Firm Heterogeneity, Capital Misallocation and Optimal Monetary Policy

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The views expressed in this presentation are those of the authors and do not necessarily represent the views of the Bank of Spain, the ECB, the Eurosystem or the IMF.
How does firm heterogeneity affect the optimal conduct of monetary policy

- Firm heterogeneity affects the transmission of monetary policy (e.g. Ottonello and Winberry, 2020; Jeenas, 2019; Koby and Wolf, 2020; Jungherr et al., 2022, ...)

- One particular channel of interest is through changes in the allocation of capital when financial frictions matter (Reis 2013, Gopinath et al 2017, Asriyan et al. 2021,...).

- Which are the implications of firm heterogeneity and financial frictions for the optimal conduct of monetary policy?
  - Challenge: net-worth / productivity distribution is an infinite-dimensional object.
What we do: analyze monetary policy in a model with heterogeneous firms and capital misallocation

- Benchmark model to understand the impact of monetary policy on misallocation and endogenous TFP.
  - Standard New Keynesian block.
  - Heterogeneous firms block as in Moll (2014).
What we do: analyze monetary policy in a model with heterogeneous firms and capital misallocation

- Benchmark model to understand the impact of monetary policy on misallocation and endogenous TFP.
  - Standard New Keynesian block.
  - Heterogeneous firms block as in Moll (2014).

- New algorithm to solve nonlinearly for Ramsey optimal policies with heterogeneous agents using continuous time and Dynare.
What we find

- Transmission: an expansionary monetary policy shock increases TFP.
What we find

- **Transmission**: an expansionary monetary policy shock increases TFP.
  - In line with aggregate responses found in the literature (Moran and Queralto, 2018; Baqee, Farhi and Sangani, 2021; Meier and Reinelt, 2021..), but different mechanism
    - Expansionary monetary policy increases investment of more productive firms relatively more, channeling resources towards high-productivity constrained firms ("misallocation channel")
  - **Empirical support** for the mechanism based on Spanish firm-level micro data.
What we find

➢ Transmission: an expansionary monetary policy shock increases TFP.

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★ Expansionary monetary policy increases investment of more productive firms relatively more, channeling resources towards high-productivity constrained firms (“misallocation channel”)

➢ Empirical support for the mechanism based on Spanish firm-level micro data.

➢ Optimal monetary policy:

➢ Misallocation creates a time inconsistent motive to temporarily expand the economy.

➢ Timeless response to demand shocks: “divine coincidence”...

➢ ... but at the ZLB: low for much longer.
Model
The model in a nutshell

Household
- Input good
- Monopolistic Competition
- Price stickiness
- Retailers
- Entrepreneurs \((z,a)\)
- Rent
- Labor
- Rent
- Capital

Capital producer
- Capital adjustment costs
- Rent Capital
- Entrepreneurs \((z,a)\)
- Operating
- Financial constraints
- Not Operating
- Sell Capital

Retailers
- Input good
- Monopolistic Competition
- Price stickiness
- Rent Capital

Final goods producer
- Retail goods

Central bank
- Capital
Continuum of heterogeneous firms operated by entrepreneurs

- Heterogeneity in entrepreneurs’ net worth \((a_t)\) and productivity (follows OU-diffusion process, \(d \log(z_t) = -\log z \ dt + \sigma \ dW_t\)).

- Firms produce the input good using labor \((l_t)\) and capital \((k_t)\).

- Entrepreneurs can borrow capital \(b_t = k_t - a_t\), subject to a borrowing constraint \(k_t \leq \gamma a_t\).
Continuum of heterogeneous firms operated by entrepreneurs

- Heterogeneity in entrepreneurs’ net worth ($a_t$) and productivity ($z_t$) follows OU-diffusion process, $d \log(z_t) = -\log z \ dt + \sigma \ dW_t$.
- Firms produce the input good using labor ($l_t$) and capital ($k_t$).
- Entrepreneurs can borrow capital $b_t = k_t - a_t$, subject to a borrowing constraint $k_t \leq \gamma a_t$.
- Firms maximize profits:

$$\Phi_t(z_t, a_t) = \max_{k_t, l_t} \left\{ m_t f_t(z_t, k_t, l_t) - w_t l_t - R_t k_t \right\}$$

s.t. $k_t \leq \gamma a_t$

- $m_t$: real price of input good $p_t^r / P_t$
- $f_t(z_t, k_t, l_t) \equiv (z_t k_t)^{\alpha} (l_t)^{1-\alpha}$
- $w_t$: real wage
- $R_t$: real rental rate of capital
- $\gamma > 1$: borrowing constraint
Entrepreneurs’ optimal production plan

\[ k_t(z, a) = \begin{cases} 
\gamma a, & \text{if } z \geq z^*_t, \\
0, & \text{if } z < z^*_t, 
\end{cases} \]

\[ z^*_t = \frac{R_t}{\alpha \left( \frac{(1-\alpha)}{w_t} \right)^{(1-\alpha)/\alpha} \cdot m_t^{\frac{1}{\alpha}}} \]
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\[ z_t^* = \frac{R_t}{\alpha \left( \frac{(1-\alpha)}{w_t} \right)^{(1-\alpha)/\alpha} m_t^{\frac{1}{\alpha}}} \]

- If \( z < z_t^* \), optimal size is \( k_t(z, a) = k_t^*(z) = 0 \) → Entrepreneur is unconstrained
  - She lends her net worth to other entrepreneurs.
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- Optimal choices and profits are linear in capital/net worth

\[ \Phi_t(z, a) = \left( z \alpha \left(\frac{1-\alpha}{w_t}\right)^{(1-\alpha)/\alpha} m_t^{1/\alpha} - R_t \right) q_t \gamma a. \]
Entrepreneurs maximize the discounted flow of dividends

- Entrepreneurs can pay dividends $d_t$ or accumulate net worth $a_t$. 
Entrepreneurs maximize the discounted flow of dividends

- Entrepreneurs can pay dividends $d_t$ or accumulate net worth $a_t$.
- Entrepreneurs are household’s members (as in Gertler & Karadi, 2011, unlike Moll, 2014).
- They retire at rate $\eta$. 

\[
V_0(z, a) = \max a_t, d_t \geq 0 \E_0 \int_0^\infty e^{-\int_0^t (r_s + \eta)} ds \left[ d_t + \text{liquidation value} \right]
\]

\[
\dot{a_t}q_t + d_t = \left[ \text{operating profits} \right] \max \{ \tilde{\Phi}_t(z), 0 \} \gamma + \text{return on capital} \left( R_t - \delta q_t \right)
\]
Entrepreneurs maximize the discounted flow of dividends

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\]

s.t.

\[
\dot{a}_t q_t + d_t = \max\{\Phi(z) / q_t - \delta q_t / q_t, 0\} + (R_t - \delta q_t / q_t) \frac{q_t a_t}{S_t(z)}
\]

- $d_t$: dividends
- $R_t$: rental rate of capital
- $q_t$: price of capital
- $a_t$: net worth (capital owned by firm)
Distribution in net worth shares and aggregation

- Entrepreneur’s behavior is linear in net worth but nonlinear in productivity.

- Joint distribution of net worth and productivity

- Only need the distribution of net worth shares \( \omega_t(z) = \frac{1}{A_t} \int_0^\infty ag_t(z, a) da. \)

\[
\frac{\partial \omega_t(z)}{\partial t} = \left[ s_t(z) - \frac{\dot{A}_t}{A_t} - (1 - \psi) \eta \right] \omega_t(z) - \frac{\partial}{\partial z} \mu(z) \omega_t(z) + \frac{1}{2} \frac{\partial^2}{\partial z^2} \sigma^2(z) \omega_t(z)
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\]

Model is isomorphic to standard RANK with endogenous TFP $\tilde{Z}_t$.

Aggregate output $Y_t$ and TFP $\tilde{Z}_t$ are

\[
Y_t = \tilde{Z}_t K_t^\alpha L_t^{1-\alpha}, \quad \tilde{Z}_t = \left( \underbrace{\mathbb{E}_{\omega_t(z)} [z \mid z > z_\ast]}_{\text{Endogenous TFP}} \right)^\alpha.
\]
Monetary policy transmission
Expansionary monetary policy shock increases TFP...
... by reducing misallocation

Monetary policy affects TFP by reducing the share of constrained firms in the economy ($z^*_t$ - productivity-threshold channel) and by redistributing resources towards high productivity firms ($\omega_t(z)$ - net-worth distribution channel)

Empirical evidence supporting the mechanism in the data
A demand shock can also increase misallocation (ex. decline in natural rate)
Optimal Monetary Policy
Central Bank’s Ramsey problem

\[
\max_{\{\omega_t(z), \text{Prices}_t, \text{Quantities}_t\}_{t \in [0, \infty)}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} u(C_t, L_t) dt
\]

subject to private equilibrium conditions \( \forall t \in [0, \infty) \) and initial conditions
Central Bank’s Ramsey problem

\[
\max_{\{\omega_t(z), \text{Prices}_t, \text{Quantities}_t\}_{t \in [0, \infty)}} \mathbb{E}_0 \int_0^\infty e^{-\rho h t} u(C_t, L_t) dt
\]

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- Need to keep track of the whole distribution of firms \( \omega_t(z) \)
Central Bank’s Ramsey problem

\[
\max \{ \omega_t(z), \text{Prices}_t, \text{Quantities}_t \}_{t \in [0, \infty)} \quad \mathbb{E}_0 \int_0^\infty e^{-\rho h t} u(C_t, L_t) dt
\]

subject to private equilibrium conditions \( \forall t \in [0, \infty) \) and initial conditions

- Need to keep track of the whole distribution of firms \( \omega_t(z) \)

- We propose a new algorithm to solve for Ramsey optimal policies with heterogeneous agents.
  - Discretize the continuous time and continuous-space problem and use standard software (Dynare) to solve non-linearly for the optimal monetary policy in the sequence space.
Optimal Ramsey policy: a new time inconsistency

▶ Complete Markets economy (CM): zero inflation is optimal (steady state is first-best due to subsidy undoing mark-up distortion)

▶ Baseline economy: surprise inflation is optimal since it temporarily reduces capital misallocation
Timeless optimal response to a demand shock: ‘divine coincidence’
Heterogeneity and financial frictions calls for 'low for longer' compared to the complete markets case (orange)
Conclusions

- New model of heterogeneous firms, financial frictions and monetary policy
  - Including a new algorithm to solve and compute optimal policy

- Positive analysis: expansionary MP reduces misallocation through the productivity-threshold and net-worth channels

- Empirical evidence supporting higher investment of high productivity firms after expansionary monetary policy shock

- Normative analysis: important implications for optimal monetary policy

- New source of inflationary time inconsistency: undoing financial frictions.

- Divine coincidence holds when facing demand shocks (timeless)

- Zero-Lower Bound: Low for even longer
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- **Normative analysis**: important implications for optimal monetary policy
  - New source of inflationary time inconsistency: undoing financial frictions.
  - 'Divine coincidence holds when facing demand shocks (timeless)
    - Zero-Lower Bound: *Low for even longer.*
Appendix
## Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.025</td>
<td>Av. 10Y bond return of 2.5% (FRED)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.065</td>
<td>Aggregate depreciation rate (NIPA)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.1</td>
<td>Av. size at entry 10% (OECD, 2001)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.12</td>
<td>Av. real return on equity 11% (S&amp;P500)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.43</td>
<td>Corporate debt to net worth of 43% (FRED)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.3</td>
<td>Standard</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>1</td>
<td>Log utility in consumption</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>1</td>
<td>Kaplan et al. (2018)</td>
</tr>
<tr>
<td>$\Upsilon$</td>
<td>0.71</td>
<td>Normalization $L = 1$</td>
</tr>
<tr>
<td>$\phi_k$</td>
<td>10</td>
<td>VAR evidence</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>10</td>
<td>Mark-up of 11%</td>
</tr>
<tr>
<td>$\theta$</td>
<td>100</td>
<td>Slope of PC of 0.1</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.25</td>
<td>-</td>
</tr>
<tr>
<td>$\upsilon$</td>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\varsigma_z$</td>
<td>0.8</td>
<td>Persistence Gilchrist et al. (2014)</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.30</td>
<td>Volatility Gilchrist et al. (2014)</td>
</tr>
</tbody>
</table>
Representative household

Standard consumption-labor-savings choice

$$\max_{C_t, L_t, D_t, B_t^N} \mathbb{E}_0 \int_0^\infty e^{-\rho h_t} u(C_t, L_t) dt$$

s.t.

$$\dot{D}_t q_t + \dot{B}_t^N + C_t = (R_t - \delta q_t) D_t + (i_t - \pi_t) B_t^N + w_t L_t + T_t$$

- $C_t$: consumption
- $D_t$: capital holdings
- $B_t^N$: holdings of nominal bonds (zero net supply)
- $L_t$: labor supply
- $i_t$: nominal interest rate
- $T_t$: profits of retailers, capital good producer and net dividends from firms
Capital good producer

Produces capital and sells it to the household and the firms at price $q_t$

\[
\max_{\iota_t, K_t} \mathbb{E}_0 \int_0^\infty e^{-\int_0^t r_s ds} \left( q_t \iota_t - \iota_t - \Xi(\iota_t) \right) K_t dt.
\]

\[\text{s.t. } \dot{K}_t = (\iota_t - \delta) K_t.\quad \text{LOM of } K_t\]

- $\iota_t$: investment rate,
- $\Xi(\iota_t) = \frac{\phi^k}{2} (\iota_t - \delta)^2$: quadratic adjustment costs.
Final good producers aggregate varieties \( j \in [0, 1] \). Cost minimization implies demand for variety \( j \) is given by

\[
y_{j,t}(p_{j,t}) = \left( \frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t, \text{ where } P_t = \left( \int_0^1 p_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}.
\]

Retailers maximize

\[
\max_{p_{j,t}} \int_0^\infty e^{-\int_0^t r_s ds} \left\{ \left( \frac{p_{j,t}}{P_t} - m_t \right) \left( \frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t - \frac{\theta}{2} \left( \frac{\dot{p}_{j,t}}{p_{j,t}} \right)^2 Y_t \right\} dt
\]

- \( \varepsilon \): elasticity of substitution across goods
  - \( \varepsilon > 0 \).
- \( \theta \): price adjustment cost parameter.
- \( p_{j,t} \): price of variety \( j \).
The symmetric solution to the pricing problem yields the New Keynesian Phillips curve

\[
\left( r_t - \frac{\dot{Y}_t}{\bar{Y}_t} \right) \pi_t = \frac{\varepsilon}{\theta} (m_t - m^*) + \dot{\pi}_t, \quad m^* = \frac{\varepsilon - 1}{\varepsilon},
\]

\( \pi_t = \frac{\dot{P}_t}{P_t} \) is inflation,

\( m_t \) are relative prices of intermediate good (inverse mark-ups of retailers),

\( m^* \) is the optimal inverse mark-up,

Real rates are defined as \( r_t \equiv \frac{R_t - \delta q_t + \dot{q}_t}{q_t} \).
The evolution of the joint distribution of net worth and productivity \( g_t(z, a) \) is given by the KFE:

\[
\frac{\partial g_t(z, a)}{\partial t} = -\frac{\partial}{\partial a}[g_t(z, a)s_t(z)a] - \frac{\partial}{\partial z}[g_t(z, a)\mu(z)] + \frac{1}{2} \frac{\partial^2}{\partial z^2}[g_t(z, a)\sigma^2(z)]
\]

- Entrepreneurs' savings
- Idiosyncratic TFP shocks

\[-\eta g_t(z, a) + \eta g_t(z, a/\psi)/\psi\]

- Entrepreneurs retire
- New entrepreneurs
RANK vs HANK

**RANK**
- All capital is owned by HH $D_t = K_t$
- No financial frictions.
- TFP is exogenous $Z = 1$

**HANK**
- Capital is owned by HH and entrepreneurs: $D_t + A_t = K_t$
- Financial frictions: $k_t \leq \gamma a_t$
- TFP is endogenous $Z = (\mathbb{E}_t [z | z > z^*_t])^\alpha$

- Introduce subsidies in both economies, such that the SS mark-up distortion is undone.
Sketch of solution algorithm

1. **Discretize** the time space ($\Delta t$); and the state space ($\Delta z$) into $J$ grid points using **finite differences** (Achdou et al., 2017):
   - system of $2J$ equations and $2J$ unknowns for the HJB and the KFE equation (we don’t have a HJB).
   - 
     \[
     \begin{bmatrix}
     \frac{1}{\Delta t}(v^{n+1} - v^n) + \rho v^{n+1} = u^{n+1} + A^{n+1} v^{n+1} \\
     \frac{g^{n+1} - g^n}{\Delta t} = (A^{n+1})^T g^{n+1}
     \end{bmatrix}
     \]
   - set of $X$ equilibrium conditions (MC, FOCs of representative agents)

2. **Compute the planner’s optimality conditions** on discretized problem: $(2J + X) + (2J + X + 1)$ equations using **symbolic differentiation**

3. **Solve the transitional dynamics** up to horizon $T$ using a **Newton algorithm** to solve a large equation set of $[(2J + X) + (2J + X + 1)] T$ equations (cf. Auclert et al., 2020)

Using Dynare
Use Dynare to solve the OMP problem in Discrete Time / Discrete Space non-linearly

Provide

- the SS of the problem conditional on the policy instrument,
- the set of discretized non-linear equilibrium conditions of the private economy,
- the planner's objective function.
Use Dynare to solve the OMP problem in Discrete Time / Discrete Space non-linearly

- Provide
  - the SS of the problem conditional on the policy instrument,
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  - the planner's objective function.

- Use `ramsey_model` command:
  - Dynare computes FOCs for the Ramsey problem by symbolic differentiation.

- Use `steady` command:
  - Dynare computes SS of the Ramsey problem.

- Use `perfect_forecast_solver` command:
  - Uses Newton method to solve simultaneously all the non-linear equations for every period, using sparse matrices.
Use Dynare to solve the OMP problem in Discrete Time / Discrete Space non-linearly

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Easy to use and general!
Can we see this pattern in the data after an expansionary MP shock?

After a monetary policy expansion, high productivity firms increase their investment relatively more than low productivity firms.
Can we see this pattern in the data after an expansionary MP shock?

After a monetary policy expansion, high productivity firms increase their investment relatively more than low productivity firms.

▶ **Data:** yearly balance sheet and cash flow data for the quasi-universe of Spanish firms.
▶ **Monetary policy shocks** identified à la Jarocinski and Karadi (2020).
▶ Use MRPK as proxy for productivity.
▶ Empirical specification following Ottonello and Winberry (2020):

\[
\Delta \log k_{j,t} = \alpha_j + \alpha_{st} + \beta (MRPK_{j,t-1} - \mathbb{E}_j [MRPK_j]) \varepsilon_{t,MP}^j + \Lambda' Z_{j,t-1} + u_{j,t}.
\]
Can we see this pattern in the data after an expansionary MP shock?

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\]

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_t^{MP1} \times MRPK_{t-1} )</td>
<td>0.141**</td>
<td>0.293***</td>
</tr>
<tr>
<td>(0.06)</td>
<td>(0.07)</td>
<td></td>
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<tr>
<td>Observations</td>
<td>5,567,706</td>
<td>4,169,950</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.267</td>
<td>0.285</td>
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<tr>
<td>MRPK control</td>
<td>YES</td>
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<td>Controls</td>
<td>NO</td>
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<td>Time-sector clustering</td>
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</table>

**Yes!**
Empirical evidence: Details

MP shock

- high-frequency data and sign restrictions in a SVAR to identify monetary policy shocks in the Euro area at the monthly level, aggregated at a yearly frequency.
- renormalized so that $\varepsilon_t^{MP}$ is a 100bps expansionary monetary policy shock.
Empirical evidence: Details

MP shock

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- renormalized so that $\varepsilon_{t}^{MP}$ is a 100bps expansionary monetary policy shock.

Productivity

- $MRPK_{t} = \frac{\partial m_{t} f_{t}(z,k,l^{*})}{\partial k} = \left[ \left( \frac{1-\alpha}{w_{t}} \right)^{1-\alpha} m_{t}^{\frac{1}{\alpha}} \right] z \propto z$.
- Demean MRPK to ensure that the results are not driven by permanent heterogeneity in responsiveness across firms.
- Controls $Z_{j,t-1}$ include: MRPK, total assets, leverage, sales growth, net financial assets as a share of total assets, $MRPK \times GDP$ growth.
**Empirical evidence: Robustness**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
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<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
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</thead>
<tbody>
<tr>
<td>$\epsilon_{t}^{MP} \times MRPK_{t-1}$</td>
<td>0.238***</td>
<td>0.299***</td>
<td>0.177**</td>
<td>0.432***</td>
<td></td>
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<tr>
<td></td>
<td>(0.06)</td>
<td>(0.07)</td>
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<td>(0.09)</td>
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<tr>
<td>$Inv_{t-1}$</td>
<td>-0.0310***</td>
<td>-0.0259***</td>
<td></td>
<td></td>
<td>0.166*</td>
<td>0.345***</td>
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<td>$\epsilon_{t}^{MP2} \times MRPK_{t-1}$</td>
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<td>0.0906**</td>
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<td>Observations</td>
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<td>283,835</td>
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<td>$R^2$</td>
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</table>
MP shocks

Panel 1 - Baseline weighting - $\varepsilon_t^{MP}$

Panel 2 - Alternative weighting - $\varepsilon_t^{MP2}$