The Long-Run Phillips Curve is... a Curve

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Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank
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An old debate: is there any trade-off between inflation and output/unemployment in the long run?

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⇒ the long-run Phillips curve is vertical
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- ...and practice: working assumption of central banks in the implementation of monetary policy

⇒ “Inflation is a monetary phenomenon”
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It is somewhat surprising to note that:

- **Empirically**: There is little econometric work devoted to test the absence of a long-run trade-off.

  Related literature: King and Watson (1994, 1997); Svensson (2015); Beyer and Farmer (2007); Berentsen et al. (2011); Haug and King (2014); Benati (2015)
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⇒ “Non-superneutrality”

▶ For example, modern macroeconomic sticky price frameworks generally do not imply the absence of a long-run relation. The Generalized NK model delivers a negative relationship between steady state inflation and output. E.g., Ascari (2004), Ascari and Sbordone (2014)
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   Search for a structural model / interpretation
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   - The LRPC is not vertical, it is negatively sloped (higher inflation is related to lower output in the LR)

   - When trend inflation $\leq 4\%$ the LRPC is vertical
   - When trend inflation $\geq 4\%$ the LRPC is negatively sloped: every percentage point increase in trend inflation is related to about 1% decrease in potential output per year

2. Structural model
   - Not very far: GNK model (Ascari 2004; Ascari and Sbordone, 2014): higher trend inflation causes lower output in the LR
   - The model has the two key features from the statistical analysis: non-linear and negatively sloped LRPC
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Three observables: GDP per capita, inflation and interest rate
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\[ X_t = \bar{X}_t + \hat{X}_t \]
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- $X_t$ is a $(n \times 1)$ vector with observed variables at time $t$
- $\bar{X}_t$ is the vector with the long-run values of $X_t$
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**Cyclical component:**

\( \hat{X}_t \) described by an unrestricted VAR as in (1): stable component with unconditional expectation equal to zero

\[
A(L) (X_t - \bar{X}_t) = \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_{\varepsilon,t})
\] (1)
The model for the long-run

\[ \tilde{y}_t = y_t^* + \delta(\tilde{\pi}_t) \]  
the trend output as a function of trend inflation

\[ \delta(\tilde{\pi}_t): \delta(0) = 0 \]

\[ y_t^* = y_{t-1}^* + g_t + \eta_t^y \]

\[ g_t = g_{t-1} + \eta_t^g \]

\[ \tilde{\pi}_t = \tilde{\pi}_{t-1} + \eta_t^\pi \]  
trend inflation is random walk

\[ \tilde{i}_t - \tilde{\pi}_t = cg_t + z_t \]  
long-run Fisher equation

\[ z_t = z_{t-1} + \eta_t^z \]
A non-linear long-run Phillips curve

Our choice of \( \delta(\bar{\pi}_t) \) is a piecewise linear function:

\[
\bar{y}_t = y_t^* + \delta(\bar{\pi}_t)
\]

\[
\delta(\bar{\pi}_t) = \begin{cases} 
     k_1 \bar{\pi}_t & \text{if } \bar{\pi}_t \leq \tau \\
     k_2 \bar{\pi}_t + c_k & \text{if } \bar{\pi}_t > \tau 
\end{cases}
\]

- It can approximate potential non-linearity without imposing strong assumptions on the functional form
- It is easy to interpret
- It is simpler to treat: methodological contribution
A piecewise linear approach

The model can be written as:

\[
\bar{X}_t = D(\theta_t) + H(\theta_t) \theta_t \\
\theta_t = M(\theta_t) + G(\theta_t) \theta_{t-1} + P(\theta_t) \eta_t
\]

where, in particular

\[
(D, H, M, G, P) = \begin{cases} 
(D_1, H_1, M_1, G_1, P_1) & \text{if } \bar{\pi}_t \leq \tau \\
(D_2, H_2, M_2, G_2, P_2) & \text{if } \bar{\pi}_t > \tau
\end{cases}
\]

- Methodological contribution: we characterize the likelihood and the posterior distribution of \( \theta_t = (\bar{y}_t, \bar{\pi}_t, g_t, z_t) \) analytically

- Compromise between efficiency and misspecification
Black: no misspecification - likelihood and posterior not available analytically
Red: misspecification - likelihood and posterior available analytically
Green: less misspecification - likelihood and posterior available analytically
Estimation

Bayesian approach, US data, sample from 1960Q1 to 2008Q2

Two sources of non linearity: stochastic volatility and a piecewise linear LRPC \( \Rightarrow \) Particle filtering approach

1. **Latent processes**: “Rao-Blackwellization”, thanks to the analytical results on the piecewise linear model \( \Rightarrow \) we can analytically characterize the likelihood function and posterior distribution of the latent states

2. **Parameters**: Particle filtering allows to jointly approximate the latent states and the posterior distribution of the parameters
   - Particle learning by Carvalho et al. (2010); see also Mertens and Nason (2020)
   - Mixture of Normal distributions as approximation of the posterior of \( \tau \) (Liu and West, 2001)
Parameter estimates - prior and posterior distributions

<table>
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<th>Parameter</th>
<th>Density</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Model L</th>
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<td>[0.13² 0.16²]</td>
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Posterior median and the 90% probability interval in brackets
Estimation results - Linear model

A vertical (or flat) long-run Phillips curve

Figure: Posterior distributions of the slope of the LRPC - Linear model.
Estimation results - Linear model

**Figure:** Inference of the slope $k_1$ - Linear model.
Estimation results - Piecewise linear model

Non linear and negatively sloped long-run Phillips curve

Figure: Posterior distributions of the slopes of the LRPC - Piecewise linear model.
Estimation results - Piecewise linear model

Figure: Inference of the slopes $k_1$ and $k_2$ - Piecewise linear model.
Long-run Phillips curve - Piecewise linear model

Figure: LRPC - Piecewise linear model. Median and 90% probability interval.
Trend inflation estimate - Piecewise linear model

Figure: Inflation and trend inflation - Piecewise linear model.
The cost of trend inflation: the long-run output gap

Figure: Long-run output gap estimated through the piecewise linear model.
The structural model

- A variant of Ascari and Ropele (2009) GNK model:
  - (external) habit formation in consumption
  - Generalized NKPC with trend inflation (no indexation)
  - Taylor-type monetary policy rule

- Time varying stochastic trend inflation $\implies$ methodological contribution, estimate a DSGE model with time-varying steady state

- Four structural shocks: discount factor, technology, monetary policy and trend inflation (allow for stochastic volatility)

- LRPC is:
  - Non-linear
  - Negatively sloped
The costs of trend inflation

- Higher trend inflation increases the average markup thus reducing aggregate output

- Price stickiness $\Rightarrow$ price dispersion, dispersion in the demand for goods and therefore inefficiency in the quantity produced

\[
N_t = \int_0^1 N_{i,t} \, di = \int_0^1 \left( \frac{Y_{i,t}}{A_t} \right)^{\frac{1}{1-\alpha}} \, di = \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \, di
\]

Aggregate output is:

\[
Y_t = \frac{A_t}{s_t^{1-\alpha}} N_t^{1-\alpha}
\]

with long-run price dispersion: $\bar{s}_t = g(\bar{\pi}_t)$

- Higher trend inflation leads to higher average markup and price dispersion and therefore increases output inefficiency
Estimation

The parameters of the model depend on trend inflation:

\[ \Gamma_0(\pi_t)\hat{Z}_t = \Gamma_1(\pi_t)\hat{Z}_{t-1} + \Psi(\pi_t)\varepsilon_t + \Pi(\pi_t)\eta_t, \]  

(5)

So the state space has time varying coefficients:

\[ y_t = c_1 + F\hat{Z}_t \]
\[ \hat{Z}_t = c_{2,t} + M_{z,t}\hat{Z}_{t-1} + M_{\varepsilon,t}\varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_{\varepsilon,t}) \]  

(6)

▶ Two sources of non linearity: stochastic volatility and time-varying trend inflation

▶ We use the same particle filtering strategy as before
Trend inflation estimate - GNK model

Figure: Inflation and trend inflation - GNK model.
Stochastic volatility estimates

Figure: Stochastic volatility of the structural shocks
Comparing long-run Phillips curve: VAR and GNK

Figure: LRPC: median (continuous line) and 90% probability interval (dashed lines) - comparison between VAR (blue) and GNK (black)
Comparing long-run output gap: VAR and GNK

Figure: Comparison between long-run output gap estimates: VAR (blue) and GNK (black).
Conclusions

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2. Structural model
   - We interpret these findings through the lens of a GNK model (Ascari 2004; Ascari and Sbordone, 2014): higher trend inflation causes lower output in the LR
   - The model has the two key features from the statistical analysis: non-linear and negatively sloped LRPC
   - The estimate from this structural model yields costs implied by the LRPC statistically consistent with the time series model
EXTRA
Econometric strategy

We use a particle filtering strategy to approximate the joint posterior distribution of latent processes and parameters:

**Latent processes:** a “conditional piecewise linear model”

\[
p (\theta_t, \log s_t | \theta_{t-1}, \log s_{t-1}, X_{1:t}) = p (\theta_t | \log s_t, \theta_{t-1}, \log s_{t-1}, X_{1:t}) \quad \text{"Full conditional posterior"}
\]

\[
p (\log s_t | \theta_{t-1}, \log s_{t-1}, Y_{1:t}) \quad \text{"blind proposal"}
\]

**Parameters:**

- Particle learning by Carvalho Johannes Lopes and Polson (2010); see also Mertens and Nason (2020)
- Mixture of Normal distributions as approximation of the posterior of \(\tau\) (Liu and West, 2001)

Fully adapted particle filter

At $t - 1$: $\left\{ \theta_{t-1}^{(i)} \right\}_{i=1}^{N}$ with corresponding weights $\left\{ w_{t-1}^{(i)} \right\}_{i=1}^{N}$ approximate $p(\theta_{t-1} \mid X_{1:t-1})$

(1) RESAMPLE

(a) Compute $\tilde{w}_{t}^{(i)} \propto p \left( X_{t} \mid \theta_{t-1}^{(i)}, X_{1:t-1} \right)$

(b) Resample $\left\{ \theta_{t-1}^{(i)} \right\}_{i=1}^{N}$ using $\left\{ \tilde{w}_{t}^{(i)} \right\}_{i=1}^{N}$ and get $\left\{ \tilde{\theta}_{t-1}^{(i)} \right\}_{i=1}^{N}$

(2) PROPAGATE

Draw $\theta_{t}^{(i)} \sim p \left( \theta_{t} \mid \tilde{\theta}_{t-1}^{(i)}, X_{1:t} \right)$

- $p \left( X_{t} \mid \theta_{t-1}^{(i)}, X_{1:t-1} \right)$ is a weighted sum of Unified Skew Normal distributions (Arellano-Valle and Azzalini, 2006)
- $p \left( \theta_{t} \mid \tilde{\theta}_{t-1}^{(i)}, X_{1:t-1} \right)$ is a weighted sum of multivariate truncated Normal distributions
Partially adapted particle filter

At $t - 1$: \( \{ \theta^{(i)}_{t-1}, \log s^{(i)}_{t-1} \}_{i=1}^{N} \) with corresponding weights \( \{ w^{(i)}_{t-1} \}_{i=1}^{N} \) approximate \( p(\theta_{t-1}, \log s_{t-1}|X_{1:t-1}) \)

(1) RESAMPLE

(a) Compute \( \tilde{w}^{(i)}_{t} \propto w^{(i)}_{t-1} p(X_{t}|\theta^{(i)}_{t-1}, g(\log s^{(i)}_{t-1}), X_{1:t-1}) \)

(b) Resample \( \{ \theta^{(i)}_{t-1}, \log s^{(i)}_{t-1} \}_{i=1}^{N} \) using \( \{ \tilde{w}^{(i)}_{t} \}_{i=1}^{N} \)

Let the new particles be \( \{ \tilde{\theta}^{(i)}_{t-1}, \log \tilde{s}^{(i)}_{t-1} \}_{i=1}^{N} \).
Partially adapted particle filter

(2) PROPAGATE
   (a) Draw \( \log s_t^{(i)} \sim N \left( \log \tilde{s}_{t-1}^{(i)}, \Sigma_v \right) \)
   (b) Draw \( \theta_t^{(i)} \sim p \left( \theta_t | \log s_t^{(i)}, \tilde{\theta}_{t-1}^{(i)}, X_{1:t} \right) \)

(3) NEW WEIGHTS
   Compute \( w_t^{(i)} \propto \frac{p \left( X_t | \tilde{\theta}_{t-1}^{(i)}, \log s_t^{(i)}, X_{1:t-1} \right)}{p \left( X_t | \tilde{\theta}_{t-1}^{(i)}, g \left( \log \tilde{s}_{t-1}^{(i)} \right), X_{1:t-1} \right)} \)
Household

The economy is populated by a representative agent with utility

\[ E_0 \sum_{t=0}^{\infty} \beta^t d_t \left[ \ln (C_t - hC_{t-1}) - d_n \frac{N_t^{1+\varphi}}{1 + \varphi} \right] \]

Budget constraint is given by

\[ P_t C_t + R_t^{-1} B_t = W_t N_t + D_t + B_{t-1} \]

d\_t \text{ is a discount factor shock which follows an AR(1) process}

\[ \ln d_t = \rho_d \ln d_{t-1} + \epsilon_{d,t} \]
Final good firm

Perfectly competitive final good firms combine intermediate inputs

\[ Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{\varepsilon - 1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon - 1}} \quad \varepsilon > 1 \]

Price index is a CES aggregate of intermediate input prices

\[ P_t = \left[ \int_0^1 P_{i,t}^{1-\varepsilon} \, di \right]^{\frac{1}{1-\varepsilon}} \]

The demand schedule for intermediate input

\[ Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} Y_t \]
Intermediate good firm

Each firm $i$ produces according to the production function

$$Y_{i,t} = A_t N_{i,t}^{1-\alpha}$$

where $A_t$ denotes the level of technology and its growth rate $g_t \equiv A_t / A_{t-1}$ follows

$$\ln g_t = \ln \bar{g} + \epsilon_{g,t}$$
Price setting

Firms adjust prices $P^*_i, t$ to maximize expected discounted profits with probability $0 < 1 - \theta < 1$

$$E_t \sum_{j=0}^{\infty} \theta^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left[ \frac{P^*_i, t}{P_{t+j}} Y_{i, t+j} - \frac{W_{t+j}}{P_{t+j}} \left[ \frac{Y_{i, t+j}}{A_{t+j}} \right]^{\frac{1}{1-\alpha}} \right]$$

subject to the demand schedule

$$Y_{i, t+j} = \left[ \frac{P^*_i, t}{P_{t+j}} \right]^{-\varepsilon} Y_{t+j},$$

where $\lambda_t$ is the marginal utility of consumption.
The Phillips curve

The first order condition for the optimized relative price \( x_t(= \frac{P_{i,t}^*}{P_t}) \) is given by

\[
(x_t)^{1+\frac{\varepsilon\alpha}{1-\alpha}} = \frac{\varepsilon}{(\varepsilon - 1)(1 - \alpha)} \frac{E_t \sum_{j=0}^{\infty} (\theta \beta)^j \lambda_{t+j} \frac{W_{t+j}}{P_{t+j}} \left[ \frac{Y_{t+j}}{A_{t+j}} \right]^{1-\alpha}}{E_t \sum_{j=0}^{\infty} (\theta \beta)^j \lambda_{t+j} \pi_{t|t+j}^{\varepsilon-1} Y_{t+j}} \pi_{t|t+j}^{\frac{\varepsilon}{1-\alpha}}.
\]

where \( \pi_{t|t+j} = \frac{P_{t+1}}{P_t} \times \ldots \times \frac{P_{t+j}}{P_{t+j-1}} \) for \( j \geq 1 \) and \( \pi_{t|t} = \pi_t \).
Aggregate price level evolves according to

\[ P_t = \left[ \int_0^1 P_{i,t}^{1-\varepsilon} \, di \right]^{\frac{1}{1-\varepsilon}} \Rightarrow \]

\[ x_t = \left[ \frac{1 - \theta \pi_t^{\varepsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}}. \]

Finally, price dispersion \( s_t \equiv \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \, di \) can be written recursively as:

\[ s_t = (1 - \theta)x_t^{-\varepsilon} + \theta \pi_t^{\varepsilon} s_{t-1} \]
Monetary policy

\[
\frac{R_t}{R_t} = \left( \frac{R_{t-1}}{R_t} \right)^\rho \left[ \left( \frac{\pi_t}{\bar{\pi}_t} \right)^\psi_\pi \left( \frac{Y_t}{Y_t^n} \right)^\psi_x \left( \frac{g_t^y}{g} \right)^\psi_{\Delta y} \right]^{1-\rho} \epsilon_{r,t}
\]

\[
\ln \bar{\pi}_t = \ln \bar{\pi}_{t-1} + \epsilon_{\pi,t}
\]

where \( \bar{\pi}_t \) denotes trend inflation, \( Y_t^n \) is the flex-price output and \( g_t^y \) is growth rate of output.
### Parameter estimates - prior and posterior distributions

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<th>Posterior</th>
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Posterior median and 90% credibility interval in brackets.
Implications for short-run output gap

Figure: Comparison between short-run output gap estimates.