The Long-Run Phillips Curve is... a Curve [1]

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[1] Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank

The question

An old debate: is there any trade-off between inflation and output/unemployment in the long run?

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  $\Rightarrow$ *the long-run Phillips curve is vertical*
- Cornerstone role in macroeconomic theory
- ...and practice: working assumption of central banks in the implementation of monetary policy

  $\Rightarrow$ *"Inflation is a monetary phenomenon"*

The question

It is somewhat surprising to note that:

- **Empirically**: There is little econometric work devoted to test the absence of a long-run trade-off.

  Related literature: King and Watson (1994, 1997); Svensson (2015); Beyer and Farmer (2007); Berentsen et al. (2011); Haug and King (2014); Benati (2015)

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- **Theoretically**: Many models imply non-vertical LRPC

  $\Rightarrow$ *"Non-superneutrality"*

  - *For example, modern macroeconomic sticky price frameworks generally do not imply the absence of a long- run relation. The Generalized NK model delivers a negative relationship between steady state inflation and output. E.g., Ascari (2004), Ascari and Sbordone (2014)*

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   Search for a structural model / interpretation

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   - The model has the two key features from the statistical analysis: non-linear and negatively sloped LRPC
   - The estimated LRPC from the structural model turns out to be quantitatively very similar (in a statistical sense) to the one from the time series analysis

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Three observables: GDP per capita, inflation and interest rate

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Trend-cycle decomposition (Generalization of Steady State VAR, Villani, 2009; Johannes and Mertens, 2021):

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- X_t is a $(n \times 1)$ vector with observed variables at time
- $\bar{X}_t$ is the vector with the long-run values of X_t
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Cyclical component:

$\hat{X}_t$ described by an unrestricted VAR as in (1): stable component with unconditional expectation equal to zero

$$A(L)\left(X_t - \bar{X}_t\right) = \varepsilon_t \quad \varepsilon_t \sim N\left(0, \Sigma_{\epsilon,t}\right) \tag{1}$$

The model for the long-run

$$\bar{y}_t = y_t^* + \delta(\bar{\pi}_t)$$ the trend output as a function of trend inflation

$$\delta(\bar{\pi}_t) : \delta(0) = 0$$

$$y_t^* = y_{t-1}^* + g_t + \eta_t^y$$

$$g_t = g_{t-1} + \eta_t^g$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \eta_t^\pi$$ trend inflation is random walk

$$\bar{i}_t - \bar{\pi}_t = c g_t + z_t$$ long-run Fisher equation

$$z_t = z_{t-1} + \eta_t^z$$

A non-linear long-run Phillips curve

Our choice of $\delta(\bar{\pi}_t)$ is a piecewise linear function:

$$\bar{y}_t = y_t^* + \delta(\bar{\pi}_t)$$

$$\delta(\bar{\pi}_t) = \begin{cases} k_1 \bar{\pi}_t & \text{if } \bar{\pi}_t \leq \tau \\ k_2 \bar{\pi}_t + c_k & \text{if } \bar{\pi}_t > \tau \end{cases}$$

- It can approximate potential non-linearity without imposing strong assumptions on the functional form
- It is easy to interpret
- It is simpler to treat: methodological contribution

A piecewise linear approach

The model can be written as:

$$\bar{X}_t = D(\theta_t) + H(\theta_t)\,\theta_t \tag{2}$$

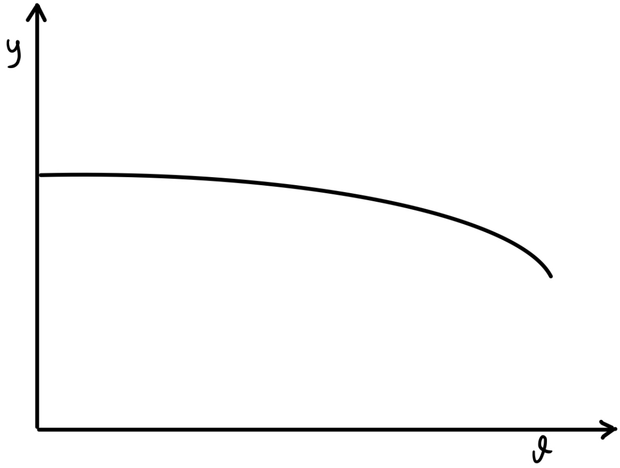
$$\theta_t = M(\theta_t) + G(\theta_t)\,\theta_{t-1} + P(\theta_t)\,\eta_t \tag{3}$$

where, in particular

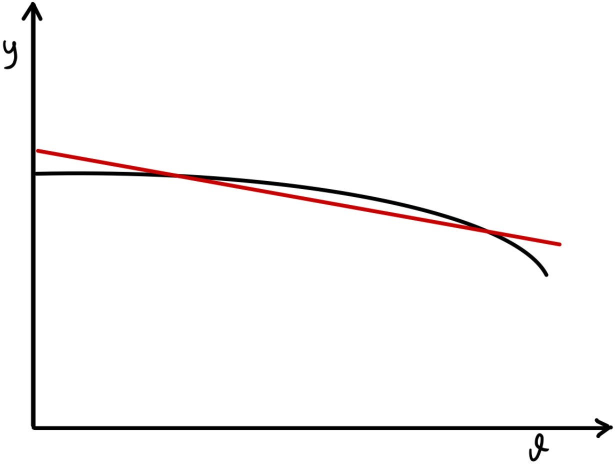
$$(D, H, M, G, P) = \begin{cases} (D_1, H_1, M_1, G_1, P_1) & \text{if } \bar{\pi}_t \leq \tau \\ (D_2, H_2, M_2, G_2, P_2) & \text{if } \bar{\pi}_t > \tau \end{cases} \tag{4}$$

- Methodological contribution: we characterize the likelihood and the posterior distribution of $\theta_t = (\bar{y}_t, \bar{\pi}_t, g_t, z_t)$ analytically
- Compromise between efficiency and misspecification

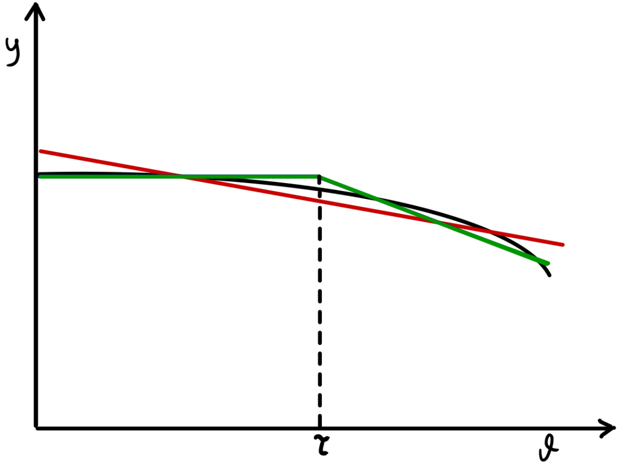
Black: no misspecification - likelihood and posterior not available analytically



Red: misspecification - likelihood and posterior available analytically



Green: less misspecification - likelihood and posterior available analytically



Estimation

Bayesian approach, US data, sample from 1960Q1 to 2008Q2

Two sources of non linearity: stochastic volatility and a piecewise linear LRPC => Particle filtering approach

1. **Latent processes**: "Rao-Blackwellization", thanks to the analytical results on the piecewise linear model $\Rightarrow$ we can analytically characterize the likelihood function and posterior distribution of the latent states

2. **Parameters**: Particle filtering allows to jointly approximate the latent states and the posterior distribution of the parameters
   - Particle learning by Carvalho et al. (2010); see also Mertens and Nason (2020)
   - Mixture of Normal distributions as approximation of the posterior of τ (Liu and West, 2001)

▸ Filter

Parameter estimates - prior and posterior distributions

Prior				Posterior	
Parameter	Density	Mean	Standard Deviation	Model L	Model PWL
k_1	Normal	0.0	0.6	-0.15 [-0.49 0.19]	-0.07 [-0.51 0.38]
k_2	Normal	0.0	0.6		-0.92 [-1.35 -0.47]
τ	Normal	4.0	0.3		4.09 [3.88 4.29]
c	Normal	4.0	0.75	3.53 [3.28 3.78]	2.93 [2.68 3.18]
	Density	Mean	Degrees of freedom		
σ_π^2	Inverse Gamma	0.25^2	15	0.2^2 [0.18^2 0.23^2]	0.23^2 [0.21^2 0.26^2]
σ_y^2	Inverse Gamma	0.5^2	15	0.49^2 [0.45^2 0.54^2]	0.59^2 [0.54^2 0.66^2]
σ_g^2	Inverse Gamma	0.05^2	15	0.043^2 [0.039^2 0.048^2]	0.05^2 [0.042^2 0.058^2]
σ_z^2	Inverse Gamma	0.15^2	15	0.14^2 [0.13^2 0.16^2]	0.17^2 [0.14^2 0.19^2]

Posterior median and the 90% probability interval in brackets

Estimation results - Linear model

A vertical (or flat) long-run Phillips curve

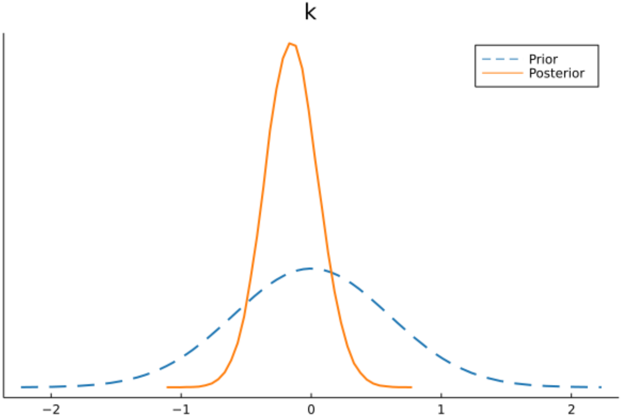


Figure: Posterior distributions of the slope of the LRPC - Linear model.

Estimation results - Linear model

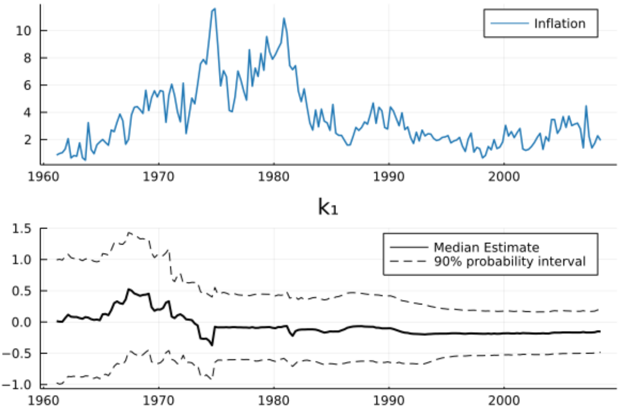


Figure: Inference of the slope k_1 - Linear model.

Estimation results - Piecewise linear model

Non linear and negatively sloped long-run Phillips curve

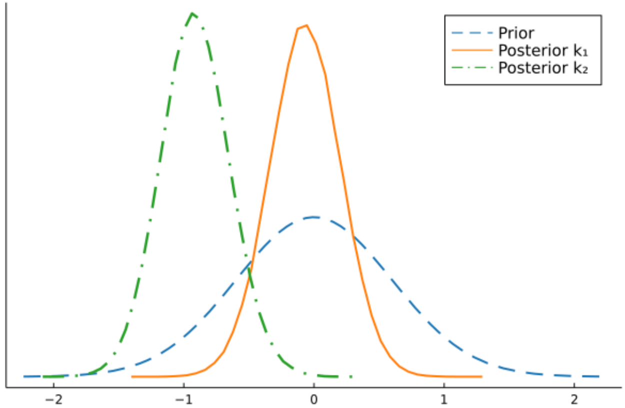


Figure: Posterior distributions of the slopes of the LRPC - Piecewise linear model.

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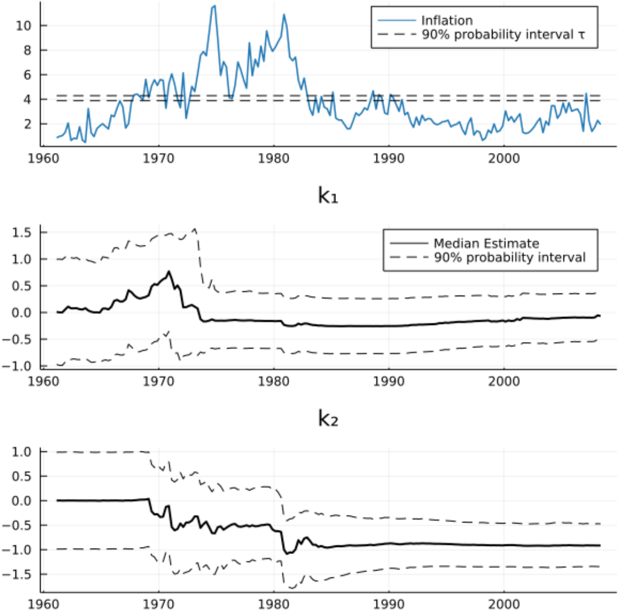

Long-run Phillips curve - Piecewise linear model

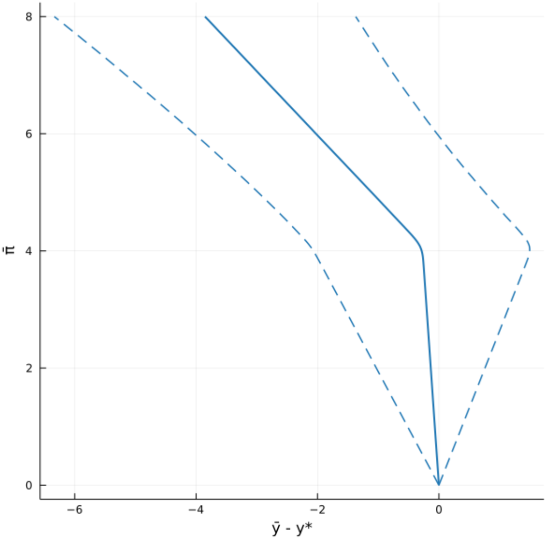

Figure: LRPC - Piecewise linear model. Median and 90% probability interval.

Trend inflation estimate - Piecewise linear model

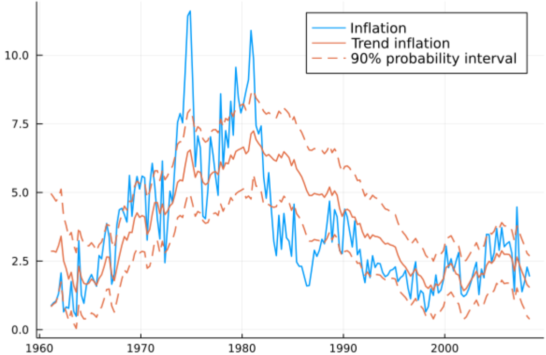

Figure: Inflation and trend inflation - Piecewise linear model.

The cost of trend inflation: the long-run output gap

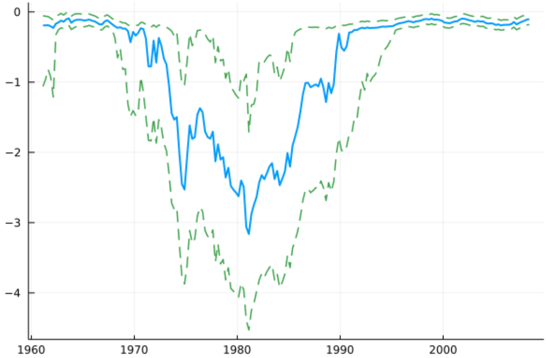

Figure: Long-run output gap estimated through the piecewise linear model.

The structural model

- A variant of Ascari and Ropele (2009) GNK model:
  - (external) habit formation in consumption
  - Generalized NKPC with trend inflation (no indexation)
  - Taylor-type monetary policy rule
- Time varying stochastic trend inflation $=>$ methodological contribution, estimate a DSGE model with time-varying steady state
- Four structural shocks: discount factor, technology, monetary policy and trend inflation (allow for stochastic volatility)
- LRPC is:
  - Non-linear
  - Negatively sloped

GNK model details

The costs of trend inflation

- Higher trend inflation increases the average markup thus reducing aggregate output

- Price stickiness => price dispersion, dispersion in the demand for goods and therefore inefficiency in the quantity produced

$$N_t = \int_0^1 N_{i,t} di = \int_0^1 \left(\frac{Y_{i,t}}{A_t} \right)^{\frac{1}{1-\alpha}} di = \underbrace{\int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{\frac{-\varepsilon}{1-\alpha}} di}_{s_t} \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}}$$

Aggregate output is:

$$Y_t = \frac{A_t}{s_t^{1-\alpha}} N_t^{1-\alpha}$$

with long-run price dispersion: $\bar{s}_t = g(\underset{+}{\bar{\pi}_t})$

- Higher trend inflation leads to higher average markup and price dispersion and therefore increases output inefficiency

Estimation

The parameters of the model depend on trend inflation:

$$\Gamma_0(\overline{\pi}_t)\hat{Z}_t = \Gamma_1(\overline{\pi}_t)\hat{Z}_{t-1} + \Psi(\overline{\pi}_t)\varepsilon_t + \Pi(\overline{\pi}_t)\eta_t, \tag{5}$$

So the state space has time varying coefficients:

$$\begin{aligned} y_t &= c_1 + F\hat{Z}_t \\ \hat{Z}_t &= c_{2,t} + M_{z,t}\hat{Z}_{t-1} + M_{\varepsilon,t}\varepsilon_t \qquad \varepsilon_t \sim N(0, \Sigma_{\varepsilon,t}) \end{aligned} \tag{6}$$

- Two sources of non linearity: stochastic volatility and time-varying trend inflation
- We use the same particle filtering strategy as before

GNK model estimates

Trend inflation estimate - GNK model

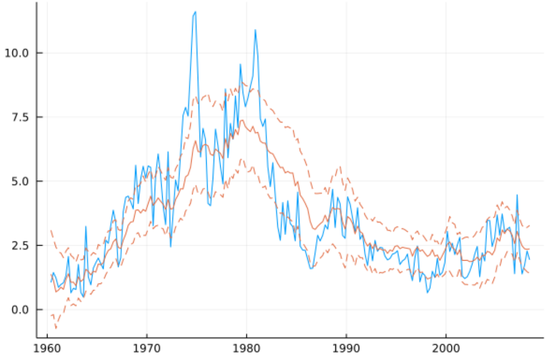

Figure: Inflation and trend inflation - GNK model.

Stochastic volatility estimates

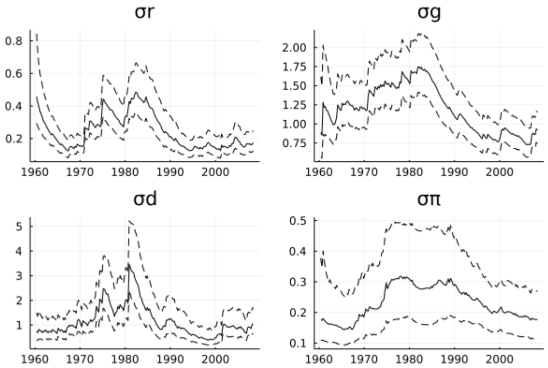

Figure: Stochastic volatility of the structural shocks

Comparing long-run Phillips curve: VAR and GNK

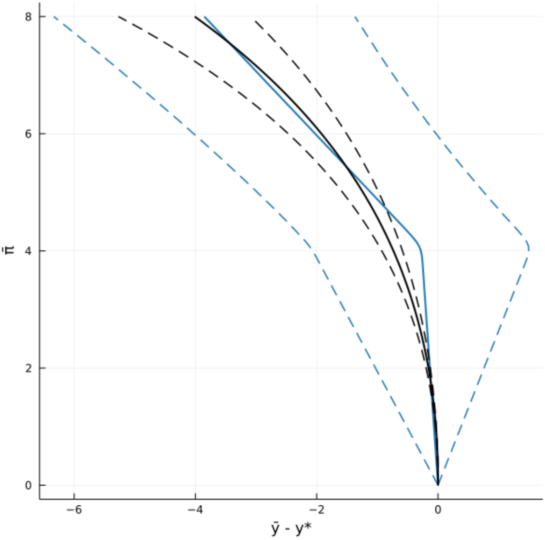

Figure: LRPC: median (continuous line) and 90% probability interval (dashed lines) - comparison between VAR (blue) and GNK (black)

Comparing long-run output gap: VAR and GNK

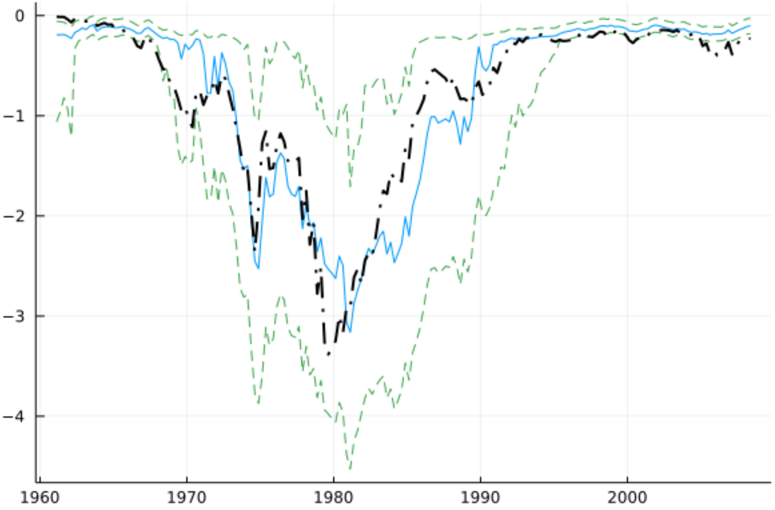

Figure: Comparison between long-run output gap estimates: VAR (blue) and GNK (black).

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   - The model has the two key features from the statistical analysis: non-linear and negatively sloped LRPC
   - The estimate from this structural model yields costs implied by the LRPC statistically consistent with the time series model

EXTRA

Econometric strategy

We use a particle filtering strategy to approximate the joint posterior distribution of latent processes and parameters:

Latent processes: a "conditional piecewise linear model"

$$p\left(\theta_t, \log s_t | \theta_{t-1}, \log s_{t-1}, X_{1:t}\right) = \underbrace{p\left(\theta_t | \log s_t, \theta_{t-1}, \log s_{t-1}, X_{1:t}\right)}_{\text{"}Full\ \ conditional\ \ posterior\text{"}} \ \underbrace{p\left(\log s_t | \theta_{t-1}, \log s_{t-1}, Y_{1:t}\right)}_{\text{"}blind\ \ proposal\text{"}}$$

Parameters:

- Particle learning by Carvalho Johannes Lopes and Polson (2010); see also Mertens and Nason (2020)
- Mixture of Normal distributions as approximation of the posterior of τ (Liu and West, 2001)

Strategy for parameter learning follows Chen, Petralia and Lopes (2010) and Ascari, Bonomolo and Lopes (2019)

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Fully adapted particle filter

At $t-1$: $\left\{\theta_{t-1}^{(i)}\right\}_{i=1}^{N}$ with corresponding weights $\left\{w_{t-1}^{(i)}\right\}_{i=1}^{N}$ approximate $p\left(\theta_{t-1}|X_{1:t-1}\right)$

(1) RESAMPLE

(a) Compute $\tilde{w}_t^{(i)} \propto p\left(X_t|\theta_{t-1}^{(i)}, X_{1:t-1}\right)$

(b) Resample $\left\{\theta_{t-1}^{(i)}\right\}_{i=1}^{N}$ using $\left\{\tilde{w}_t^{(i)}\right\}_{i=1}^{N}$ and get $\left\{\tilde{\theta}_{t-1}^{(i)}\right\}_{i=1}^{N}$

(2) PROPAGATE

Draw $\theta_t^{(i)} \sim p\left(\theta_t|\tilde{\theta}_{t-1}^{(i)}, X_{1:t}\right)$

- $p\left(X_t|\theta_{t-1}^{(i)}, X_{1:t-1}\right)$ is a weighted sum of Unified Skew Normal distributions (Arellano-Valle and Azzalini, 2006)
- $p\left(\theta_t|\tilde{\theta}_{t-1}^{(i)}, X_{1:t-1}\right)$ is a weighted sum of multivariate truncated Normal distributions

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Partially adapted particle filter

At $t-1$: $\left\{\theta_{t-1}^{(i)}, \log s_{t-1}^{(i)}\right\}_{i=1}^{N}$ with corresponding weights $\left\{w_{t-1}^{(i)}\right\}_{i=1}^{N}$ approximate $p\left(\theta_{t-1}, \log s_{t-1} | X_{1:t-1}\right)$

(1) RESAMPLE

(a) Compute $\tilde{w}_t^{(i)} \propto w_{t-1}^{(i)} \; p\left(X_t | \theta_{t-1}^{(i)}, g\left(\log s_{t-1}^{(i)}\right), X_{1:t-1}\right)$

(b) Resample $\left\{\theta_{t-1}^{(i)}, \log s_{t-1}^{(i)}\right\}_{i=1}^{N}$ using $\left\{\tilde{w}_t^{(i)}\right\}_{i=1}^{N}$

Let the new particles be $\left\{\tilde{\theta}_{t-1}^{(i)}, \log \tilde{s}_{t-1}^{(i)}\right\}_{i=1}^{N}$.

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Partially adapted particle filter

(2) PROPAGATE

(a) Draw $\log s_t^{(i)} \sim N\left(\log \tilde{s}_{t-1}^{(i)}, \Sigma_\nu\right)$

(b) Draw $\theta_t^{(i)} \sim p\left(\theta_t | \log s_t^{(i)}, \tilde{\theta}_{t-1}^{(i)}, X_{1:t}\right)$

(3) NEW WEIGHTS

Compute $w_t^{(i)} \propto \frac{p\left(X_t | \tilde{\theta}_{t-1}^{(i)}, \log s_t^{(i)}, X_{1:t-1}\right)}{p\left(X_t | \tilde{\theta}_{t-1}^{(i)}, g\left(\log \tilde{s}_{t-1}^{(i)}\right), X_{1:t-1}\right)}$

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Household

The economy is populated by a representative agent with utility

$$E_0 \sum_{t=0}^{\infty} \beta^t d_t \left[\ln\left(C_t - h\mathbf{C}_{t-1}\right) - d_n \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

Budget constraint is given by

$$P_t C_t + R_t^{-1} B_t = W_t N_t + D_t + B_{t-1}$$

d_t is a discount factor shock which follows an AR(1) process

$$\ln d_t = \rho_d \ln d_{t-1} + \epsilon_{d,t}$$

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Final good firm

Perfectly competitive final good firms combine intermediate inputs

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad \varepsilon > 1$$

Price index is a CES aggregate of intermediate input prices

$$P_t = \left[\int_0^1 {P_{i,t}}^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$$

The demand schedule for intermediate input

$$Y_{i,t} = \left[\frac{P_{i,t}}{P_t}\right]^{-\varepsilon} Y_t$$

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Intermediate good firm

Each firm i produces according to the production function

$$Y_{i,t} = A_t N_{i,t}^{1-\alpha}$$

where A_t denotes the level of technology and its growth rate $g_t \equiv A_t / A_{t-1}$ follows

$$\ln g_t = \ln \overline{g} + \epsilon_{g,t}$$

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Price setting

Firms adjust prices $P^*_{i,t}$ to maximize expected discounted profits with probability $0 < 1 - \theta < 1$

$$E_t \sum_{j=0}^{\infty} \theta^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left[\frac{P^*_{i,t}}{P_{t+j}} Y_{i,t+j} - \frac{W_{t+j}}{P_{t+j}} \left[\frac{Y_{i,t+j}}{A_{t+j}} \right]^{\frac{1}{1-\alpha}} \right]$$

subject to the demand schedule

$$Y_{i,t+j} = \left[\frac{P^*_{i,t}}{P_{t+j}} \right]^{-\varepsilon} Y_{t+j},$$

where λ_t is the marginal utility of consumption.

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The Phillips curve

The first order condition for the optimized relative price $x_t (= \frac{P^*_{i,t}}{P_t})$ is given by

$$\left(x_t\right)^{1+\frac{\varepsilon\alpha}{1-\alpha}} = \frac{\varepsilon}{(\varepsilon-1)(1-\alpha)} \frac{E_t \sum_{j=0}^{\infty} (\theta\beta)^j \lambda_{t+j} \frac{W_{t+j}}{P_{+j}} \left[\frac{Y_{t+j}}{A_{t+j}}\right]^{\frac{1}{1-\alpha}} \pi_{t|t+j}^{\frac{\varepsilon}{(1-\alpha)}}}{E_t \sum_{j=0}^{\infty} (\theta\beta)^j \lambda_{t+j} \pi_{t|t+j}^{\varepsilon-1} Y_{t+j}}.$$

where $\pi_{t|t+j} = \frac{P_{t+1}}{P_t} \times ... \times \frac{P_{t+j}}{P_{t+j-1}}$ for $j \geqslant 1$ and $\pi_{t|t} = \pi_t$.

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Price setting contd.

Aggregate price level evolves according to

$$\begin{aligned} P_t &= \left[\int_0^1 {P_{i,t}}^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \Rightarrow \\ x_t &= \left[\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}. \end{aligned}$$

Finally, price dispersion $s_t \equiv \int_0^1 (\frac{P_{i,t}}{P_t})^{-\varepsilon} di$ can be written recursively as:

$$s_t = (1-\theta)x_t^{-\varepsilon} + \theta\pi_t^{\varepsilon} s_{t-1}$$

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Monetary policy

$$\frac{R_t}{\overline{R}_t} = \left(\frac{R_{t-1}}{\overline{R}_t}\right)^{\rho} \left[\left(\frac{\pi_t}{\overline{\pi}_t}\right)^{\psi_\pi} \left(\frac{Y_t}{Y_t^n}\right)^{\psi_x} \left(\frac{g_t^y}{\overline{g}}\right)^{\psi_{\Delta y}} \right]^{1-\rho} e^{\varepsilon_{r,t}}$$

$$\ln \overline{\pi}_t = \ln \overline{\pi}_{t-1} + \epsilon_{\overline{\pi},t}$$

where $\overline{\pi}_t$ denotes trend inflation, Y_t^n is the flex-price output and g_t^y is growth rate of output.

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Parameter estimates - prior and posterior distributions

Parameter	Prior			Posterior
	Density	Mean	St Dev	
ψ_π	Gamma	1.5	0.5	2.36 [2.04 2.7]
ψ_x	Gamma	0.125	0.05	0.13 [0.08 0.21]
$\psi_{\Delta y}$	Gamma	0.125	0.05	0.34 [0.2 0.56]
ρ	Beta	0.7	0.1	0.75 [0.72 0.79]
h	Beta	0.5	0.1	0.39 [0.33 0.45]
r^*	Gamma	2	0.5	2.12 [1.87 2.41]
θ	Beta	0.5	0.1	0.49 [0.45 0.52]
ρ_d	Beta	0.7	0.1	0.79 [0.74 0.83]

Posterior median and 90% credibility interval in brackets

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Implications for short-run output gap

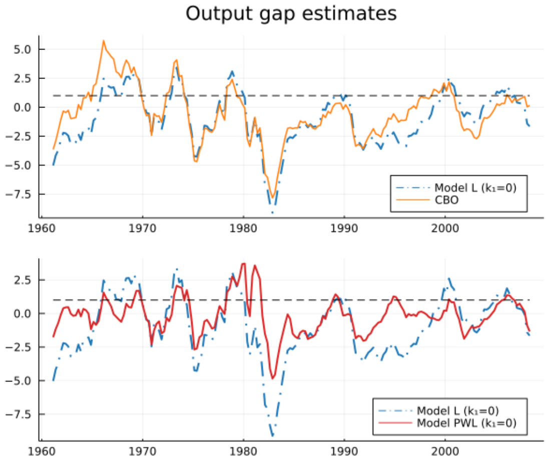

Figure: Comparison between short-run output gap estimates.