

Firm Heterogeneity, Capital Misallocation and Optimal Monetary Policy

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*The views expressed in this presentation are those of the authors and **do not** necessarily represent the views of the Bank of Spain, the ECB, the Eurosystem or the IMF.*

How does firm heterogeneity affect the optimal conduct of monetary policy

- ▶ Firm heterogeneity affects the transmission of monetary policy (e.g. [Ottonello and Winberry, 2020](#); [Jeenas, 2019](#); [Koby and Wolf, 2020](#); [Jungherr et al., 2022](#), ...)
- ▶ One particular channel of interest is through changes in the allocation of capital when financial frictions matter ([Reis 2013](#), [Gopinath et al 2017](#), [Asriyan et al. 2021](#),...).
- ▶ Which are the implications of firm heterogeneity and financial frictions for the **optimal conduct of monetary policy**?
 - ▶ Challenge: net-worth / productivity distribution is an infinite-dimensional object.

What we do: analyze monetary policy in a model with heterogeneous firms and capital misallocation

- ▶ Benchmark model to understand the [impact of monetary policy on misallocation](#) and endogenous TFP.
 - ▶ Standard New Keynesian block.
 - ▶ Heterogeneous firms block as in [Moll \(2014\)](#).

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- ▶ Benchmark model to understand the [impact of monetary policy on misallocation](#) and endogenous TFP.
 - ▶ Standard New Keynesian block.
 - ▶ Heterogeneous firms block as in [Moll \(2014\)](#).
- ▶ [New algorithm](#) to solve nonlinearly for Ramsey optimal policies with heterogeneous agents using continuous time and Dynare.
 - ▶ Previous methods by [Nuño and Thomas \(2020\)](#), [LeGrand, Martin-Baillon, and Ragot \(2019\)](#) and [Bhandari et al \(2020\)](#)

What we find

- ▶ **Transmission:** an expansionary monetary policy shock **increases** TFP.

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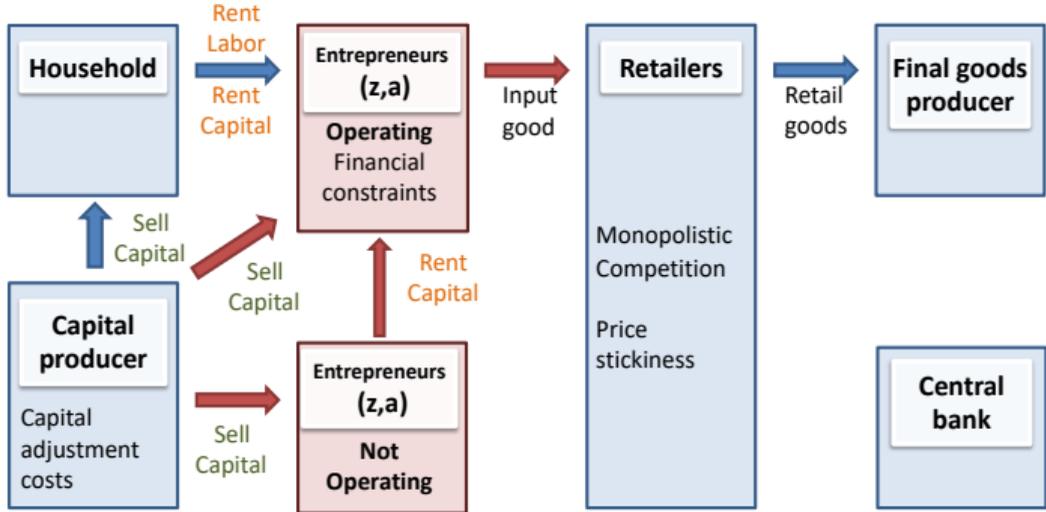
- ▶ **Transmission:** an expansionary monetary policy shock **increases** TFP.
 - ▶ In line with aggregate responses found in the literature ([Moran and Queralto, 2018](#); [Baqae, Farhi and Sangani, 2021](#); [Meier and Reinelt, 2021](#)..), but different mechanism
 - ★ Expansionary monetary policy increases investment of more productive firms relatively more, channeling resources towards high-productivity constrained firms (“[misallocation channel](#)”)
 - ▶ **Empirical support** for the mechanism based on Spanish firm-level micro data.

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 - ★ Expansionary monetary policy increases investment of more productive firms relatively more, channeling resources towards high-productivity constrained firms (“**misallocation channel**”)
 - ▶ **Empirical support** for the mechanism based on Spanish firm-level micro data.
- ▶ **Optimal monetary policy:**
 - ▶ Misallocation creates a *time inconsistent* motive to temporarily expand the economy.
 - ▶ **Timeless** response to demand shocks: “**divine coincidence**”...
 - ▶ ... but at the **ZLB: low for much longer.**

Model

The model in a nutshell



Continuum of heterogeneous firms operated by entrepreneurs

- ▶ Heterogeneity in entrepreneurs' net worth (a_t) and productivity (follows OU-diffusion process, $d \log(z_t) = -\log z dt + \sigma dW$);
- ▶ Firms produce the input good using labor (l_t) and capital (k_t).
- ▶ Entrepreneurs can borrow capital $b_t = k_t - a_t$, subject to a borrowing constraint $k_t \leq \gamma a_t$.

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- ▶ Firms maximize profits:

$$\begin{aligned}\Phi_t(z_t, a_t) &= \max_{k_t, l_t} \{ m_t f_t(z_t, k_t, l_t) - w_t l_t - R_t k_t \} \\ \text{s.t. } k_t &\leq \gamma a_t\end{aligned}$$

- ▶ m_t : real price of input good p_t^y / P_t
- ▶ $f_t(z_t, k_t, l_t) \equiv (z_t k_t)^\alpha (l_t)^{1-\alpha}$
- ▶ w_t : real wage

- ▶ R_t : real rental rate of capital
- ▶ $\gamma > 1$: borrowing constraint

Entrepreneurs' optimal production plan

$$k_t(z, a) = \begin{cases} \gamma a, & \text{if } z \geq z_t^*, \\ 0, & \text{if } z < z_t^*, \end{cases}$$

$$z_t^* = \frac{R_t}{\alpha \left(\frac{(1-\alpha)}{w_t} \right)^{(1-\alpha)/\alpha} m_t^{\frac{1}{\alpha}}}$$

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- ▶ **Optimal choices and profits** are linear in capital/net worth

$$\Phi_t(z, a) = \underbrace{\left(\frac{z \alpha \left(\frac{(1-\alpha)}{w_t} \right)^{(1-\alpha)/\alpha} m_t^{\frac{1}{\alpha}} - R_t}{q_t} \right)}_{\tilde{\Phi}_t(z)} q_t \underbrace{\gamma a}_{k_t}.$$

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$$V_0(z, a) = \max_{a_t, d_t \geq 0} \mathbb{E}_0 \int_0^{\infty} e^{-\int_0^t (r_s + \eta) ds} \left(d_t + \overbrace{\eta q_t a_t}^{\text{liquidation value}} \right) dt$$

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s.t.

$$\dot{a}_t q_t + d_t = \underbrace{\left(\overbrace{\max\{\tilde{\Phi}_t(z), 0\} \gamma}^{\text{operating profits}} + \overbrace{\left(\frac{R_t - \delta q_t}{q_t} \right)}^{\text{return on capital}} \right)}_{S_t(z)} q_t a_t$$

- ▶ d_t : dividends
- ▶ R_t : rental rate of capital

- ▶ q_t : price of capital
- ▶ a_t : net worth (capital owned by firm)

Distribution in net worth shares and aggregation

- ▶ Entrepreneur's behavior is linear in net worth but nonlinear in productivity.

▶ Joint distribution of net worth and productivity

- ▶ Only need the distribution of **net worth shares** $\omega_t(z) = \frac{1}{A_t} \int_0^\infty ag_t(z, a) da$.

$$\frac{\partial \omega_t(z)}{\partial t} = \left[s_t(z) - \frac{\dot{A}_t}{A_t} - (1 - \psi)\eta \right] \omega_t(z) - \frac{\partial}{\partial z} \mu(z) \omega_t(z) + \frac{1}{2} \frac{\partial^2}{\partial z^2} \sigma^2(z) \omega_t(z)$$

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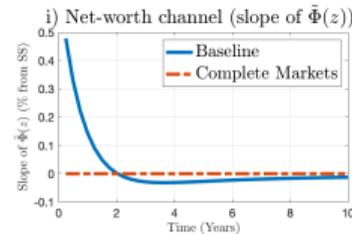
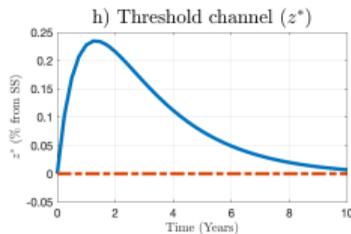
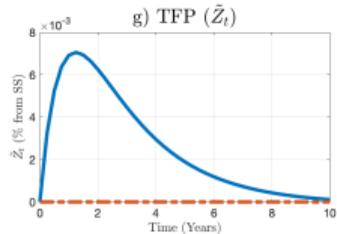
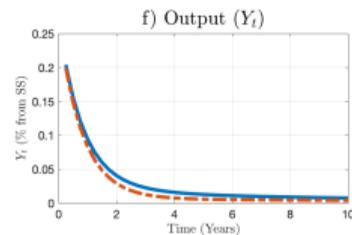
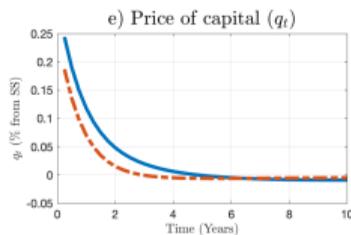
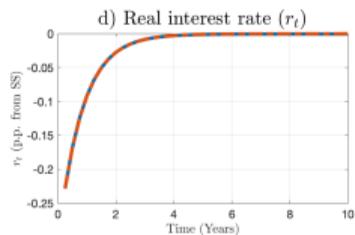
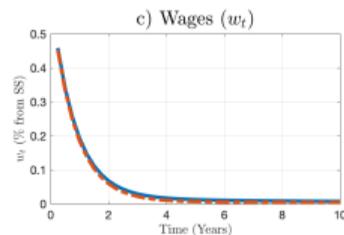
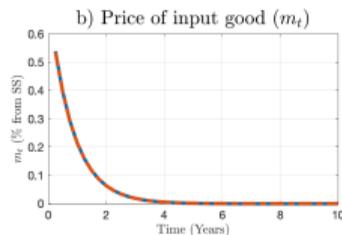
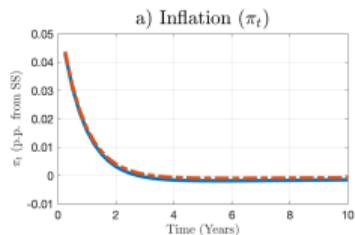
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- ▶ Model is isomorphic to standard RANK with **endogenous** TFP \tilde{Z}_t .
- ▶ Aggregate output Y_t and TFP \tilde{Z}_t are

$$Y_t = \tilde{Z}_t K_t^\alpha L_t^{1-\alpha}, \quad \tilde{Z}_t = \left(\underbrace{\mathbb{E}_{\omega_t(z)} [z \mid z > z_t^*]}_{\text{Endogenous TFP}} \right)^\alpha.$$

Monetary policy transmission

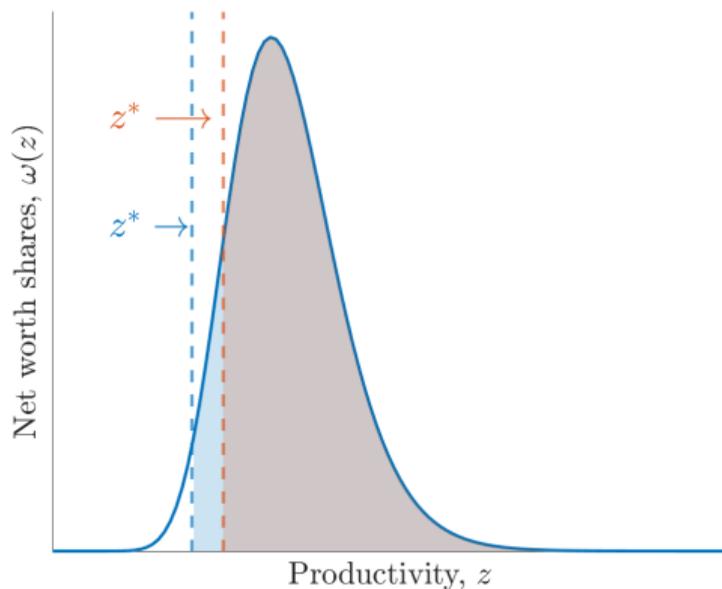
Expansionary monetary policy shock increases TFP...



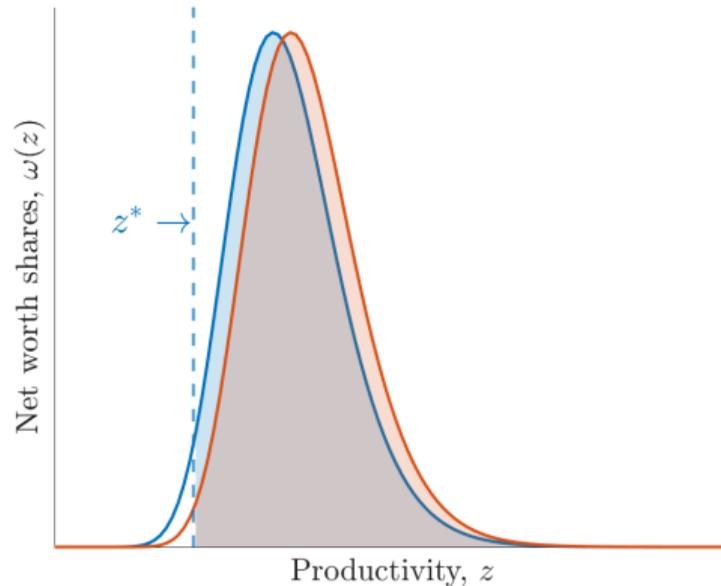
... by reducing misallocation

Monetary policy affects TFP by reducing the share of constrained firms in the economy (z_t^* - *productivity-threshold channel*) and by redistributing resources towards high productivity firms ($\omega_t(z)$ - *net-worth distribution channel*)

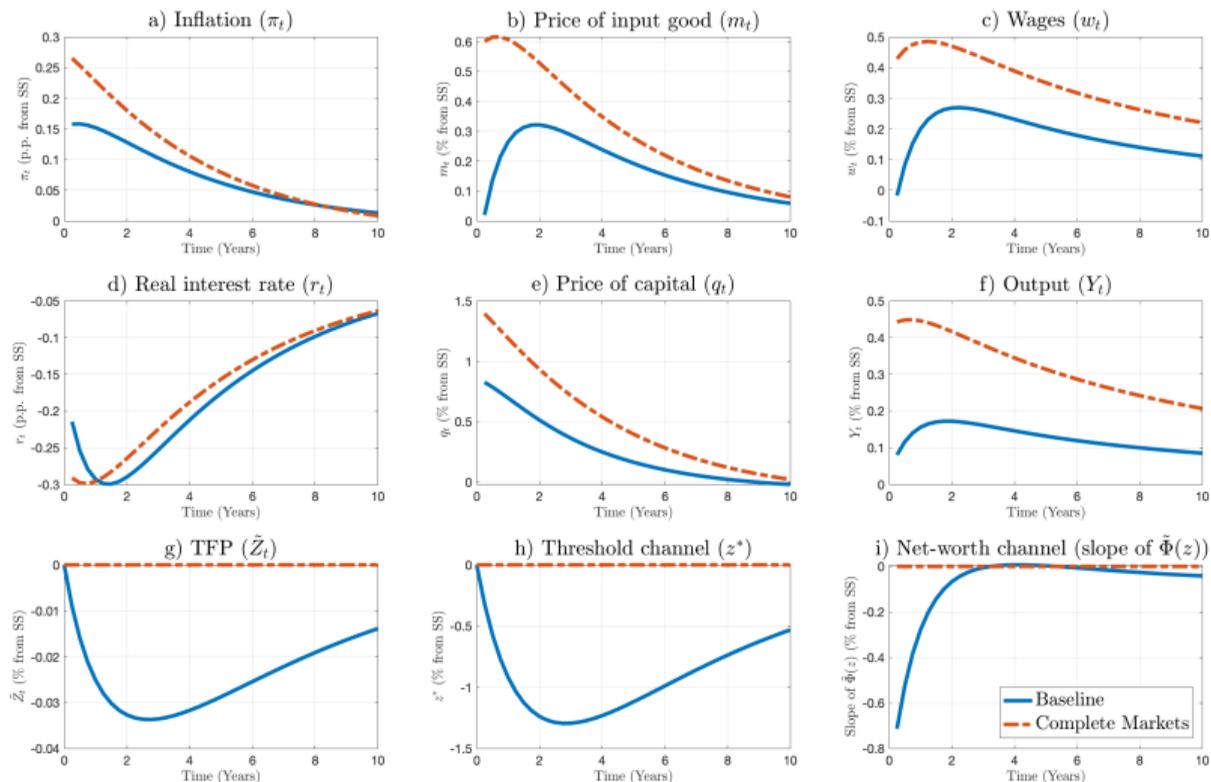
Productivity-threshold channel



Net-worth distribution channel



A demand shock can also increase misallocation (ex. decline in natural rate)



Optimal Monetary Policy

Central Bank's Ramsey problem

$$\max_{\{\omega_t(z), \text{Prices}_t, \text{Quantities}_t\}_{t \in [0, \infty)}} \mathbb{E}_0 \int_0^{\infty} e^{-\rho^h t} u(C_t, L_t) dt$$

subject to private equilibrium conditions $\forall t \in [0, \infty)$ and initial conditions

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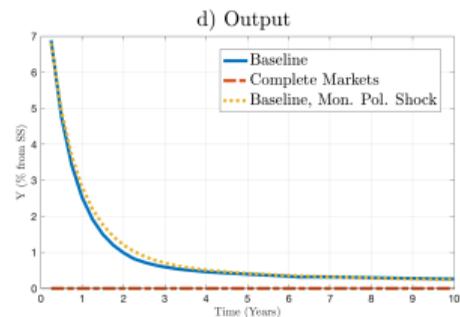
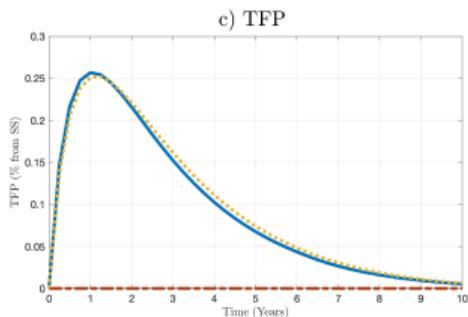
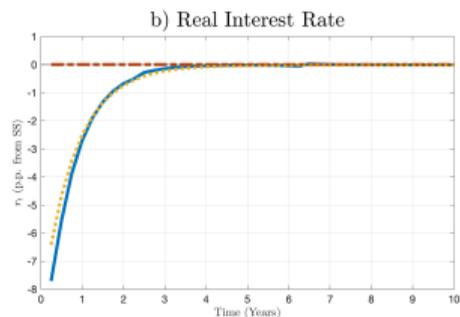
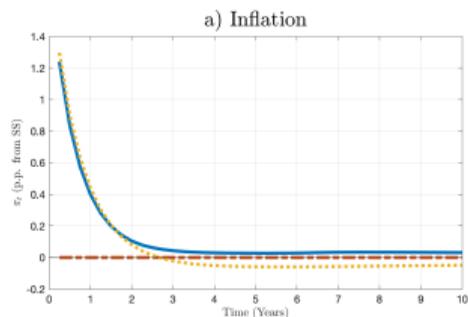
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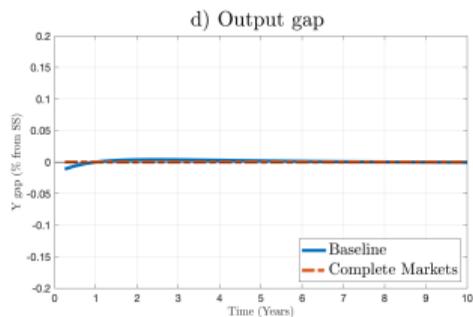
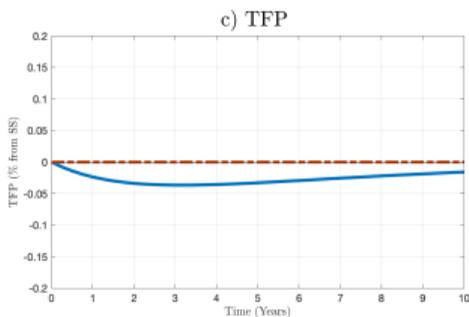
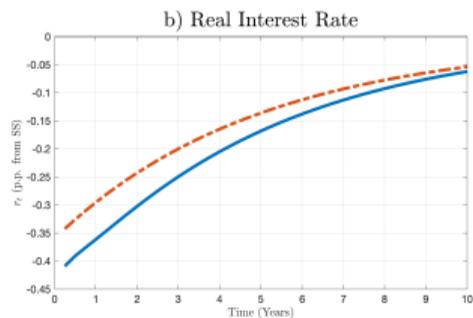
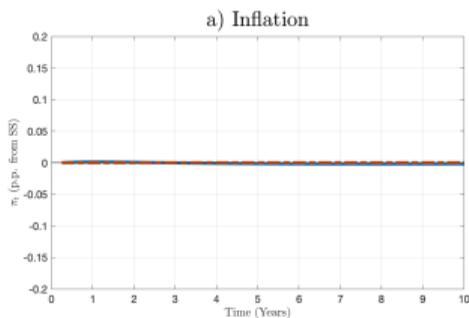
- ▶ Need to keep track of the whole distribution of firms $\omega_t(z)$
- ▶ We propose a **new algorithm** to solve for Ramsey optimal policies with heterogeneous agents.
 - ▶ Discretize the continuous time and continuous-space problem and use standard software (Dynare) to solve non-linearly for the optimal monetary policy in the sequence space. [▶ More](#)

Optimal Ramsey policy: a new time inconsistency

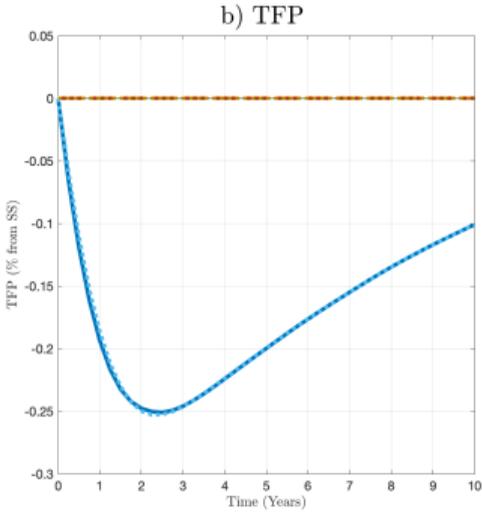
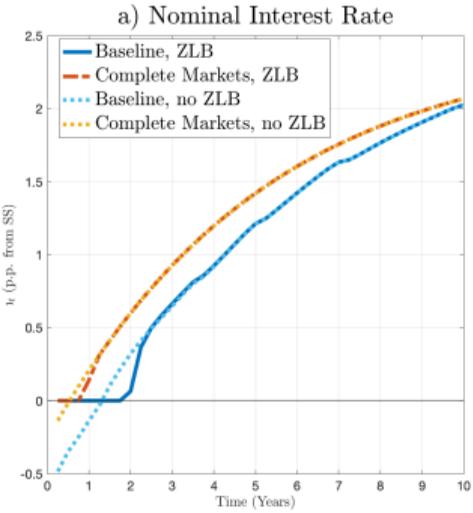


- ▶ Complete Markets economy (CM): **zero inflation** is optimal (steady state is first-best due to subsidy undoing mark-up distortion) ▶ CE vs Baseline
- ▶ Baseline economy: **surprise inflation** is optimal since it temporarily reduces capital misallocation

Timeless optimal response to a demand shock: 'divine coincidence'



Timeless optimal response to a demand shock with ZLB: low for even longer



- ▶ Heterogeneity and financial frictions calls for '*low for longer*' compared to the complete markets case (orange)

Conclusions

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 - ▶ Empirical evidence supporting higher investment of high productivity firms after expansionary monetary policy shock

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 - ▶ Including a new algorithm to solve and compute optimal policy
- ▶ **Positive analysis**: expansionary MP reduces misallocation through the productivity-threshold and net-worth channels
 - ▶ Empirical evidence supporting higher investment of high productivity firms after expansionary monetary policy shock
- ▶ **Normative analysis**: important implications for optimal monetary policy
 - ▶ New source of inflationary time inconsistency: undoing financial frictions.
 - ▶ 'Divine coincidence holds when facing demand shocks (timeless)
 - ★ Zero-Lower Bound: *Low for even longer.*

Appendix

Calibration

	Parameter	Value	Source/target
ρ^h	Household's discount factor	0.025	Av. 10Y bond return of 2.5% (FRED)
δ	Capital depreciation rate	0.065	Aggregate depreciation rate (NIPA)
ψ	Fraction firms' assets at entry	0.1	Av. size at entry 10% (OECD, 2001)
η	Firms' death rate	0.12	Av. real return on equity 11% (S&P500)
γ	Borrowing constraint parameter	1.43	Corporate debt to net worth of 43% (FRED)
α	Capital share in production function	0.3	Standard
ζ	Relative risk aversion parameter HH	1	Log utility in consumption
ϑ	Inverse Frisch Elasticity	1	Kaplan et al. (2018)
Υ	Constant in disutility of labor	0.71	Normalization $L = 1$
ϕ^k	Capital adjustment costs	10	VAR evidence
ϵ	Elasticity of substitution retail goods	10	Mark-up of 11%
θ	Price adjustment costs	100	Slope of PC of 0.1
$\bar{\pi}$	Inflation target	0	-
ϕ	Slope Taylor rule	1.25	-
ν	Persistence Taylor rule	0.8	-
Γ	SS Aggregate Productivity	1	Normalization
ς_z	Mean reverting parameter	0.8	Persistence Gilchrist et al. (2014)
σ_z	Volatility of the shock	0.30	Volatility Gilchrist et al. (2014)

Representative household

▶ Back

Standard consumption-labor-savings choice

$$\max_{C_t, L_t, D_t, B_t^N} \mathbb{E}_0 \int_0^{\infty} e^{-\rho^h t} u(C_t, L_t) dt$$

s.t.

$$\dot{D}_t q_t + \dot{B}_t^N + C_t = (R_t - \delta q_t) D_t + (i_t - \pi_t) B_t^N + w_t L_t + T_t$$

▶ C_t : consumption

▶ D_t : capital holdings

▶ B_t^N holdings of nominal bonds (zero net supply)

▶ L_t : labor supply

▶ i_t : nominal interest rate

▶ T_t : profits of *retailers*, *capital good producer* and *net dividends* from firms

Capital good producer

Produces capital and sells it to the household and the firms at price q_t

▶ Back

$$\max_{\iota_t, K_t} \mathbb{E}_0 \int_0^{\infty} e^{-\int_0^t r_s ds} (q_t \iota_t - \iota_t - \Xi(\iota_t)) K_t dt.$$
$$\text{s.t. } \underbrace{\dot{K}_t = (\iota_t - \delta) K_t}_{\text{LOM of } K_t}.$$

- ▶ ι_t : investment rate,
- ▶ $\Xi(\iota_t) = \frac{\phi^k}{2} (\iota_t - \delta)^2$: quadratic adjustment costs.

New Keynesian block

▶ Back

- ▶ **Final good producers** aggregate varieties $j \in [0, 1]$. Cost minimization implies demand for variety j is given by

$$y_{j,t}(p_{j,t}) = \left(\frac{p_{j,t}}{P_t}\right)^{-\epsilon} Y_t, \text{ where } P_t = \underbrace{\left(\int_0^1 p_{j,t}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}}_{\text{Agg. Price index}}.$$

- ▶ **Retailers** maximize

$$\max_{p_{j,t}} \int_0^\infty e^{-\int_0^t r_s ds} \left\{ \underbrace{\left(\frac{p_{j,t}}{P_t} - m_t\right)}_{\text{Mark-up}} \left(\frac{p_{j,t}}{P_t}\right)^{-\epsilon} Y_t - \frac{\theta}{2} \left(\frac{\dot{p}_{j,t}}{p_{j,t}}\right)^2 Y_t \right\} dt$$

- ▶ ϵ : elasticity of substitution across goods

$\epsilon > 0$.

- ▶ θ : price adjustment cost parameter.

- ▶ $p_{j,t}$: price of variety j .

New Keynesian block

▶ Back

- ▶ The symmetric solution to the pricing problem yields the **New Keynesian Phillips curve**

$$\left(r_t - \frac{\dot{Y}_t}{Y_t}\right) \pi_t = \frac{\varepsilon}{\theta} (m_t - m^*) + \dot{\pi}_t, \quad m^* = \frac{\varepsilon - 1}{\varepsilon},$$

- ▶ $\pi_t = \frac{\dot{P}}{P_t}$ is inflation,
- ▶ m_t are relative prices of intermediate good (inverse mark-ups of retailers),
- ▶ m^* is the optimal inverse mark-up,
- ▶ Real rates are defined as $r_t \equiv \frac{R_t - \delta q_t + \dot{q}_t}{q_t}$.

Distribution of entrepreneurs

- ▶ The evolution of the **joint distribution** of net worth and productivity $g_t(z, a)$ is given by the KFE:

$$\frac{\partial g_t(z, a)}{\partial t} = \underbrace{-\frac{\partial}{\partial a} [g_t(z, a) s_t(z) a]}_{\text{Entrepreneurs' savings}} \underbrace{-\frac{\partial}{\partial z} [g_t(z, a) \mu(z)] + \frac{1}{2} \frac{\partial^2}{\partial z^2} [g_t(z, a) \sigma^2(z)]}_{\text{idiosyncratic TFP shocks}} \\ \underbrace{-\eta g_t(z, a)}_{\text{Entrepreneurs retire}} \underbrace{+\eta g_t(z, a/\psi)/\psi}_{\text{New entrepreneurs}}$$

RANK vs HANK

▶ Back

RANK

- ▶ All capital is owned by HH $D_t = K_t$
- ▶ No financial frictions.
- ▶ TFP is exogenous
 $Z = 1$

HANK

- ▶ Capital is owned by HH and entrepreneurs: $D_t + A_t = K_t$
- ▶ Financial frictions: $k_t \leq \gamma a_t$
- ▶ TFP is endogenous
 $Z = (\mathbb{E}_t [z \mid z > z^*])^\alpha$

- ▶ Introduce subsidies in both economies, such that the SS mark-up distortion is undone.

Sketch of solution algorithm

▶ Back

- 1 **Discretize** the time space (Δt); and the state space (Δz) into J grid points using **finite differences** (Achdou et al, 2017):

▶ system of $2J$ equations and $2J$ unknowns for the HJB and the KFE equation (we don't have a HJB).

$$\left(\begin{array}{l} \frac{1}{\Delta t} (\mathbf{v}^{n+1} - \mathbf{v}^n) + \rho \mathbf{v}^{n+1} = \mathbf{u}^{n+1} + \mathbf{A}^{n+1} \mathbf{v}^{n+1} \\ \frac{\mathbf{g}^{n+1} - \mathbf{g}^n}{\Delta t} = (\mathbf{A}^{n+1})^T \mathbf{g}^{n+1} \end{array} \right)$$

▶ set of X equilibrium conditions (MC, FOCs of representative agents)

- 2 Compute the **planner's optimality conditions** on discretized problem : $(2J + X) + (2J + X + 1)$ equations using **symbolic differentiation**
- 3 Solve the transitional dynamics up to horizon T using a **Newton algorithm** to solve a large equation set of $[(2J + X) + (2J + X + 1)] T$ equations (cf. Auclert et al., 2020)

▶ Using Dynare

Use Dynare to solve the OMP problem in Discrete Time / Discrete Space non-linearly

▶ Back

▶ Provide

- ▶ the **SS of the problem** conditional on the policy instrument,
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Use Dynare to solve the OMP problem in Discrete Time / Discrete Space non-linearly

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Easy to use and general!

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- ▶ **Data:** yearly balance sheet and cash flow data for the quasi-universe of Spanish firms.
- ▶ **Monetary policy shocks** identified à la Jarociński and Karadi (2020). [▶ more](#)
- ▶ Use MRPK as proxy for productivity.
- ▶ Empirical specification following Ottonello and Winberry (2020):

$$\Delta \log k_{j,t} = \alpha_j + \alpha_{st} + \beta (MRPK_{j,t-1} - \mathbb{E}_j [MRPK_j]) \varepsilon_t^{MP} + \Lambda' Z_{j,t-1} + u_{j,t}.$$

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	(1)	(2)
$\varepsilon_t^{MP1} \times MRPK_{t-1}$	0.141** (0.06)	0.293*** (0.07)
Observations	5,567,706	4,169,950
R^2	0.267	0.285
MRPK control	YES	YES
Controls	NO	YES
Time-sector FE	YES	YES
Time-sector clustering	YES	YES

Yes!

Empirical evidence: Details

MP shock

- ▶ high-frequency data and sign restrictions in a SVAR to identify monetary policy shocks in the Euro area at the monthly level, aggregated at a yearly frequency.
- ▶ renormalized so that ε_t^{MP} is a 100bps expansionary monetary policy shock.

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Productivity

- ▶ $MRPK_t = \frac{\partial m_t f_t(z, k, l^*)}{\partial k} = \left[\left(\frac{1-\alpha}{w_t} \right)^{\frac{1-\alpha}{\alpha}} m_t^{\frac{1}{\alpha}} \right] z \propto z$.
- ▶ Demean MRPK to ensure that the results are not driven by permanent heterogeneity in responsiveness across firms.
- ▶ Controls $Z_{j,t-1}$ include: MRPK, total assets, leverage, sales growth, net financial assets as a share of total assets, MRPK \times GDP growth.

Empirical evidence: Robustness

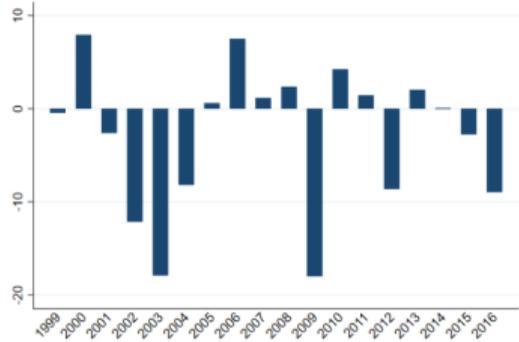
▶ Back

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\epsilon_t^{MP} \times MRPK_{t-1}$	0.238*** (0.06)	0.299*** (0.07)	0.177** (0.07)	0.432*** (0.09)				
Inv_{t-1}	-0.0310*** (0.00)	-0.0259*** (0.00)						
$\epsilon_t^{MP2} \times MRPK_{t-1}$					0.166* (0.10)	0.345*** (0.10)		
$\epsilon_t^{MP} \times MRPK_{t-1}$ (not demeaned)							0.0906** (0.04)	0.243*** (0.04)
Observations	4,162,114	4,094,537	283,835	263,397	5,567,706	4,169,950	5,567,706	4,169,950
R^2	0.279	0.283	0.153	0.162	0.267	0.285	0.267	0.286
MRPK control	YES	YES	YES	YES	YES	YES	YES	YES
Controls	NO	YES	NO	YES	NO	YES	NO	YES
Time-sector FE	YES	YES	YES	YES	YES	YES	YES	YES
Time-sector clustering	YES	YES	YES	YES	YES	YES	YES	YES
Panel	FULL	FULL	BALANCED	BALANCED	FULL	FULL	FULL	FULL

MP shocks

▶ Back

Panel 1 - Baseline weighting - ϵ_t^{MP}



Panel 2 - Alternative weighting - ϵ_t^{MP2}

