Can Deficits Finance Themselves?

Marios Angeletos  Chen Lian  Christian Wolf
Northwestern    Berkeley    MIT

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How are deficits financed? (when $r > g$)

\[
\text{gov't debt} = \text{PDV of primary surpluses}
\]

- **Standard margin**: fiscal adjustment—raise taxes (or cut spending) in the future

- "Self-financing"—close shortfall via eq'm changes in prices & quantities

| $Q$: how important is such self-financing? can there ever be full self-financing? | Angeletos, Lian, and Wolf |
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- We’ll investigate another margin that arises with liquidity frictions [HANK, OLG, …]
  - Simple mechanism: deficit today $\rightarrow$ demand-driven boom $\rightarrow$ tax base ↑, inflation ↑
    Will operate even if fiscal policy is “passive/Ricardian”, and even if the Taylor principle is satisfied.
  - “Self-financing”—close shortfall via eq’m changes in prices & quantities
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How big can “self-financing” be?

Environment: finite lives/liquidity constraints + nominal rigidities
Policy: full fiscal adjustment promised at future date $H$ + monetary policy is “neutral” (fix $\mathbb{E}(r)$)
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- **Main result**: if fiscal adjustment is delayed enough, then get full **self-financing**
  1. **Monotonicity**: as $H$ increases, the actual required future tax hike gets smaller and smaller
  2. **Limit**: the future tax hike vanishes, i.e., we converge to full self-financing

Split depends on nominal rigidities. All via output/tax base ↑ if rigid, all via prices ↑ if flexible.
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  Why is the limit one of exact self-financing? Keynesian cross arithmetic.

- **Practical relevance**: holds in many environments & quantitatively powerful
  general aggregate demand (incl. HANK), active monetary policy, investment, distortionary taxation, …
Environment
• Aggregate demand
  ○ Unit continuum of OLG households with survival probability $\omega \in (0, 1]$. Nests standard PIH model with $\omega = 1$, and mimics HANK with $\omega < 1$. 

\[
c_t = (1 - \beta \omega) \left| \text{MPC} \right| (1 - \beta \omega) \left| \text{wealth} \right| + E^{\infty} \sum_{k=0}^{\infty} \left( \beta \omega \right)^k (y_t + k - t) - \gamma E^{\infty} \sum_{k=0}^{\infty} \left( \beta \omega \right)^k r_t.
\] 

Key features: (i) elevated MPC + (ii) addt'l discounting of future income & taxes

• Aggregate supply
  ○ Nominal rigidities + union bargaining gives a standard NKPC relation:

\[
\pi_t = \kappa y_t + \beta E_t [\pi_t + 1].
\] 

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Non-policy block

• Aggregate demand
  ○ Unit continuum of OLG households with survival probability \( \omega \in (0, 1] \). Nests standard PIH model with \( \omega = 1 \), and mimics HANK with \( \omega < 1 \).
  ○ Optimal consumption-savings behavior yields aggregate demand relation: Details

\[
c_t = (1 - \beta \omega) \times \left( d_t + \mathbb{E}_t \left[ \sum_{k=0}^{\infty} (\beta \omega)^k (y_{t+k} - t_{t+k}) \right] - \gamma \mathbb{E}_t \left[ \sum_{k=0}^{\infty} (\beta \omega)^k r_{t+k} \right] \right)
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(2)
• Monetary policy

  ○ Monetary authority responds to output fluctuations:

  \[
  i_t - \mathbb{E}_t [\pi_{t+1}] = \phi \times y_t \equiv r_t
  \]

  ○ First consider “neutral” monetary policy with \( \phi = 0 \)—no monetary help. Later generalize.
Policy

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- **Fiscal policy**
  - Issue nominal debt. Log-linearized government budget constraint (in real terms):
    \[
    d_{t+1} = (1 + \bar{r}) \times (d_t - t_t) + \frac{\bar{d}}{\bar{y}} r_t - \frac{\bar{d}}{\bar{y}} (\pi_{t+1} - \mathbb{E}_t [\pi_{t+1}])
    \]  
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      \[ (4) \]
  - Taxes adjust **gradually** to balance gov’t budget, where \( \tau_d \) parameterizes **delay**:
    \[ t_t = \tau_d \times (d_t + \varepsilon_t) + \tau_y y_t - \varepsilon_t \]  
      \[ \text{fiscal adjustment} \quad \text{tax base financing} \quad \text{“stimulus checks”} \]  
      \[ (5) \]


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    \[\text{(5)}\]
    
    For transparent intuition look at \(H\)-rule: \(\tau_{d.t} = 0\) initially, then \(= 1\) after \(H\) so \(d_{H+1} = 0\).
Proposition

Suppose that $\omega < 1$ and $\tau_y > 0$. The economy (1) - (5) has a unique bounded eq’m.
Equilibrium & sources of financing

- Eq’m existence & uniqueness ➤ Full eq’m characterization

Proposition

Suppose that \( \omega < 1 \) and \( \tau_y > 0 \). The economy (1) - (5) has a unique bounded eq’m.

- Our Q: how are fiscal deficits in this eq’m financed?
  - From the intertemporal gov’t budget constraint:
    
    \[
    \varepsilon_0 = \tau_d \times \left( \varepsilon_0 + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_0 (d_k) \right) + \frac{\bar{d}}{\bar{y}} (\pi_0 - \mathbb{E}_{-1} (\pi_0)) + \sum_{k=0}^{\infty} \beta^k \tau_y \mathbb{E}_0 (y_k)
    \]
    
    - fiscal adjustment: \( (1 - \nu) \times \varepsilon_0 \)
    - debt erosion
    - tax base expansion
    - self-financing: \( \nu \times \varepsilon_0 \)

  - Next: characterize \( \nu \) as a function of fiscal adjustment delay (\( \tau_d \) or \( H \))
The Self-Financing Result
The self-financing result

**Theorem**

Suppose that $\omega < 1$ and $\tau_y > 0$. The **self-financing share** $\nu$ has the following properties:

1. **Monotonicity**: It is increasing in the delay of fiscal adjustment (i.e., increasing in $H$ and decreasing in $\tau_d$).

2. **Limit**: As fiscal financing is delayed more and more (i.e., as $H \to \infty$ or $\tau_d \to 0$), $\nu$ converges to 1. In words, delaying the tax hike makes it vanish.

In this limiting eq’m:

- **a)** Gov’t debt returns to steady state even without any fiscal adjustment: 
  \[ E_t \left[ d_t + 1 \right] = \rho_d (d_t + \varepsilon_t) \]
  where $\rho_d \in (0, 1)$.

- **b)** The share of self-financing coming from the tax base expansion is increasing in the strength of nominal rigidities. With rigid prices the cumulative output multiplier is $1/\tau_y$. 

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The self-financing result

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A graphical illustration

Output $y_t$

Gov’t Debt $d_t$

Self-Financing Share $\nu$

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A graphical illustration

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Immediate Financing

$H = 5$

$H = 10$

Date-0 Inflation

Tax Base

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A graphical illustration

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A graphical illustration

if fiscal adjustment is delayed, then financing will come via eq’m prices & quantities
The Self-Financing Result

Intuition
Economic intuition

- Background: self-financing in a “static” Keynesian cross
  - Transfer at $t = 0$, tax (if needed) at $t = 1$, assume static KC at $t = 0$.  

\[
y_0 = \frac{1}{1 - \text{mpc}} (1 - \tau) y_0 \Rightarrow \nu = \tau \frac{1}{1 - \text{mpc}} (1 - \tau) y_0
\]

- We see: $\nu$ is increasing in the mpc, with $\nu \to 1$ for $\text{mpc} \to 1$

- Our th'm: features of static model have analogues in dynamic economy (for now: $\kappa = 0$)

PE: Largely discount date-angle H, tax hike + spend date-angle 0 gain quickly, so short-run PE effect reaches 1 far before H — akin to numerator above. Then get later demand bust around H.
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Economic intuition

"short run"  "long run"

PE

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Economic intuition

”short run”

”long run”

$1/\tau_y$

$-1$

$1$

$0$

$0.5$

$0.4$

$0.3$

$0.2$

$0.1$

$0$

$-0.1$

$-0.2$

$0$

$20$

$40$

$60$

$80$

$100$

$\%$

$P_E \quad G_E$

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With imperfectly rigid prices: boom partially leaks into prices instead of quantities.
Practical Relevance
1. Policy
   - Fiscal policy: distortionary taxes, gov't purchases
   - Monetary response
     - Intuition: $\phi < 0$ accelerates the Keynesian cross, $\phi > 0$ delays it
     - Length of eq’m boom is increasing in $\phi$. Full self-financing as long as $\phi$ is not too big.

2. Economic environment
   - Rest of the economy: different NKPC, wage rigidity, investment
   - Demand relation
     - Need discounting—break Ricardian equivalence + front-load spending
     - Next: what happens in quantitative model consistent with evidence on consumer demand?
Extensions & generality

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Key targets: (i) consumer spending behavior [IMPCs] & (ii) fiscal adjustment speed

For now continue to set $\phi = 0$. In paper also investigate classical “active” monetary policy.
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  Later consider alternatives—other calibration targets, behavioral models, and a HANK model.

  (ii) Consider range of $\tau_d$ consistent with literature on fiscal adjustment rule estimation
  Galí-López-Salido-Vallés, Bianchi-Melosi, Auclert-Rognlie, …
Model & calibration strategy

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  Nominal rigidities: today standard flat NKPC, in paper explore steeper slope.
Self-financing in the quantitative model

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Takeaways
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• **Implications**
  
a) **Theory**: grounded in a classical failure of Ricardian equivalence, robust to information perturbations, consistent with Taylor principle, + emphasize \( y \) vs. \( \pi \) channel

b) **Practice**: self-sustaining stimulus may be less implausible than commonly believed
   In particular if supply constraints are slack—get self-financing via output boom.
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• **Ongoing work**: (optimal) policy implications for fiscal-monetary interaction
Appendix
Aggregate demand

- **Consumption-savings problem**
  - OLG hh’s with survival probability $\omega \in (0, 1]$ [can interpret as $\approx 1$ - prob. of liq. constraint]
    \[
    E_t \left[ \sum_{k=0}^{\infty} (\beta \omega)^k (u(C_{i,t+k}) - v(L_{i,t+k})) \right]
    \]
  - Invest in actuarially fair annuities. Budget constraint:
    \[
    A_{i,t+1} = \frac{I_t}{\omega} (A_{i,t} + P_t \cdot (W_{t}L_{i,t} + Q_{i,t} - C_{i,t} - T_{i,t} + \text{transfer to newborns}))
    \]

- **Aggregate demand relation**
  \[
  c_t = (1 - \beta \omega) \times \left( d_{t, \text{wealth}} + E_t \left[ \sum_{k=0}^{\infty} (\beta \omega)^k (y_{t+k} - t_{t+k}) \right] - \gamma E_t \left[ \sum_{k=0}^{\infty} (\beta \omega)^k r_{t+k} \right] \right)
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  Key features: (i) elevated MPC + (ii) addt’l discounting of future income & taxes

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Aggregate supply

- **Unions** equalize post-tax wage and average consumption-labor MRS. This gives

\[
(1 - \tau_y)W_t = \frac{\chi \int_0^1 L_{i,t}^{1/\varphi} di}{\int_0^1 C_{i,t}^{-1/\sigma} di}
\]

Log-linearizing:

\[
\frac{1}{\varphi} \ell_t = w_t - \frac{1}{\sigma} c_t
\]

- Combining with optimal firm pricing decisions we get the **NKPC**:

\[
\pi_t = \kappa y_t + \beta \mathbb{E}_t [\pi_{t+1}]
\]

- Note: no time-varying wedge since distortionary taxes $\tau_y$ are fixed
Equilibrium characterization

• First step to eq’m characterization is a more concise representation of agg. demand

• Combining (1), (3), (4), (5), and output market-clearing, we get

\[ y_t = \mathcal{F}_1 \cdot (d_t + \epsilon_t) + \mathcal{F}_2 \cdot \mathbb{E}_t \left[ \sum_{k=0}^{\infty} (\beta \omega)^k y_{t+k} \right] \]  

(7)

○ Here: \( \mathcal{F}_1 \equiv \frac{(1-\beta \omega)(1-\omega)(1-\tau_d)}{1-\omega(1-\tau_d)} \) and \( \mathcal{F}_2 = (1 - \beta \omega) \left( 1 - \frac{(1-\omega)\tau_y}{1-\omega(1-\tau_d)} \right) \)

○ Note: \( \mathcal{F}_1 = 0 \) if \( \omega = 1 \)—reflects lack of direct effect of deficit on consumer spending/aggregate demand under Ricardian equivalence

• Equilibrium: (2), (7) and law of motion for government debt
We will look for **bounded equilibria** in the sense of Blanchard-Kahn

- Note: in our case—with \( \omega < 1 \) and \( \tau_y > 0 \)—this is enough to rule out sunspot solutions. Recover same eq’m through limit \( \phi \to 0^+ \).

**The unique bounded eq’m takes a particularly simple form:**

\[
 y_t = \chi(d_t + \varepsilon_t), \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t + \varepsilon_t)
\]

where \( \chi > 0 \) (deficits trigger boom) and \( 0 < \rho_d < 1 \) (debt goes back to steady state).
Distortionary fiscal financing

• Environment
  ○ Fiscal adjustment now instead through distortionary tax adjustments. Specifically:
    \[ \tau_{y,t} = \tau_y + \tau_{d,t}(D_t - D^{ss}) \]
  ○ Only effect is to change (2) to
    \[ \pi_t = \kappa y_t + \beta E_t[\pi_{t+1}] + \zeta_t d_t \]

• Self-financing result
  ○ Easy to see: exactly the same limiting self-financing eq’m as before
  ○ Why? tax adjustment not necessary, so distortionary vs non-distortionary is irrelevant
Government purchases

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Monetary policy reaction

- **Intuition**: $\phi < 0$ accelerates the Keynesian cross, $\phi > 0$ delays it

**Proposition**

There exists a $\Phi > 0$ such that:

1. An equilibrium with full self-financing exists if and only if $\phi < \Phi$.
2. The persistence of $\rho_d(\phi)$ of gov't debt (and output) in the equilibrium with full self-financing is increasing in $\phi$, with $\rho_d(0) \in (0, 1)$ and $\rho_d(\Phi) = 1$.

Note: same logic for standard Taylor-type rules like $i_t = \phi \times \pi_t$.

- What happens if $\phi > \Phi$? Depends on **fiscal adjustment**:
  - If too delayed then no bounded eq’m exists. For such an aggressive monetary policy fiscal adjustment needs to be fast enough.
  - If adjustment is fast enough then there is partial but not complete self-financing.
Leeper regions
Leeper regions

\[ \tau_d \]

\[ \phi \]

Legend:
- None
- Unique
- Multiple
A generalized aggregate demand relation

- **Important**: our results are *not* tied to the particular OLG microfoundations
- Instead: it’s all about two empirically plausible features of **consumer demand**
  1. *Discounting*: households at date $t = 0$ respond little to expectations of far-ahead tax hikes
  2. *Front-loaded spending*: transfer receipt (and higher-order GE income) is spent quickly

  In OLG both of these are ensured by $\omega < 1$

- Will formalize this using the following **generalized AD relation**:

  $$c_t = M_d d_t + M_y \left( y_t - t_t + \delta \mathbb{E}_t \left[ \sum_{k=0}^{\infty} (\beta \omega)^k (y_{t+k} - t_{t+k}) \right] \right)$$

Rich enough to nest PIH, OLG, spender-saver, spender-OLG, behavioral discounting, .... Also can provide very close reduced-form fit to consumer behavior in quantitative HANK models.
A generalized aggregate demand relation

- **Headline result**: sufficient conditions for self-financing

  **A1 Discounting**

  \[ \omega < 1 \]

  Transfer today and taxes in the future redistribute from future generations to the present.

  **A2 Front-loading**

  \[
  M_d + \frac{1 - \beta}{\tau_y} (1 - \tau_y) M_y \left( 1 + \delta \frac{\beta \omega}{1 - \beta \omega} \right) > \frac{1 - \beta}{\tau_y}
  \]

  Self-financing boom is front-loaded enough to deliver \( \rho_d < 1 \).

- **Note**: the self-financing result *fails* if there are **PIH households**
  “Deep-pocket” rational investor intuition—infinitely elastic PIH hh’s link infinite future & present.
The importance of discounting

spender-saver model

Output $y_t$

Gov’t Debt $d_t$

Self-Financing Share $\nu$

- Immediate Financing
- $H = 5$
- $H = 10$
- $H = 20$
- $H = 40$

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The importance of discounting

hybrid spender-OLG model

Output $y_t$

Gov’t Debt $d_t$

- Immediate Financing
- $H = 5$
- $H = 10$
- $H = 20$
- $H = 40$

Self-Financing Share $\nu$

- Date-0 Inflation
- Tax Base

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Adding investment

• Environment

  ○ **Households**: receive labor income plus dividends $e_t$. Pay taxes $\tau y$ on both.
  ○ **Production**: standard DSGE production block. Key twist: no tax payments anywhere.

• Self-financing result

  ○ For rigid prices exactly the same self-financing eq’m as before. Why? Keynesian cross &
    gov’t budget both have $c_t$ rather than $y_t$ in them, so same pair of equations as before.
  ○ Partially sticky prices: more complicated mapping from $\{c_t\}_{t=0}^{\infty}$ back to $\pi_0$, so fixed point
    is more complicated, but can still show that self-financing eq’m exists.
    Perfectly analogous to change in NKPC. Just change mapping into $\pi_0$. 

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Alternative calibration strategies

**Baseline**: match impact and short-run MPCs, then extrapolate

Note: also consistent with evidence on long-run elasticity of asset supply.
Extension: two-type OLG + spender model to match cumulative MPC time profile

This gives $\omega_2 = 0.97$, and thus counterfactually elastic asset supply ($\approx 7 \times$ emp. upper bound).
More flexible prices

**Steeper NKPC:** arguably more informative about post-covid episode

Takeaways: (i) change $\nu_y/\nu_p$ split & (ii) faster convergence to self-financing limit
**Monetary response**: consider standard Taylor rule $i_t = \phi \times \pi_t$

Takeaways: (i) slower convergence & (ii) no self-financing eq’m exists for sufficiently large $\phi$
Other models

**Environment**: baseline + behavioral friction [strong cognitive discounting]

![Graphs showing output, inflation, and self-financing share](image-url)

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Other models

**Environment:** HANK model [similar to Wolf (2022)]

![Graph showing output, inflation, and self-financing share](image)

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