\tilde{y}^N}, l_{\tilde{y}}) + \nu_{y^N},$$

$$l_{y^N}^{1/\varphi} = \chi w y_0^N,$$

$$\tilde{a}_{y^N} = \sum_{\hat{y}^N \in \mathcal{Y}^N} \frac{S_{\hat{y}^N}}{S_{y^N}} \Pi_{\hat{y}^N, y^N} a_{\hat{y}^N},$$

$$\hat{\psi}_{y^N} = \beta\mathbb{E}(1+r)\hat{\psi}_{\tilde{y}^N} \text{ if } \nu_{y^N} = 0 \text{ and } \lambda_{y^N} = 0 \text{ otherwise},$$

$$\sum_{y^N \in \mathcal{Y}^N} S_{y^N} \hat{\psi}_{y^N} y_{y^N} l_{t,y^N} = \varphi\mu(L - (1-\alpha)\frac{Y}{w}),$$

$$\sum_{y^N \in \mathcal{Y}^N} S_{y^N} \hat{\psi}_{y^N} \tilde{a}_{y^N} = -\sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \tilde{\lambda}_{y^N} \xi_{y^N} U_c(c_{y^N}, l_{y^N}),$$

$$\mu = \beta\mathbb{E}\left[\mu(1+\tilde{r}^K)\right], \text{ and } \sum_{y^N \in \mathcal{Y}^N} S_{y^N} \hat{\psi}_{y^N} = 0,$$

$$K = \sum_{y^N \in \mathcal{Y}^N} S_{y^N} a_{y^N}, L = \sum_{y^N \in \mathcal{Y}^N} S_{y^N} y_0^N l_{y^N}.$$

Step 3: Simulation of the dynamics Go back

We simulate the optimal dynamics on the truncated model, using perturbation method, while assuming:

- ξ_{y^N} equal to their steady-state values,
- credit-constrained histories fixed at their steady state value (small aggregate shocks).

Consistent approximation:

- $N \to \infty$, $\xi_{y^N} \to 1$, and the allocation arbitrarily close to the true one (LeGrand and Ragot, 2021).
- Quantitative issue (tractable N).

Perturbation method (simple: Dynare, Julia/JuMP). Full model specif.

Algorithm

Step 3: Dynamic model Go back

$$a_{y^N,t} + c_{y^N,t} = w_t y_0^N l_{y^N,t} + (1 + r_t)\tilde{a}_{y^N,t} + T_t,$$

$$\xi_{y^N} U_c(c_{y^N,t}, l_{y^N,t}) = \beta\mathbb{E}(1 + r_{t+1})\xi_{\tilde{y}^N} U_c(c_{\tilde{y}^N,t}, l_{\tilde{y},t}) + \nu_{y^N,t},$$

$$l_{y^N,t}^{1/\varphi} = \chi w_t y_0^N,$$

$$\tilde{a}_{y^N,t} = \sum_{\hat{y}^N \in \mathcal{Y}^N} \frac{S_{\hat{y}^N}}{S_{y^N}} \Pi_{\hat{y}^N,y^N} a_{\hat{y}^N,t},$$

$$\hat{\psi}_{y^N,t} = \beta\mathbb{E}(1 + r_{t+1})\hat{\psi}_{\tilde{y}^N,t+1} \text{ if } \nu_{y^N} = 0 \text{ and } \lambda_{y^N} = 0 \text{ otherwise},$$

$$\sum_{y^N \in \mathcal{Y}^N} S_{y^N} \hat{\psi}_{y^N,t} y_{y^N} l_{t,y^N,t} = \varphi\mu(L_t - (1 - \alpha)\frac{Y_t}{w_t}),$$

$$\sum_{y^N \in \mathcal{Y}^N} S_{y^N} \hat{\psi}_{y^N,t} \tilde{a}_{y^N,t} = - \sum_{y^N \in \mathcal{Y}^N} S_{y^N} \tilde{\lambda}_{y^N,t} \xi_{y^N} U_c(c_{y^N,t}, l_{y^N,t}),$$

$$\mu_t = \beta\mathbb{E}\left[\mu_{t+1}(1 + \tilde{r}_{t+1}^K)\right], \text{ and } \sum_{y^N \in \mathcal{Y}^N} S_{y^N} \hat{\psi}_{y^N,t} = 0,$$

$$K_t = \sum_{y^N \in \mathcal{Y}^N} S_{y^N} a_{y^N,t}, L_t = \sum_{y^N \in \mathcal{Y}^N} S_{y^N} y_0^N l_{y^N,t}.$$

Feasability, Existence, Stability Go back

Assumptions $T_t \geq -\bar{T} > -\infty$ and $a_t \geq -\bar{a}$, above natural limit.

1. Can the planner saciate the economy with liquidity?

   No, requires $B \to +\infty$ and $T \to -\infty$ to pay interest on debt.

2. Does a steady-state equilibrium exist with positive capital tax (Chien, Chen and Yang)?

   Yes, but requires credit constraints to bind

3. Does a steady-state eqilibrium exist without binding credit-constraints?

   No, but other equlibrium exists (BEGS, 2021).

4. Is the steady-state equilibrium locally stable (Straub-Werning, 2020)?

   Open question (Chari, Nicolini, Teles 2020). Here, yes : Numerical simulations.