Should monetary policy care about redistribution?
Optimal fiscal and monetary policy with heterogeneous agents

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Introduction

- Monetary policy generates redistributive effects through various channels. (from Friedman 1968; Bewley, 1980; Kehoe Levine Woodford 1992; Erosa and Ventura 2002; Bilbiie 2008; Algan Ragot 2010; Kaplan, Moll, and Violante 2018; Auclert 2019, among many others).

- Research question: Should monetary policy care about redistribution or only focus on monetary objectives?

- Our strategy:
  - Derive **optimal fiscal and monetary policy** in a HANK model (het-agent economy with aggregate shocks and nominal rigidities).
  - Fiscal policy: distorting labor and capital taxes, lump-sum transfer and one-period riskless public debt.
  - Monetary policy: New-Keynesian setup.
Results’ preview

- **Theoretical results.**
  
  Irrelevance result: No redistributive role for monetary policy when capital and labor taxes are available with commitment (both timeless and time-0 perspective).

- **Quantitative results.**
  
  1. Monetary policy (timeless perspective; McCallum and Nelson 2000; Woodford, 2003): Even with incomplete fiscal tools (no optimal capital tax), inflation has little role to play for redistribution.
  
  2. Monetary policy (time-0 perspective): Inflation is an imperfect substitute to missing capital tax.
  
  3. Fiscal policy (both perspectives): Public debt is more countercyclical and capital tax much less volatile than in the standard complete market setup.
How? Truncating individual histories

Computing Ramsey policies in HANK is challenging. We do so thanks to a methodological contribution (Le Grand and Ragot, 2021).

- We use the Lagrangian approach of Marcet and Marimon (2019).
- We consider a “truncated representation” of idiosyncratic histories.

Allows for simple and accurate quantitative investigation. See Le Grand and Ragot (2022), for results.
Literature (Selected)


- Closest papers: Bhandari, Evans, Golosov, and Sargent (2021), Acharya, Challe, and Dogra (2020), Nuno and Thomas (2020). Slightly different conclusions (more on this later).
Outline of the presentation

1. The environment

2. Optimal fiscal-monetary policy

3. Numerical simulations: Timeless

4. Numerical simulations: Time-0 problem
1 - The environment

- Aggregate state $Z_t$, affects TFP or public consumption. First-order Markov process.
- Unit mass of agents facing uninsurable productivity risk.
- Productivity levels $y \in Y$: constant discrete first-order Markov process with transition matrix $\Pi_{y\tilde{y}}$. Constant share $S_y$.
- Credit constraint $a_t \geq -\bar{a}$
- GHH utility function over consumption and labor supply (works with more general utility functions):

$$U(c, l) = u \left( c - \chi^{-1} \frac{l^{1+1/\varphi}}{1 + 1/\varphi} \right).$$
Production

Standard NK production sector with capital.

- Rotemberg cost: \( \frac{\kappa}{2} \left( \frac{p_t(j)}{p_{t-1}(j)} - 1 \right)^2 Y_t + \text{production subsidy} \)

- Phillips curve:

\[
\Pi_t(\Pi_t - 1) = \varepsilon - \frac{1}{\kappa} (\zeta_t - 1) + \beta \mathbb{E}_t \Pi_{t+1}(\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \frac{M_{t+1}}{M_t}
\]

with marginal cost

\[
\zeta_t = \frac{1}{Z_t} \left( \frac{\tilde{r}_t K}{\alpha} \right)^{\alpha} \left( \frac{\tilde{w}_t}{1 - \alpha} \right)^{1-\alpha}.
\]
Asset markets

- Risk-neutral fund collects capital and public debt and issues claims to agents with rate $\tilde{r}_t$ (and borrowing limits). (as Gornemann et al. 2012)
  - no actual portfolio choice by agents.

- Funds hold two assets:
  - Capital stock with pre-tax tax net rate $\tilde{r}_t^K$.
  - Public debt with nominal pre-tax gross rate $\tilde{R}_{t-1}^{B,N}$ that is predetermined.
Government

Has to finance exogenous $G_t$ and transfers $T_t$, with fiscal tools:

- Distorting taxes on capital and labor ($\tau^K_t$ and $\tau^L_t$).
- Public debt issuance $B_t$.
- Firms’ profits are fully taxed.


$$G_t + \frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} B_{t-1} + T_t \leq \tau^L_t \tilde{w}_t L_t + \tau^K_t \left( \tilde{r}_t K_{t-1} + \left( \frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1 \right) B_{t-1} \right)$$

$$+ \left( 1 - \zeta_t - \frac{\kappa}{2} \right) \pi^2_t Y_t + B_t.$$
2 - Optimal fiscal-monetary policy

- Aggregate welfare criterion: \( \sum_{t=0}^{\infty} \beta^t \int \omega_i U(c^i, l^i) \ell(di) \), with weights \( \omega^i_t = \omega_t(a_0, y^{i, t}) \).

- Consistently, pricing kernel \( M_t = \int \omega^i U(c^i, l^i) \ell(di) \).

**Ramsey program:** Find \( \tau^L_t, \tau^K_t, T_t, B_t, \tilde{R}^{B,N}_t, \pi_t \) for \( t \geq 0 \) that maximize aggregate welfare among competitive equilibria. I.e. maximize aggregate welfare subject to:
  - individual and governmental budget constraints,
  - individual Euler equations,
  - Phillips curve, fund conditions
  - market clearing conditions,
  - factor price relationships.
The Lagrangian approach

- Rich framework for normative questions.

→ Rely on Lagrangian approach of Marcet and Marimon (2019), adjusted in LeGrand and Ragot (2021)

- Enables the derivation of first-order conditions. Useful for:
  - Interpretation and understanding of mechanisms.
  - Simulations of solutions.

- Two Lagrange multipliers of interest:
  - $\lambda_t^i$: on agents’ Euler equation between $t$ and $t+1$.
  - $\mu_t$: on governmental budget constraint.
Nominal economy with all fiscal tools

An irrelevance result
With capital, labor tax and public debt, the planner implements $\Pi = 1$ and there is no further role for monetary policy.

**Intuition.** There are sufficient independent instruments to set the mark-up wedge of firms to $\zeta_t = 1$ (as in the real case). Inflation variations are then only costly and $\Pi_t = 1$. Valid for both timeless and time-0 perspectives. Spirit of Correia, Nicolini, and Teles (2008)

**Very general:**
- General utility function
- Portfolio choice (as many taxes as assets)
- Different shocks (cost-push shocks...)
- Steady-state may not exist

**Question.** What happens with missing fiscal instruments?
2 - Optimal fiscal-monetary policy

Steps

1. Consider a steady state (Bewley model) with a realistic fiscal system.

2. Assume that the actual fiscal system is optimal.
   \[ \Rightarrow \text{From allocations to Pareto weights. Invert Optimal approach Heathcote and Tsujiyama, 2021.} \]

3. Analyze the optimal dynamics with aggregate shocks (Ramsey allocation) around the steady state.
   \[ \Rightarrow \text{From Pareto weights to allocations in the dynamics.} \]

4. Simulate the dynamics with a timeless or a time-0 perspective.

To do so, we use a **truncated representation**. Only the idiosyncratic history over the last \( N \) periods matters (\( N \) is the truncation length).
### 3 - Numerical simulations: Timeless

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td><strong>Preference and technology (quarterly)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Capital share</td>
<td>0.36</td>
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<tr>
<td>( \delta )</td>
<td>Depreciation rate</td>
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<td>( \bar{a} )</td>
<td>Credit limit</td>
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<td>( \varphi )</td>
<td>Frisch elasticity labor supply</td>
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<th><strong>Shock process</strong></th>
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<td>( \rho_z )</td>
<td>Autocorrelation TFP</td>
<td>0.95</td>
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<tr>
<td>( \sigma_z )</td>
<td>Standard deviation TFP shock</td>
<td>0.31%</td>
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<tr>
<td>( \rho_y )</td>
<td>Autocorrelation idio. income</td>
<td>0.99</td>
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<tr>
<td>( \sigma_y )</td>
<td>Standard dev. idio. income</td>
<td>14%</td>
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<td>( n_y )</td>
<td>Numb. idio. states, from Rouwenhorst</td>
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<th><strong>Tax system</strong></th>
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<tr>
<td>( \tau_K )</td>
<td>Capital tax</td>
<td>36%</td>
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<tr>
<td>( \tau_L )</td>
<td>Labor tax</td>
<td>28%</td>
</tr>
<tr>
<td>( T )</td>
<td>Transfer over GDP</td>
<td>8%</td>
</tr>
<tr>
<td>( B/Y )</td>
<td>Public debt over yearly GDP</td>
<td>60%</td>
</tr>
<tr>
<td>( G/Y )</td>
<td>Public spending over yearly GDP</td>
<td>12.4%</td>
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<td>( \kappa )</td>
<td>Price adjustment cost</td>
<td>100</td>
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<td>( \varepsilon )</td>
<td>Elasticity of sub.</td>
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Distribution

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<th>Data</th>
<th>Model</th>
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<td>PSID, 06</td>
<td>SCF, 07</td>
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<td>Q1</td>
<td>-0.9</td>
<td>-0.2</td>
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<tr>
<td>Q2</td>
<td>0.8</td>
<td>1.2</td>
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<tr>
<td>Q3</td>
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<td>Q4</td>
<td>13.0</td>
<td>11.9</td>
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<tr>
<td>Q5</td>
<td>82.7</td>
<td>82.5</td>
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<tr>
<td>Top 5%</td>
<td>36.5</td>
<td>36.4</td>
</tr>
<tr>
<td>Top 1%</td>
<td>30.9</td>
<td>33.5</td>
</tr>
<tr>
<td>Gini</td>
<td>0.77</td>
<td>0.78</td>
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Truncated model calibration

**Truncation length.** \( N = 5 \rightarrow 5^5 = 3125 \) truncated histories.
Dynamics: Timeless perspective

Steady-state + planner cannot renege on past commitments (Woodford 1999; McCallum and Nelson, 2000): closest to a rule.

We compare several cases:

1. Complete market with a representative agent.
2. Incomplete market with full set of fiscal instruments. Our setup with monetary irrelevance.
3. Incomplete market with missing capital tax. Possible role for monetary policy.
Full set incomplete markets vs. complete markets

1. $Z$
2. $C$
3. $K$
4. $Y$
5. $w$
6. $r$
7. $T$
8. $\tau^K$
9. $\tau^L$
10. $\mu$
11. $B$
12. $\pi$
Missing capital tax vs. full set (incomplete markets)

1. $Z$
2. $C$
3. $K$
4. $Y$
5. $w$
6. $r$
7. $T$
8. $\tau^K$
9. $\tau^L$
10. $\mu$
11. $B$
12. $\pi$
Second-order moments

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<th></th>
<th>CM</th>
<th>Full</th>
<th>No cap.tax</th>
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<tr>
<td>$C$ mean</td>
<td>0.754</td>
<td>0.754</td>
<td>0.754</td>
</tr>
<tr>
<td>$C$ std</td>
<td>0.026</td>
<td>0.027</td>
<td>0.027</td>
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<tr>
<td>$K$ mean</td>
<td>11.056</td>
<td>11.054</td>
<td>11.054</td>
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<tr>
<td>$K$ std</td>
<td>0.027</td>
<td>0.027</td>
<td>0.029</td>
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<tr>
<td>$Y$ mean</td>
<td>1.176</td>
<td>1.176</td>
<td>1.176</td>
</tr>
<tr>
<td>$Y$ std</td>
<td>0.026</td>
<td>0.027</td>
<td>0.027</td>
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<tr>
<td>$\tau^K$ mean</td>
<td>0.000</td>
<td>0.360</td>
<td>0.360</td>
</tr>
<tr>
<td>$\tau^K$ std</td>
<td>0.885</td>
<td>0.014</td>
<td>0.000</td>
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<td>$\tau^L$ mean</td>
<td>0.000</td>
<td>0.280</td>
<td>0.280</td>
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<tr>
<td>$\tau^L$ std</td>
<td>0.000</td>
<td>0.002</td>
<td>0.001</td>
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<tr>
<td>$B$ mean</td>
<td>$-10.933$</td>
<td>2.843</td>
<td>2.843</td>
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<tr>
<td>$B$ std</td>
<td>0.015</td>
<td>0.046</td>
<td>0.054</td>
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<tr>
<td>$T$ mean</td>
<td>0.000</td>
<td>0.094</td>
<td>0.094</td>
</tr>
<tr>
<td>$T$ std</td>
<td>0.000</td>
<td>0.061</td>
<td>0.064</td>
</tr>
<tr>
<td>$\pi$ mean</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$\pi$ std</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>$corr(\tau^K, Y)$</td>
<td>$-0.208$</td>
<td>$-0.487$</td>
<td>0.000</td>
</tr>
<tr>
<td>$corr(\tau^L, Y)$</td>
<td>0.927</td>
<td>$-0.637$</td>
<td>$-0.925$</td>
</tr>
<tr>
<td>$corr(B, Y)$</td>
<td>$-0.835$</td>
<td>$-0.759$</td>
<td>$-0.829$</td>
</tr>
<tr>
<td>$corr(Y, Y_{-1})$</td>
<td>0.978</td>
<td>0.978</td>
<td>0.978</td>
</tr>
<tr>
<td>$corr(B, B_{-1})$</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
</tr>
</tbody>
</table>
4 - Numerical simulations: Time-0 problem

Model and other results

- Model without capital, to compare other papers (no fund)
- We set initial Lagrange multipliers to 0, when the shock hits (as Bhandari, et al. 2020; Asharya, Challe, and Dogra, 2020; Nuno and Thomas, 2020): Planner reoptimizes.

- Our equivalence result holds in all periods (also for a time-0 problem): To have inflation dynamics, we must have $\tau^K$ not optimal.

Inflation used as a missing capital tax (Dyrda and Pedroni, 2018).
Robustness checks

- Validation of the truncation method:
  
  Comparison with other methods (Reiter and Boppart et al.)

- Modification of the pricing kernel / of parameters for nominal rigidities.

- Public spending shock.

- Fixed labor tax vs. fixed labor and capital taxes.

Other results
Conclusion

- **Take-away**
  - **Irrelevance result** for the monetary policy when the full set of fiscal tools is available. Confirmed in simulations.
  - **Incomplete markets matter.** Public debt is actual debt and capital tax volatility is reduced by two orders of magnitude.
  - **Importance of public debt.** Even in absence of capital tax, shock is mostly smoothed out by public debt.
  - **Monetary policy.** In absence of capital tax, monetary policy plays a role only for a time-0 problem.

- **Work in Progress**
  - Optimal Monetary Policy with sub-optimal fiscal policy
  - Separable Utility Function
  - Utilitarian Welfare Function
Ramsey program formulation

\[
\max_{(T_t, w_t, \tau_t, \tilde{w}_t, \tau_{K_t}, \tilde{R}_t^{B,N}, \tau_{K_t}, \tau_{L_t}, B_t, K_t, L_t, \Pi_t, (a^i_t, c^i_t, l^i_t))} \quad \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \int_i \omega^i_t U(c^i_t, l^i_t) \ell(di) \right],
\]

\[
G_t + B_{t-1} + r_t(B_t + K_t - 1) + w_t L_t + T_t = B_t + (1 - \kappa)X_t - \delta K_{t-1},
\]

for all \( i \):

\[
a^i_t + c^i_t = (1 + r_t)a^i_{t-1} + w_t y^i_t l^i_t,
\]

\[
a^i_t \geq -\bar{a}, (a^i_t + \bar{a})\nu^i_t = 0
\]

\[
U_c(c^i_t, l^i_t) = \beta \mathbb{E}_t \left[ U_c(c^i_{t+1}, l^i_{t+1}) (1 + r_{t+1}) \right] + \nu^i_t,
\]

\[
l^{i,1/\varphi}_t = \chi w y^i_t,
\]

\[
\Pi_t(\Pi_t - 1) = \frac{\varepsilon - 1}{\kappa} (\zeta_t - 1) + \beta \mathbb{E}_t \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \frac{M_{t+1}}{M_t},
\]

\[
K_t + B_t = \int_i a^i_t \ell(di), \quad L_t = \int_i y^i_t l^i_t \ell(di),
\]

\[
r_t = (1 - \tau^K_t) \frac{\tilde{r}^K_t K_{t-1} + \frac{\tilde{R}_t^{B,N}}{\Pi_t} - 1 B_{t-1}}{K_{t-1} + B_{t-1}},
\]

\[
\mathbb{E}_t \left[ \frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}} \right] = \mathbb{E}_t [1 + \tilde{r}^{K}_{t+1}].
\]
Key concept: **social valuation of liquidity** $\psi^i_t$ for agent $i$.

$$\psi^i_t \equiv \omega^i_t U_c(c^i_t, l^i_t) - U_{cc}(c^i_t, l^i_t) (\lambda^i_t - (1 + r_t)\lambda^i_{t-1})$$

$$- \left( (\gamma^i_t - \gamma^i_{t-1}) \Pi_t (\Pi_t - 1) - \frac{\varepsilon - 1}{\kappa} \gamma^i_t (\zeta^i_t - 1) \right) Y_t \omega^i_t U_{cc}(c^i_t, l^i_t).$$

→ Benefit, from planner’s perspective, of transferring an extra unit of consumption to agent $i$.

Related concept: **net social valuation**.

$$\hat{\psi}^i_t = \psi^i_t - \mu_t.$$
Reformulation

Lagrangian simplification:

\[
J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int \omega_t^i U_t^i \ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int \lambda_t^i \omega_t^i \left( U_{c,t}^i - \nu_t^i - \beta \mathbb{E}_t \left[ (1 + r_{t+1}) U_{c,t+1}^i \right] \right) \ell(di)
\]

\[
- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_t \left( \Pi_t \left( \Pi_t - 1 \right) Y_t M_t - \frac{\varepsilon - 1}{\kappa} (\zeta_t - 1) Y_t M_t - \beta \mathbb{E}_t \left[ \Pi_{t+1} \left( \Pi_{t+1} - 1 \right) Y_{t+1} M_{t+1} \right] \right).
\]

We get

\[
J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int \omega_t^i U_t^i \ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int \left( \omega_t^i \lambda_t^i - (1 + r_t) \lambda_{t-1}^i \omega_{t-1}^i \right) U_{c,t}^i \ell(di).
\]
Optimal fiscal policy

- Euler equation for $T_t$:
  $$\int_i \hat{\psi}_t^i \ell(d\ell) = 0$$

- Capital tax:
  $$\int_i \hat{\psi}_t^i a_{t-1}^i \ell(d\ell) = - \int_i \lambda_{t-1}^i U(c_t^i, l_t^i) \ell(d\ell) .$$
  = redistributive effects
  = savings distortions

- Labor wage:
  $$\int_i \hat{\psi}_t^i y_t^i l_t^i \ell(d\ell) = \varphi \mu_t \left( L_t - (1 - \alpha) \frac{Y_t}{w_t} \right) .$$
  = redistributive effects
  = labor supply distortions
\[ \hat{\psi}_{y^N} = \omega_{y^N} \xi_{y^N} U_c(c_{y^N}, l_{y^N}) - (\lambda_{y^N} - \tilde{\lambda}_{y^N} (1 + r)) \xi_{y^N} U_{cc}(c_{y^N}, l_{y^N}) - \mu \]

and

\[ \hat{\psi}_{y^N} = \beta \mathbb{E}(1 + r) \hat{\psi}_{\tilde{y}^N} \text{ if } \nu_{y^N} = 0 \text{ and } \lambda_{y^N} = 0 \text{ otherwise}, \]

\[ \sum_{y^N \in \mathcal{Y}^N} S_{y^N} \hat{\psi}_{y^N} y^N \ell_{t,y^N} = \varphi \mu(L - (1 - \alpha) \frac{Y}{w}), \]

\[ \sum_{y^N \in \mathcal{Y}^N} S_{y^N} \hat{\psi}_{y^N} \tilde{a}_{y^N} = - \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \tilde{\lambda}_{y^N} \xi_{y^N} U_c(c_{y^N}, l_{y^N}), \]

\[ \mu = \beta \mathbb{E} \left[ \mu(1 + \tilde{r}^K) \right], \text{ and } \sum_{y^N \in \mathcal{Y}^N} S_{y^N} \hat{\psi}_{y^N} = 0. \]
Dynamic truncated model (all fiscal tools)

\[
\begin{align*}
\alpha_{t,yN} + c_{t,yN} = w_t y_0 \lambda_{t,yN} + (1 + r_t) \bar{a}_{t,yN} + T_t, \\
\xi_{yN} U(c_{t,yN}, l_{t,yN}) = \beta (1 + r_{t+1}) \mathbb{E} \xi_{yN} U(c_{t+1,yN}, l_{t+1,yN}) + \nu_{yN}, \text{ and } l_{t,yN}^{1/\varphi} = \chi w_t y_0, \\
\bar{a}_{t,yN} = \sum_{\hat{y}_N \in \mathcal{Y}_N} \frac{S_{yN}}{S_{\hat{y}_N}} \Pi_{\hat{y}_N} y_N \alpha_{t-1,\hat{y}_N}, \\
\hat{\psi}_{t,yN} = \omega_{yN} \xi_{yN} U(c_{t,yN}, l_{t,yN}) - \left( \lambda_{t,yN} - \bar{\lambda}_{t,yN} (1 + r_t) \right) \xi_{yN} U(c_{t,yN}, l_{t,yN}) - \mu_t, \\
G_t + B_{t-1} + r_t \left( B_{t-1} + K_{t-1} \right) + w_t L_t + T_t = B_t + Y_t - \delta K_{t-1}, \\
\hat{\psi}_{t,yN} = \beta \mathbb{E} (1 + r_{t+1}) \hat{\psi}_{t+1,yN} \text{ if } \nu_{t,yN} = 0 \text{ and } \lambda_{t,yN} = 0 \text{ otherwise,} \\
\sum_{yN \in \mathcal{Y}_N} S_{yN} \hat{\psi}_{t,yN} y_N l_{t,yN} = \varphi \mu_t \left( L_t - (1 - \alpha) \frac{Y_t}{w_t} \right), \\
\sum_{yN \in \mathcal{Y}_N} S_{yN} \hat{\psi}_{t,yN} \bar{a}_{t,yN} = - \sum_{yN \in \mathcal{Y}_N} S_{t,yN} \tilde{\lambda}_{t,yN} \xi_{yN} U(c_{t,yN}, l_{t,yN}), \\
\mu_t = \beta \mathbb{E} \left[ \mu_{t+1} (1 + r_{t+1}) \right], \text{ and } \sum_{yN \in \mathcal{Y}_N} S_{yN} \hat{\psi}_{t,yN} y_N = 0, \\
K_t = \sum_{yN \in \mathcal{Y}_N} S_{yN} \alpha_{t,yN}, L_t = \sum_{yN \in \mathcal{Y}_N} S_{yN} y_0 l_{t,yN}, Y_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha}.
\end{align*}
\]
Model without fiscal policy.

<table>
<thead>
<tr>
<th></th>
<th>Reiter</th>
<th>BKM</th>
<th>Trunc</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP $Y$</td>
<td>Mean</td>
<td>1.397</td>
<td>1.397</td>
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<tr>
<td></td>
<td>Std</td>
<td>0.017</td>
<td>0.017</td>
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<tr>
<td>Capital $K$</td>
<td>Mean</td>
<td>18.34</td>
<td>18.34</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>Aggregate consumption $C$</td>
<td>Mean</td>
<td>0.939</td>
<td>0.939</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.014</td>
<td>0.014</td>
</tr>
<tr>
<td>Aggregate labor $L$</td>
<td>Mean</td>
<td>0.328</td>
<td>0.328</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>$corr(C, Y)$</td>
<td></td>
<td>0.954</td>
<td>0.958</td>
</tr>
<tr>
<td>$corr(Y, Y_{-1})$</td>
<td></td>
<td>0.973</td>
<td>0.974</td>
</tr>
</tbody>
</table>

Table: Method Comparison
To compare with the literature:

- model without capital (as Acharya et al., 2020; Bhandari et al., 2020),
- model with time-varying idiosyncratic risk (as Acharya et al., 2020),
- model with nominal debt (no fund), (to compare with Nuno and Thomas, 2020).

<table>
<thead>
<tr>
<th>Economy</th>
<th>$\kappa$</th>
<th>$\bar{a}$</th>
<th>$\phi^{TV}$</th>
<th>$\Delta \pi$ (%)</th>
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</thead>
<tbody>
<tr>
<td>(0)</td>
<td>30</td>
<td>5</td>
<td>0</td>
<td>1.19</td>
</tr>
<tr>
<td>(i)</td>
<td>60</td>
<td>5</td>
<td>0</td>
<td>0.79</td>
</tr>
<tr>
<td>(ii)</td>
<td>30</td>
<td>2</td>
<td>0</td>
<td>0.78</td>
</tr>
<tr>
<td>(iii)</td>
<td>30</td>
<td>5</td>
<td>3</td>
<td>1.25</td>
</tr>
</tbody>
</table>

**Table:** Comparative statics about the inflation on impact.
Algorithm

1. Choose the steady-state fiscal system
2. Solve the Bewley model
3. Choose a $N$, construct the Aggregated model
   Compute the $\xi$s (Simple linear algebra)
4. Construct the FOC of the planner on the Truncated model
   Compute the $\omega$s (Simple linear algebra)
5. Write the dynamic truncated model, with fixed $\xi$s (Dynare, Julia/JuMP)
6. Simulate the truncated model (exogenous instruments) compare with other methods, increase $N$ if necessary
7. Simulate the truncated model with endogenous instruments
Step 1:  

For any Bewley model (with a realistic fiscal system $\tau^K, \tau^L, T, G, \Pi = 1$).

- Steady-state distribution $\Lambda(a, y_k)$ and policy rules $g_a(a, y_k), g_c(a, y_k)$ for $a \geq -\bar{a}$ and $k = 1...Y$.

- For $k, k' = 1...Y$. Iterating $g_c(a, y_{k'})$ on $\Lambda(a, y_k)$, find the distribution $\Lambda(a, \{y_k, y_{k'}\})$

- Consider a truncated history of length $N$
  
  $y^N = (y_{-N+1}, \ldots, y_0) \in \mathcal{Y}^N$, (there are $Y^N$ histories).

  $\Rightarrow$ We can find the distribution $\widetilde{\Lambda}(a, y^N)$

- We can then compute for histories $y^N \in \mathcal{Y}^N$ ($a_{y^N}$ is the end-of-period wealth):

  $$S_{y^N}, c_{y^N}, l_{y^N}, a_{y^N}, \nu_{y^N}, u'_{c, y^N}.$$
Step 1: Aggregation

- Law of motion across histories:

\[ \Pi \hat{y}_N y_N = 1_{y^N \preceq \hat{y}^N} \Pi \hat{y}_0^N y_0^N. \]

- Check that

\[ S_{y^N} \equiv \sum_{\hat{y}^N \in Y^N} \Pi \hat{y}^N, y^N S_{\hat{y}^N}. \]

- Beginning-of-period wealth \( \tilde{a}_{y^N} \) is:

\[ \tilde{a}_{y^N} \equiv \sum_{\hat{y}^N \in Y^N} \frac{S_{\hat{y}^N}}{S_{y^N}} \Pi \hat{y}^N, y^N a_{\hat{y}^N}. \]

- Budget constraint:

\[ a_{y^N} + c_{y^N} = w y_0^N l_{y^N} + (1 + r) \tilde{a}_{y^N} + T, \text{ for } y^N \in Y^N. \]
Step 1: Aggregation

Euler equations $u'_{c,y} \neq U_c(c_y, l_y)$, Jensen inequality due to distribution within truncated histories.

We can find $\xi_{yN}$ such that (related to Werning, 2015):

$$\xi_{yN} U_c(c_{yN}, l_{yN}) = \beta(1 + r) \mathbb{E}_{\tilde{y}N} \xi_{\tilde{y}N} U_c(c_{\tilde{y}N}, l_{\tilde{y}N}) + \nu_{yN}.$$

$\xi_{yN}$ in closed-form from steady-state allocation.
Step 1: Aggregation

Aggregated model, for \( y^N \in \mathcal{Y}^N \). Finite state space

\[
a_y^N + c_y^N = w y_0^N l_y^N + (1 + r) \tilde{a}_y^N + T,
\]

\[
\xi_y^N U_c(c_y^N, l_y^N) = \beta (1 + r) E_{\tilde{y}^N} \xi_{\tilde{y}^N} U_c(c_{\tilde{y}^N}, l_{\tilde{y}}) + \nu_y^N ,
\]

\[
l_{y^N}^{1/\varphi} = \chi w y_0^N ,
\]

\[
\tilde{a}_y^N = \sum_{\hat{y}^N \in \mathcal{Y}^N} \frac{S_{\hat{y}^N}}{S_{y^N}} \Pi_{\hat{y}^N, y^N} a_{\hat{y}^N} ,
\]

and

\[
K = \sum_{y^N \in \mathcal{Y}^N} S_{y^N} a_{y^N} , \quad L = \sum_{y^N \in \mathcal{Y}^N} S_{y^N} y_0^N l_{y^N} .
\]

Remark: exact aggregation, no simplifying assumption.
Step 2: Identification of Pareto weights

Compute optimal policy on aggregated model. Finite state space objective:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{y^N \in \mathcal{Y}^N} S'_{y^N} \omega_{y^N} \xi_{y^N} U(c_{t,y^N}, l_{t,y^N}).$$

Assumption. The planner doesn’t consider within-history heterogeneity: $\xi_{y^N}$ are constant.
Step 2: Identification of Pareto weights

We can find $\omega_{y^N}$ for $y^N \in \mathcal{Y}^N$, such that the planner’s FOCs are fulfilled for the truncated model given a fiscal system. We choose the weights minimizing:

$$\min_{\omega_{y^N}} \left\| (\omega_{y^N}) y^N - 1 y^N \right\|_2$$

subject to planner’s FOCs fulfilled with actual fiscal system.

→ can be computed with basic linear algebra (Linear-Quadratic problem)
Step 2: Consistent truncated model of Pareto weights

\[
\alpha_{yN} + c_{yN} = w y_0^N l_{yN} + (1 + r) \bar{a}_{yN} + T,
\]

\[
\xi_{yN} U_c(c_{yN}, l_{yN}) = \beta (1 + r) \xi \tilde{y}_N U_c(c_{yN}, \tilde{l}_y) + \nu_{yN},
\]

\[
l^{1/\varphi}_{yN} = x w y_0^N,
\]

\[
\bar{a}_{yN} = \sum_{\tilde{y}_N \in \mathcal{V}_N} S_{yN} \Pi_{\tilde{y}_N, yN} \alpha_{yN},
\]

\[
\hat{\psi}_{yN} = \beta \mathbb{E}(1 + r) \hat{\psi}_{\tilde{y}_N} \text{ if } \nu_{yN} = 0 \text{ and } \lambda_{yN} = 0 \text{ otherwise},
\]

\[
\sum_{yN \in \mathcal{V}_N} S_{yN} \hat{y}_N y_{yN} l_{t, yN} = \varphi \mu (L - (1 - \alpha) \frac{Y}{w}),
\]

\[
\sum_{yN \in \mathcal{V}_N} S_{yN} \hat{\psi}_{yN} \bar{a}_{yN} = - \sum_{yN \in \mathcal{V}_N} S_{t, yN} \tilde{\lambda}_{yN} \xi_{yN} U_c(c_{yN}, l_{yN}),
\]

\[
\mu = \beta \mathbb{E} \left[ \mu (1 + r) K \right], \text{ and } \sum_{yN \in \mathcal{V}_N} S_{yN} \hat{\psi}_{yN} = 0,
\]

\[
K = \sum_{yN \in \mathcal{V}_N} S_{yN} \alpha_{yN}, L = \sum_{yN \in \mathcal{V}_N} S_{yN} y_0^N l_{yN}.
\]
Step 3: Simulation of the dynamics

We simulate the optimal dynamics on the truncated model, using perturbation method, while assuming:

- \( \xi_y^N \) equal to their steady-state values,
- credit-constrained histories fixed at their steady state value (small aggregate shocks).

Consistent approximation:

- \( N \to \infty, \xi_y^N \to 1 \), and the allocation arbitrarily close to the true one (LeGrand and Ragot, 2021).
- Quantitative issue (tractable \( N \)).

Perturbation method (simple: Dynare, Julia/JuMP).
Step 3: Dynamic model

\[ a_{yN,t} + c_{yN,t} = w_{t} y_{0}^{N} l_{yN,t} + (1 + r_{t}) \tilde{a}_{yN,t} + T_{t}, \]

\[ \xi_{yN} U_{c}(c_{yN,t}, l_{yN,t}) = \beta E(1 + r_{t+1}) \xi_{yN} U_{c}(c_{yN,t}, \tilde{l}_{y}, t) + \nu_{yN,t}, \]

\[ l^{1/\varphi}_{yN,t} = \chi w_{t} y_{0}^{N}, \]

\[ \tilde{a}_{yN,t} = \sum_{\tilde{y}N \in \mathcal{Y}_{N}} \frac{S_{yN}}{S_{yN}} \prod_{yN,yN^{a} \tilde{y}N,t} \tilde{a}_{yN,t}, \]

\[ \hat{\psi}_{yN,t} = \beta E(1 + r_{t+1}) \hat{\psi}_{yN,t+1} \text{ if } \nu_{yN} = 0 \text{ and } \lambda_{yN} = 0 \text{ otherwise,} \]

\[ \sum_{yN \in \mathcal{Y}_{N}} S_{yN} \hat{\psi}_{yN,t} y_{N,t} y_{N,t} = \varphi \mu (L_{t} - (1 - \alpha) \frac{Y_{t}}{w_{t}}), \]

\[ \sum_{yN \in \mathcal{Y}_{N}} S_{yN} \hat{\psi}_{yN,t} \tilde{a}_{yN,t} = - \sum_{yN \in \mathcal{Y}_{N}} S_{yN} \tilde{\lambda}_{yN,t} \xi_{yN} U_{c}(c_{yN,t}, l_{yN,t}), \]

\[ \mu_{t} = \beta E \left[ \mu_{t+1} (1 + \tilde{r}_{t+1}^{K}) \right], \text{ and } \sum_{yN \in \mathcal{Y}_{N}} S_{yN} \hat{\psi}_{yN,t} = 0, \]

\[ K_{t} = \sum_{yN \in \mathcal{Y}_{N}} S_{yN} a_{yN,t} L_{t} = \sum_{yN \in \mathcal{Y}_{N}} S_{yN} y_{0}^{N} l_{yN,t}. \]
Assumptions $T_t \geq -\bar{T} > -\infty$ and $a_t \geq -\bar{a}$, above natural limit.

1. Can the planner saciate the economy with liquidity?
   No, requires $B \to +\infty$ and $T \to -\infty$ to pay interest on debt.

2. Does a steady-state equilibrium exist with positive capital tax (Chien, Chen and Yang)?
   Yes, but requires credit constraints to bind

3. Does a steady-state equilibrium exist without binding credit-constraints?
   No, but other equilibrium exists (BEGS, 2021).

4. Is the steady-state equilibrium locally stable (Straub-Werning, 2020)?
   Open question (Chari, Nicolini, Teles 2020). Here, yes : Numerical simulations.