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The Bank of Amsterdam and the limits of fiat money
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Abstract

Central banks can operate with negative equity, and many have done so in history without undermining trust in fiat money. However, there are limits. How negative can central bank equity be before fiat money loses credibility? We address this question using a global games approach motivated by the fall of the Bank of Amsterdam (1609–1820). We solve for the unique break point where negative equity and asset illiquidity renders fiat money worthless. We draw lessons on the role of fiscal support and central bank capital in sustaining trust in fiat money.

Key words: central banks, negative equity, fiat money, trust

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1 Introduction

Money is a social convention. One party accepts it as payment in the expectation that others will also do so in the future. In theory, anything could serve as money provided that this convention is sustained as an equilibrium. Yet this bare definition of money does not leave much room for the institutions underpinning the monetary system, especially the role of central banks. Over the ages, various private currencies have come and gone; while some have lasted longer than others, they have invariably given way to central bank money.\footnote{For rich historical accounts, see Giannini (2011), Martin (2013) and Desan (2014).}

In this paper, we draw lessons on the central bank underpinnings of money by examining the limits beyond which trust in fiat money falls away. We examine a global games model motivated by the fall of the Bank of Amsterdam (1609-1820), perhaps the best known of the public deposit banks in Europe, and arguably an early precursor to a modern central bank in that it issued fiat money and conducted monetary policy to stabilise its value. By examining what it takes for an issuer of fiat money to fail, we may draw lessons on the central bank underpinnings of the institution of money. These lessons will have resonance and relevance even today.

How then does a central bank fail? Indeed, how could a central bank fail when it can always “print” more money? It is well-known that central banks can operate with negative equity, and many have done so throughout history without undermining trust in fiat money (see eg Stella (1997); Archer and Moser-Boehm (2013)). However, this is not to say that there are no limits. We can pose the question in the following precise way: how negative can central bank equity be before fiat money loses credibility?

We address this question using a global game model motivated by the features that eventually led to the failure of the Bank of Amsterdam. The Bank started out with full backing of its liabilities with metal coins, but over time took on functions of a modern
central bank, issuing fiat money and maintaining a target exchange rate through open market operations.

At the core of our paper is a model of the limits to maintaining the value of fiat currency by adjusting the money supply, and how this limit depends on the extent of negative equity and the economic fundamentals. The value to users of Bank money depends on its value in the settlement of wholesale trade transactions, and therefore on the volume of trade and the general buoyancy of the economy. In the face of a negative shock to the economy that reduces the value to users, there would be reduced demand for Bank money. The network effects in the use of Bank money would further amplify the decline in the value to users of holding Bank money. Other things equal, the excess supply of Bank money would put downward pressure on the exchange rate (the “agio”).

In principle, the Bank of Amsterdam could respond to the negative shock by reducing the money supply to restore the agio to the desired level. It could do so by selling coins in the open market and debiting the accounts of the buyers, thereby reducing the money supply. However, in the presence of illiquid loans on the balance sheet, there is a hard limit to the reduction in the money stock. The hard limit binds more as losses mount and negative equity eats into the asset value of the Bank. Once the Bank has sold all the liquid assets (the coins), it only has the illiquid assets (the loans). The sales needed to stabilise the agio cannot go further, as there is no more capacity for a reduction in the deposits.

Crucially, unlike a modern central bank, the Bank of Amsterdam did not have fiscal backing from a sovereign with (adequate) power to tax. We solve for the unique break point of the global game where negative equity and asset illiquidity renders fiat money worthless. The model allows us to draw lessons on the role of fiscal support and central bank capital in sustaining trust in fiat money.
Three key features stand out from our model, which resonate even for debates of today.

First, while the network effects of fiat money allow monetary regimes to persist for quite some time, there are limits to how resilient such arrangements can be. Being able to issue fiat money gives the central bank considerable latitude to leverage up its balance sheet without loss of confidence in the value of money. Yet the Bank of Amsterdam’s failure is a vivid lesson in how a central bank that loses public trust can push its luck too far, beyond the threshold for failure. In our model, there is a unique break point at which trust in fiat money breaks down.

Second, this break point binds harder when central bank equity becomes more negative and when economic fundamentals are weaker. The ultimate backing for the value of money is the fiscal sustainability of the consolidated public sector - consisting of the central bank and fiscal authorities (Sims (1994), Cúrdia and Woodford (2011), Reis (2015)). In this sense, fiat currencies need backing, and modern central banks need the ultimate fiscal backing of the government that flows from the sustainability of public finances. Negative central bank equity limits the room for manoeuvre to conduct open market operations that shore up the value of fiat money. The credibility of fiat money may then be at risk, especially if the holders of fiat money doubt the willingness of the fiscal authority to recapitalise the central bank. The loss of confidence manifests itself in a switch in the portfolio of monetary instruments used in the economy. Bank money (ie the deposits issued by the Bank of Amsterdam) maintains value until the loss of trust results in a coordinated switch away from deposits to an alternative form of money – in our model, metal coins. In a modern context such a switch in the portfolio of monetary instruments could occur through dollarisation, as we have seen in the case of many emerging market economies. Where users have access not to only foreign currencies but also to other digital assets such as cryptocurrencies or private
stablecoins, the dollarisation could take place in the form of “crypto-isation” where the portfolio decision in the money system tilts towards crypto.

Third, our global game model delivers sharp predictions on the nature of the break point. As the fundamental uncertainty among investors dissipates, the relationship between the state of the economy and trust in fiat money approaches a step function: there is a discrete jump at the break point below which fiat money becomes worthless.

While the Bank of Amsterdam is not the only historical antecedent of modern central banks, we argue that it is particularly instructive. First, its money (the Bank guilder) persisted for nearly two centuries, and became widely used not only in the Netherlands but for trade across Europe and around the world - it is sometimes seen as the first global reserve currency. Second, the substantial institutional evolution of the bank over time sheds light on the move to fiat money and how trust in such fiat money is maintained. Third, unlike the Bank of England, which also saw changes between a more rigid and elastic structure of fiat money over time, the Bank of Amsterdam failed - and was thus not able to become a modern central bank as we know them today.

By drawing on this earlier, and relatively simple period of competition between fiat money and alternatives, we can shed light on the economics underlying fiat currency. Our main contribution is the modelling exercise on how trust in fiat money can be lost, using a portfolio choice approach between competing forms of money. We show

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2This distinguishes it, for instance, from the goldsmiths of medieval Italian cities, and early public deposit banks in Italy, notably the Banco di Rialto of Venice and similar banks in Rome, Genoa and Naples (Bindseil (2019)).

3This clearly distinguishes the Bank of Amsterdam from Stockholms Banco (1657-1667), which is seen as a precursor of today’s Swedish Riksbank. See Edvinsson et al (2018).

4For instance, the 1844 Bank Charter Act, which enforced a 100% marginal gold backing for the Bank of England’s banknotes, can be seen as a move from an elastic to a rigid stablecoin structure (see below) - but such requirements were subsequently relaxed during crisis periods, with fiscal guarantees from the government. See Ugolini (2013). The resulting international role of sterling, in turn, bestowed exorbitant privilege - i.e the UK government was able to borrow more than warranted by macro fundamentals (Chen et al (2022)).
that these have a bearing on much broader issues of central bank solvency and the governance of fiat money. We introduce the distinction between rigid and elastic “stablecoins”, and show their relevance for understanding the role of fiat money. We discuss key implications for central banks of today, in the context of above-target inflation, quantitative tightening and large losses on central bank balance sheets.

The rest of this paper is organised as follows. Section 2 briefly introduces the Bank of Amsterdam and its downfall. In Section 3, we present our core global games model of the Bank’s downfall as an equilibrium outcome. Section 4 draws implications for modern central banks, highlighting the conditions under which a loss of trust may or may not occur. Section 5 concludes.

2 The Bank of Amsterdam and its downfall

The Bank of Amsterdam (Wisselbank, or “Exchange Bank”) was a public giro or payments bank owned by the municipality of Amsterdam. The Bank was founded in the context of a large number of circulating metal coins in the early 17th century, and the debasement of those coins by the deliberate mixing of base metals into gold and silver coins (Kindleberger and Aliber (2005); Schnabel and Shin (2004; 2018)). In the Bank’s founding decree, it was given a mandate to “check all agio (of the current money) and confusion of coin, and to be of use to all persons who are in need of any kind of coin in business”.

The Bank operated as follows. Customers would physically deposit metal coins with the Bank and account balances were recorded in a central ledger. These deposit

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5 The appendix contains an additional proof.
6 The “agio” referred at the time to the premium on different types of currency, ie the difference between the rate of exchange in the market and the nominal value. As will be shown later, the agio or premium on Bank guilders relative to current guilders came to be an important indicator of confidence in the stability of the Bank of Amsterdam.
balances could be transferred to other account holders without cost, or withdrawn for a small fee.

In this sense, the early Bank of Amsterdam guilder resembled what we now know as a stablecoin - where account-based money is backed by assets of stable value. The term “stablecoin” entered the lexicon of monetary economics through Facebook’s Libra proposal in 2019. They are now better known by connection with the crypto universe, where they provide a unit of account in terms of conventional money and hence a point of contact between the crypto universe and the conventional monetary system.7

2.1 Proto-central bank

Over time, the Bank departed from the strict application of full backing - without initially undermining its credibility (Uittenbogaard (2009)). It engaged in liquidity operations familiar to modern central banks, as well as outright lending.

A key date is 1683, when the Bank ended the policy of redeemability of deposits into coin (Quinn and Roberds (2014)). In this sense, the Bank started issuing fiat money. This change was crucial for the role the Bank would play at the heart of the international payment system.

However, the shift from a rigid to an elastic structure was not a complete shift. At the same time as removing the redeemability of deposits into coin, the Bank introduced a separate “receipt” system that allowed coin holders to sell their coins to the Bank with the option to repurchase the same coins after a fixed period - typically six months - for a small fee (¼ percent for silver coins and ½ percent for gold coins). During this period, the coin sellers would have a deposit claim at the Bank, and the coins under receipt would be earmarked (“encumbered”) for potential withdrawal.

7See Coeuré (2019), G7 Working Group on Stablecoins (2019), FSB (2020) and Arner et al (2020). Stablecoins are private cryptocurrencies (or “crypto-assets”) that seek to maintain a stable value against assets or fiat currencies.
With this policy change, the bank moved from a “rigid” to an “elastic” stablecoin, which combined redeemability with fiat money. The ability to purchase underlying coins by crediting the account of the seller meant that the Bank could increase the stock of bank money through the outright purchase of coins, just as a modern central bank could increase base money through an asset purchase programme and quantitative easing (QE).

As part of its monetary operations, the Bank of Amsterdam engaged in asset purchases and sales to stabilise the value of the agio. The Bank expanded the money stock through the purchase of coins when the agio rose, and contracted the money stock through the sale of coins when the agio fell. The Bank of Amsterdam sought to keep the agio of Bank guilders to current guilders (metal coins) in a target range between 4 and 5%, and thus to ensure Bank guilders could serve as a stable unit of account. For many years, this policy target was implicit and was not publicly communicated, although in 1782 the Amsterdam executive council formalised the arrangement by instructing the Bank’s commissioners to maintain this target range (van Dillen (1925), pp 433-4; Quinn and Roberds (2019), p 751).

The Bank maintained a close relationship with another key institution of the time – the Dutch East India Company (Verenigde Oostindische Compagnie, VOC). The VOC, founded in Amsterdam in 1602, is often considered the world’s first joint-stock company, and played a crucial role in European trade with Asia for nearly two centuries. Financed by equity from its shareholders, which included the largest merchants of the day, the VOC sent nearly 5,000 ships between Europe and Asia in the 17th and 18th century. It would load these ships in ports in the Netherlands and elsewhere in Europe with precious metal coins from the mines of the New World, and exchange these for goods from Asia (De Vries (2003)).

Shareholders of the VOC were also among the
largest depositors at the Bank of Amsterdam. Moreover, given the seasonal patterns of trade by the VOC, the Bank regularly lent to the VOC to provide settlement liquidity for the wholesale payment system. Finally, the VOC itself held deposits at the Bank, and made wholesale transactions in Bank money. Figure 1 shows selected relationships between the two institutions.

2.2 Downfall of the Bank of Amsterdam

The resilience of the Bank of Amsterdam and its success over many decades came under pressure in the late 1770s. Under the economic stresses generated by war with the English, the Bank departed more seriously from sound practice by lending on a more substantial scale to the VOC, in a sustained and non-transparent way.

Crucially, unlike a modern central bank that has the fiscal backing of the sovereign, the Bank of Amsterdam lacked fiscal backing. While the Bank’s public sector ownership by the city of Amsterdam gave it some degree of fiscal support from the city tax authorities (and also the ability to mutualise losses across segments of Amsterdam officers (Lucassen and van Rossum (2016); Pol (1985)).
society), this was not sufficient for the large scale of activities of the Bank given the
large volume of international trade through Amsterdam. The Bank operated with
slightly negative equity for most of its existence, but did not have safeguards for when
equity turned more deeply negative. In particular, the actions of municipal authorities
to receive profit distributions of the Bank without a symmetric recapitalisation flow in
times of losses cast doubt on the value of the municipal backing for the sustainability
of the Bank’s solvency.\footnote{Additionally, the weak governance of the Bank meant that the safeguards and governance
structure needed to support a durable fiat currency were sorely lacking. Janssen (2015) and van ’t Hart
(2009) relate how the excessively cosy relationship between the Amsterdam municipal authorities, the
VOC and the commissioners of the Bank made the latter susceptible to pressure to act in disregard
of its charter.}

The pivotal event was the shock of the Fourth Anglo-Dutch war (1780-84) which led
to extensive naval confrontations between the Dutch Republic and England in several
theatres of conflict - in European, West Indian and Asian waters. This conflict was an
economic shock that strained the VOC, which had become the main borrower of the
Bank of Amsterdam. Shipping volumes by the VOC fell dramatically; sales of trade
goods in the Netherlands dropped from 20.9 million guilders in 1780 to only 5.9 million
in 1781 (Figure 2).

Amid dire and deteriorating economic conditions, the Bank commissioners made
the fateful decision to start granting even larger overdrafts to the VOC. As a result,
the credit exposure of the Bank rose from 0 in June 1779 to 4.8 million guilders in
1781. The Bank then temporarily stopped new lending, but the stock of loans to the
VOC remained high. The slump in the VOC trade continued (Jonker and Sluyterman
(2000)). High losses of ships meant that loans that were already extended could no
longer be repaid.\footnote{The scale of this exogenous economic shock is perhaps best illustrated by the fate of VOC ships.
In May 1781, VOC ships such as the Amsterdarn, Batavia and Indiaan, on their way back to port,
were sent to Mauritius to assist the French in the war against the English; all were either damaged
or went missing. In July 1781, the ships Honkoop, Hoogkarspel, Middelburg, Parel and Dankbaarheid}
In May 1782, the commissioners decided to swap the suspended loans to the VOC into longer-term bonds (van Dillen (1934)). Throughout 1782, the Bank steadily ramped up its lending to the VOC; outstanding loans rose to a peak of 7.8 million guilders in February 1783. As loans increased (to a full 71% of the Bank’s assets), the metal stock fell, from 17.6 million guilders in 1776 to 7.8 million in 1783 (Figure 3). This was because account holders with receipts redeemed coins by allowing their receipts to expire.

During the first half of 1783, the Bank responded to downward pressure on the agio were seized or burnt by the English in Saldanha Bay (modern South Africa). The loss of this many ships imposed catastrophic financial and operational losses for the VOC.
by selling 3.5 million worth of guilder coins into the market (Quinn and Roberds, 2016). By the summer of 1783, guilders were now only backed by metal coins for 28% of their value, from 97% just four years earlier. With the conclusion of the war in May 1784, the Bank had accumulated a large credit exposure which soon become non-performing.

The Bank’s insolvency - and the inability of the city authorities to recapitalise it - are important elements in its downfall. The Bank’s income sources comprised mainly fees from the receipt system and interest margins on loans. However, while the loans to the VOC became non-performing, the bank had not been rebuilding capital to cover these losses, as profits were regularly distributed to the city. Moreover, it had neither the seigniorage income of modern central banks, nor an adequate fiscal backstop. The city of Amsterdam did make limited attempts to recapitalise the Bank, but the funds were quickly diverted back to city coffers (Quinn and Roberds (2016)). From the perspective of modern central banking theory, the City of Amsterdam’s fiscal capacity was insufficient to provide the sovereign backing of an institution that had become a proto-central bank.

The extent of lending exposures remained opaque for a further decade, but market developments as indicated by the agio suggest that market participants were sceptical about the solvency of the Bank of Amsterdam. In July 1789, as the Bastille was stormed in Paris and uncertainty spread across Europe, the agio on the Bank guilder dropped to 2%, and eventually turned negative in October 1790–February 1791. It recovered briefly following a bond issue by the City of 6 million guilders for recapitalisation (van Dillen (1964)). Yet this recapitalisation was unsuccessful, in part because the city authorities - lacking adequate tax resources - soon diverted these resources to other uses (Quinn and Roberds (2015)).

The final chapter came in 1795, after the invasion of the Netherlands by French revolutionary armies. It was then that the true extent of the Bank’s insolvency came
to light. The new authorities decreed that the Bank’s accounts would be made public, revealing the low metal stock. The agio on Bank guilders dropped to nearly -30% on the revelation. From 1795 to 1820, the Bank lived on as a severely weakened institution. After William, Prince of Orange-Nassau, proclaimed himself King William I in 1813, he founded the De Nederlandsche Bank, today’s central bank of the Netherlands (Vanhoor, 2006; Uittenbogaard, 2015). The Bank of Amsterdam was finally closed in 1820.

The economic fallout from the war, compounded by the downfall of the VOC and the failure of the Bank of Amsterdam, was severe. Income per capita fell by 17% between 1794 and 1807 (Zanden and van Leeuwen (2012)). The Bank guilder lost its role in international finance, and the centre of gravity in European finance shifted irrevocably to London (Carlos and Neal, 2011).

3 Model of Bank money and coins

At the heart of our model is the portfolio decision of merchants who have the choice between holding Bank of Amsterdam (fiat) money or coins. Bank money gives them access to a wholesale payment system, allowing clearing and settlement of the financial instruments underpinning trade and manufacturing. The value of Bank money to a particular merchant is assumed to be increasing in the total stock of Bank money held by other merchants. The rationale is that the network effects of money will enhance the coordination value of using a common means for the settlement of transactions. The value of Bank money is also increasing in the buoyancy or fundamentals of the economy, reflecting the increased need for international bills of exchange with greater economic activity. The alternative to Bank money is circulating coins, which can be used to make daily payments, but which are more cumbersome for the settlement of
trade transactions. Demand for Bank money therefore depends on its price (in coins), the general state of the economy (as a proxy for the volume of trade settled through the books of the Bank) and a positive network effect of using Bank money as a settlement asset.

The monetary policy objective of the Bank of Amsterdam is to maintain a stable premium of Bank money relative to coins (the agio). This objective is attained by adjusting the money stock through asset purchases or sales so that Bank money supply is set equal to Bank money demand at the target agio. However, the Bank of Amsterdam faces a hard limit on the sale of assets, which determines a “break point” at which merchants lose their trust in the value of Bank money.

Using a global game approach, we show that there is a unique break point, defined as the state of the economy at which the agio breaks below the target band and the value of Bank money falls to zero. We will show that the loss of trust in fiat money is more likely to occur when the external shocks are graver, the central bank engages in more (illiquid) lending and central bank equity is more deeply negative – particularly where fiscal support for the central bank is lacking. And indeed, this latter factor was particularly crucial. The City of Amsterdam was not able to play the role of a modern fiscal authority and thus to maintain trust in fiat money.

3.1 Set-up

There are three periods \{0, 1, 2\}, two types of money (Bank money and coins), and a continuum \([0,1]\) of risk-neutral merchants. Bank money is supplied by the Bank of Amsterdam, while coins circulate in abundance. On its balance sheet, the Bank of Amsterdam has coins \(C\) and (illiquid) loans \(L\) as assets, and Bank money \(M\) and
equity $E$ as liabilities. The balance sheet identity reads:

$$C + L = M + E.$$ 

Bank money can be bought by merchants at a unit price $p$; coins are the ‘numeraire’ in our model with a unit price of 1. We denote by $p = 1 + \gamma$ the price of Bank money in terms of coins (the ‘exchange rate’), where $\gamma$ denotes the agio.

We assume that economic fundamentals $\Theta$ (the ‘state of the economy’) is lognormally distributed, and $\theta = \log \Theta$ normally distributed with mean $y$ and standard deviation $1/\sqrt{\alpha}$, which are common knowledge. We may interpret the dynamic process of economic fundamentals $\{\theta_t\}$ to follow a random walk with Gaussian increments, where $\mathbb{E}(\theta_t) = \theta_{t-1} = y$.

In period 0 each merchant is ‘born’ and endowed with one unit of wealth $W_0$. The Bank of Amsterdam sets the price of Bank money at $\bar{p} = 1 + \bar{\gamma}$, where $\bar{\gamma}$ denotes the target agio.

At the beginning of period 1, before merchants make their portfolio decision, the state of the economy $\theta$ is drawn. The Bank of Amsterdam observes the state $\theta$, but merchants do not. Instead, each merchant $i$ learns his ‘own type’ $v_i$ that is strongly correlated with the fundamentals of the economy. Specifically, it is assumed that the type $v_i$ of merchant $i$ is given by:

$$v_i = \theta + \varepsilon_i,$$

with $\varepsilon_i$ being the idiosyncratic component of merchant $i$’s type. The idiosyncratic component of merchant types is independently drawn from a normal distribution with mean 0 and standard deviation $1/\sqrt{\beta}$. A merchant $i$’s own type $v_i$ is private information, and reflects the specific individual circumstances that give rise to small differences.
in the utility enjoyed by individual merchants from holding Bank money.

After the merchants have learned their own type, they have to decide on their portfolio allocation. A merchant $i$’s gross utility, $u_i^B$, from holding one unit of Bank money is specified as follows:

$$u_i^B(m) = v_i \cdot f(m),$$ \hspace{1cm} (1)

where type $v_i$ represents (direct) utility from having access to a wholesale payment system and $f(m)$ is a bounded and increasing function of aggregate Bank money holding that measures (indirect) network effects, with $m \in [0, 1]$ and assuming $f(0) > 0$. Here, aggregate Bank money $m$ denotes the proportion of merchants that holds Bank money. For convenience, a merchant $i$’s utility, $u_i^C$, from holding a unit of coins is (normalised to) 1:

$$u_i^C = 1.$$ \hspace{1cm} (2)

We now turn to the monetary policy operation of the Bank of Amsterdam. Motivated by the historical discussion, we assume that the Bank of Amsterdam conducts monetary policy by selling or buying assets in the open market (‘open market operations’) in order to maintain the target agio at $\bar{\gamma} > 0$. The sale and purchase of assets has a one-for-one impact on the stock of Bank money, as the purchase of an asset is paid for by creating a deposit for the seller of the asset. Conversely, the sale of an asset entails the debit of the buyer’s account. In this way, the target agio is maintained by adjusting the stock of Bank money so that the market clearing price of Bank money is at the target agio. However, as we will show, there is a hard limit on the sale of assets to raise the agio, which restricts the Bank’s maneuverability to reduce its money
supply.

Finally, in period 2, the state of the economy is revealed to all, and the target agio is realised. The implied target price is the market clearing price so that demand for Bank money equals the supply. Moreover, illiquid loans mature and merchants enjoy their utility from either Bank money or coins. Then, they ‘die’, leaving the record of their transactions for posterity.

3.2 Money demand

After the merchants have learned their own type at the beginning of period 1, a merchant $i$ must decide which fraction, $1 - \omega$, of his wealth $W_0$ is held in coins and which fraction $\omega$ to hold in Bank money, so as to maximise (expected) wealth $W_1$. That is, given agio $\bar{\gamma}$, merchant $i$ maximises:

$$\max_{\omega} \mathbb{E}[W_1] \quad \text{s.t.} \quad W_1 = (1 - \omega) \cdot W_0 \cdot 1 + \omega \cdot \frac{W_0}{1 + \bar{\gamma}} \cdot u_i(m).$$

Under risk neutrality, merchant $i$ prefers to allocate 100% of his wealth to Bank money if and only if $\mathbb{E}(u_i(m)/(1 + \bar{\gamma})) \geq 1$, which implies $v_i \geq (1 + \bar{\gamma})/\mathbb{E}(f(m))$. Otherwise, he prefers to only hold coins. The solution of the portfolio problem yields demand for Bank money. Conditional on the value of $\theta$, the proportion of merchants that prefers to hold Bank money is given by $Pr(v_i \geq (1 + \bar{\gamma})/\mathbb{E}(f(m)))$.

3.3 Unique equilibrium

It is well-known that multiple equilibria arise when the underlying state of the economy is common knowledge (eg Goldstein and Pauzner (2005)). This is a common feature in coordination games, in the same spirit as Diamond and Dybvig (1983). Here, this drawback is resolved by introducing some noise around the state of the economy.
that drives small differences in private merchant types. Since merchants do not exactly
know each other’s types, this forces them to coordinate their actions.

Network effects of Bank money introduce a coordination element to the portfolio
choice problems, with the desirability of Bank money increasing in the aggregate hold-
ing of Bank money. Formally, we derive two equations in two unknowns: a break point
$\theta^*$, below which the agio falls below target, and a marginal type of merchant $v^*$, which
separates the population into those who hold Bank money and those who hold coins.

As a first step in our solution, we confine attention to switching strategies. Suppose
there is a marginal type $v^* \in \mathbb{R}$ such that each merchant holds Bank money if and
only if $v_i \geq v^*$. Conditional on the value of $\theta$, first observe that merchant types $v_i$ are
independently and normally distributed with mean $\theta$ and standard deviation $1/\sqrt{\beta}$.
Therefore, if – given the state of the economy $\theta$ – all merchants follow this switching
strategy around $v^*$, then the proportion of merchants holding Bank money, $D = D(\theta)$,
is given by:

$$D(\theta) = \Pr(v_i \geq v^* | \theta) = \Phi(\sqrt{\beta}(\theta - v^*)),$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. Clearly, the de-
mand for Bank money $D(\theta)$ is upward sloping in $\theta$.

As a second step, we solve for the marginal merchant with type $v^*$ using the fact that
the marginal type is indifferent between holding Bank money and coins. Specifically,
the marginal type $v^*$ satisfies the indifference condition:

$$\frac{v^*}{1 + \gamma} \cdot F(v^*, y) = 1,$$

where $F(v^*, y)$ denotes the expected aggregate Bank money holding, conditional on $y$.
and the merchant being the marginal type $v^*$:

$$F(v^*, y) = \int_0^1 f(z) dG(z \mid v^*, y), \quad (5)$$

with $G(z \mid v^*, y)$ the (posterior) cumulative distribution function (c.d.f.) over Bank money holdings conditional on marginal type $v^*$. In the Appendix, we show that this c.d.f. is given by:

$$G(z \mid v^*, y) = \Phi\left(\frac{\alpha}{\sqrt{\alpha + \beta}}(v^* - y) + \sqrt{\frac{\alpha + \beta}{\beta}}\Phi^{-1}(z)\right). \quad (6)$$

The left-hand side of (4) is the expected utility of holding Bank money, while the right-hand side is the utility of holding coins. Consider the next lemma.

**Lemma 1** For given $\alpha$, if $\beta$ is sufficiently large then there exists a unique solution for $v^*$ that satisfies indifference condition (4).

**Proof.**

In the limit as $\beta \to \infty$ for given $\alpha$, $G(z \mid v^*, y)$ converges to the identity function in (6) so that the probability density function over Bank money holdings is the uniform density. In the limit, equation (4) becomes

$$\frac{v^*}{1 + \gamma} \cdot F = 1,$$

where $F = \int_0^1 f(z) z \, dz$, which is invariant to $v^*$ or $y$. Hence, there is a unique solution in this limiting case. By continuity, for sufficiently large $\beta$, there exists a unique solution for $v^*$ that solves (4). ■

As a third step, we introduce the monetary policy rule of the Bank of Amsterdam which is to set money supply $M = M(\theta)$ equal to money demand $D(\theta)$ so as to maintain
the agio at its target $\bar{\gamma}$. Note an important asymmetry in the conduct of monetary operations. The Bank of Amsterdam can always lower the agio through asset purchases, as there is no upper bound to the quantity of money that can be created by funding the purchase of assets. However, in order for the Bank of Amsterdam to raise the agio, it must have sufficient liquid assets (i.e., coins in its own vaults) it can sell so as to reduce the stock of Bank money. If there are illiquid loans on the balance sheet or equity turns sufficiently negative, there is a hard limit on the sale of assets to raise the agio. This hard limit determines the break point $\theta^*$, which is defined as the value of $\theta$ below which the agio falls below the target $\bar{\gamma}$. Using (3) and (4), it follows:

$$
M(\theta) = D(\theta) = \Phi \left( \sqrt{\beta} (\theta - v^*) \right) = \Phi \left( \sqrt{\beta} (\theta - (1 + \bar{\gamma}) / F(v^*, y)) \right)
$$

The break point is given by the value of $\theta$ where Bank money demand $D$ is equal to the total illiquid asset holdings $L$ minus equity $E$. This follows because there is a hard lower bound on the money supply $M$. From the balance sheet identity:

$$
C + L = M + E,
$$

we must have $M \geq L - E$ since the holdings of coins are always non-negative, $C \geq 0$. In other words, if the Bank of Amsterdam runs out of coins ($C = 0$) after a large negative shock, the break point $\theta^*$ is the solution to:

$$
\Phi \left( \sqrt{\beta} (\theta^* - v^*) \right) = L - E,
$$
or

\[ \theta^* = v^* + \frac{\Phi^{-1}(L - E)}{\sqrt{\beta}}. \]  

(7)

From (7), the break point is increasing in \( L \) and decreasing in \( E \), indicating that higher levels of illiquid assets and/or lower levels of equity can be counterbalanced only by stronger economic fundamentals. For the Bank of Amsterdam though, which did not have explicit fiscal support, \( L \) was large and \( E \) was deeply negative. It thus could not respond in any effective way to the deterioration of economic fundamentals.

Finally, for sufficiently large \( \beta \), observe there is a unique solution for \( \theta^* \) that solves (7), since \( v^* \) is unique (by Lemma 1). Effectively, the joint solution \((v^*, \theta^*)\) characterises the unique equilibrium in switching strategies.

However, we can go one step further. Morris and Shin (2003) show that when the payoff function of players in a global game satisfies strategic complementarities (ie the payoff advantage of taking one action is increasing in the measure of other players who take that same action), the equilibrium in switching strategies also proves to be the only outcome that survives the iterated deletion of strictly dominated strategies. In our model, the payoffs are such that the payoff advantage to a merchant to holding Bank money (relative to holding coins) is strictly increasing in the share of other merchants that hold Bank money. Moreover, note that for sufficiently low merchant types it is strictly dominant (‘low dominance’ region) to hold coins even if all other merchants hold Bank money (namely for \( v_i < (1 + \tilde{\gamma})/f(1) \)) and strictly dominant (‘high dominance’ region) to hold Bank money for sufficiently high types (namely for \( v_i > (1 + \tilde{\gamma})/f(0) \)). Therefore, by appealing to the result of Morris and Shin (2003) on dominance solvability, we have our first key result.

**Proposition 2 (Unique equilibrium)** For any \( \alpha \), there is a \( \beta \) sufficiently large such that there exists a unique equilibrium in switching strategies which is dominance solv-
able. The equilibrium outcome is characterised by a joint solution \((v^*, \theta^*)\) to equations (4)-(7).

Proof.

First, we have already shown that for a given \(\alpha\), the joint solution \((v^*, \theta^*)\) uniquely solves equations (4)-(7) if \(\beta\) is sufficiently large.

Second, to verify that the switching strategies constitutes an equilibrium, we have to show that, given that all other merchants adhere to their switching strategies, a merchant with type \(v_i < v^*\) has no incentive to hold Bank money, and similarly, a merchant with type \(v_i > v^*\) has no incentive to hold coins. That is, we have to show that \(v_i F(v_i, y)/(1 + \bar{\gamma}) < 1\) if \(v_i < v^*\), and \(v_i F(v_i, y)/(1 + \bar{\gamma}) > 1\) if \(v_i > v^*\). Note that \(v^* F(v^*, y)/(1 + \bar{\gamma}) = 1\) for the marginal type of merchant. Define \(V(v_i) = v_i F(v_i, y)\) and look at \(dV/dv_i = F(v_i, y) + v_i \cdot dF(v_i, y)/dv_i\). Using (6), in the limit as \(\beta \to \infty\) for given \(\alpha\), \(F(v_i, y)\) converges to \(F = \int_0^1 f(z)z \, dz\), which is invariant to \(v_i\), so that \(dF(v_i, y)/dv_i\) converges to 0. Therefore, in the limit, we get \(dV/dv_i = F > 0\), since \(f(z) > 0\) for all \(z\) in \([0, 1]\). Hence, by continuity, for sufficiently large \(\beta\), we have \(V(v_i) = v_i F(v_i, y)\), which increases in \(v_i\). This means that \(v_i F(v_i, y)/(1 + \bar{\gamma}) < v^* F(v^*, y)/(1 + \bar{\gamma}) = 1\) if \(v_i < v^*\), and \(v_i F(v_i, y)/(1 + \bar{\gamma}) > v^* F(v^*, y)/(1 + \bar{\gamma}) = 1\) if \(v_i > v^*\).

Third, starting from a dominance region, we can apply iterative elimination of dominated strategies – from above and below – and stop at the proposed switching strategy profile. For the proof that only this equilibrium survives iterated deletion of strictly dominated strategies, see Morris and Shin (2003).

Observe from (7) that in the limit as \(\beta \to \infty\), the break point \(\theta^*\) converges to the switching point \(v^*\). For our main result, we study the limiting case where \(\alpha \to \infty\) and \(\beta \to \infty\) but such that \(\sqrt{\beta}/\alpha \to k\), with \(k > 0\) some constant. Then, in this limit
as $\sqrt{\beta}/\alpha \rightarrow k$, it is straightforward to show that the distribution function $G(z \mid v^*, y)$ converges to $G_k(z \mid v^*, y)$, with:

$$G_k(z \mid v^*, y) = \Phi \left( \frac{(v^* - y)}{k} + \Phi^{-1}(z) \right),$$

and assuming that $k$ is large enough to ensure a unique solution to (4).

The next proposition addresses how the break point is reached. In the limit, the money demand function $D$ follows a 'step function' where the demand for Bank money falls to zero below the break point $\theta^*$, implying that the market clearing price of Bank money (relative to coins) must go to zero as well in this case. Moreover, we can show that the break point $\theta^*$ is decreasing in the (ex ante) mean $y$ of the state of the economy. When $y$ falls, fundamentals are commonly known to deteriorate. In this adverse economic environment, the break point for the monetary system where the demand for Bank money collapses, shifts 'upward'. Hence, a bad signal coming from a low draw of $y$ results in a more fragile monetary system where the same fundamentals $\theta$ that would have otherwise been consistent with the maintenance of the agio are now not strong enough to sustain the agio at the target level. The value of Bank money then collapses to zero.

**Proposition 3 (Break point)** In the limit as $\alpha \rightarrow \infty$ and $\beta \rightarrow \infty$ but such that $\sqrt{\beta}/\alpha \rightarrow k$, the money demand function is a step function given by:

$$D(\theta) = \begin{cases} 
0, & \text{if } \theta < \theta^*, \\
1, & \text{if } \theta \geq \theta^*,
\end{cases}$$
and the market clearing price of Bank money is the step function

\[ p = \begin{cases} 0, & \text{if } \theta < \theta^*, \\ 1 + \bar{\gamma}, & \text{if } \theta \geq \theta^*. \end{cases} \]

Moreover, we have \( \theta^* = v^* \) and the break point \( \theta^* \) is a decreasing function of the fundamentals mean \( y \).

Proof.

In the limit as \( \beta \to \infty \), from (3), the money demand function converges to a step function with the jump at \( \theta^* = v^* \). Below the break point \( \theta^* \), the demand for Bank money is zero while the supply of Bank money is at least \( L - E \). Therefore, the market clearing price of Bank money is zero. It remains to be shown that the break point \( \theta^* \) is a decreasing function of \( y \). To derive this result, first note that \( G_k(z \mid v^*, y) \) in (8) is decreasing in \( y \). Therefore, a larger \( y \) gives rise to a first-degree stochastic shift of the Bank money holding distribution to the right. Since \( f(\cdot) \) is an increasing function, we have that the first-degree stochastic shift puts more weight on higher utility of holding Bank money. Hence,

\[ \int_0^1 f(z) dG_k(z \mid v^*, y') > \int_0^1 f(z) dG_k(z \mid v^*, y) \quad \text{if } y' > y. \]

Therefore, from the indifference condition (4), the marginal type \( v^* \) is a decreasing function of \( y \). Finally, as we derived \( \theta^* = v^* \) in the limit, the break point \( \theta^* \) is decreasing in \( y \) as well. \( \blacksquare \)

Our global game model is built around a unique break point below which trust in fiat currency evaporates and fiat money becomes worthless. The comparative statics of the unique break point reveal three features. First, the break point is more likely to be...
reached when the state of the economy is poor, when the network effects of fiat money is weakened. Second, the break point is more likely to be reached when the central bank engages in lending so that the proportion of illiquid assets is higher relative to the money supply. Third, the break point is more likely to be reached when the issuer of fiat money has more deeply negative equity – particularly in the absence of adequate fiscal backing.

4 Discussion: implications for modern central banks

Our model did not consider explicitly the recapitalisation of the fiat money issuer. In the presence of fiscal backing, the granting of resources by the government would have the effect of easing the equity constraint, and increase the scope for asset sales to reduce the money supply and defend the exchange rate (the agio). However, in the absence of fiscal backing, the agio on Bank money collapses at the break point.

These findings have implications for long-standing debates on the relevance of central bank finances (see eg Stella (1997); Archer and Moser-Boehm (2013); Reis (2015) and Wessels and Broeders (2022)). The standard discussion features three broad arguments as to why central banks, in contrast to commercial banks and other private parties, can operate with negative equity: (1) the central bank can always meet its liabilities by creating new money; (2) central banks have positive franchise value, since seigniorage acts as a buffer for central banks; and (3) the government provides an ultimate backstop. Thus, the central bank can continue to operate even when formally insolvent based on common accounting procedures. Central banks in Chile, Czechia, Israel and Mexico are just a few examples of institutions that have done so for a period in recent history. Yet this does not imply that central bank equity positions do not matter for their policy effectiveness.
The literature already points to the limits to arguments (1) and (2). Creating new liabilities, or earning a positive spread, may not always be in line with the required monetary policy stance and policy objectives. Archer and Moser-Boehm (2013) summarise the evidence and conclude that central banks may not always be profitable, and that periods of financial weakness are not rare in practice. This is because central banks use their balance sheet to pursue public policy objectives, and may incur losses as a result: the required policy stance is expected to take precedence over the financial effects of central bank policies. For example, independent central banks are expected to raise policy interest rates and sell assets if this is needed from a monetary policy perspective, knowing that doing so will incur portfolio losses. These losses could become large enough to erode their equity position. Over time, profits will return in many cases, so that positive equity will be re-established. Yet in extreme cases the fiscal authority will need to recapitalise the central bank.

A key assumption underlying this line of argument is that there is sufficient demand for fiat money: it is based on a going-concern scenario. However, this may not always be the case, as users may shift to alternative forms of money. Our model indicates that the demand for fiat money may be negatively affected by economic shocks, the availability of alternatives and network effects. In the 18th century, a move away from Bank money meant a return to metal coins as currency. This move entailed substantial costs to the economy, as the benefits of efficient wholesale payments were lost. This weighed heavily on international trade.

Today, while switching costs may be higher, users could again find alternatives to a fiat currency that comes under pressure. Dollarisation and the possibility of central bank money being substituted by electronic money are discussed eg in Friedman (2000), Goodhart (2000) and Santomero and Seater (1996). If trust in a fiat currency were to break down, actors could move from a national fiat currency to the dollar or any
number of private cryptocurrencies.\textsuperscript{11} In practice, this would mean that buyers and sellers of goods and services, savers and borrowers and other actors would agree to use this currency or asset as a new numeraire.\textsuperscript{12} Episodes of dollarisation in past decades show that when this happens, it happens fast (Agur (2022)). Such a move could be very damaging to the overall economy.

The evidence in Agur (2022) mostly concerns developing economies with high inflation rates. We are not aware (yet) of a switch to an alternative currency in an advanced economy with an independent central bank and a credible fiscal backstop. While negative equity may in principle be well-understood as a by-product of the monetary policy stance, the public at large may see things differently. In case a perception arises amongst users of fiat money (both wholesale and retail) that a central bank’s finances are not sound, and that it may not be able to tighten as much as is needed, its credibility may be negatively affected. If such extreme cases were to arise in practice, a break point as predicted by our model would come closer, unless the central bank were recapitalised. In sum, capital adequacy may still be important to be a credible, independent central bank over a medium-term horizon, even if central banks cannot default formally (Wessels and Broeders (2022)).

This brings us to the role of argument (3), the fiscal backstop. The demise of the Bank of Amsterdam shows how a loss of trust in fiat money is a real possibility if a credible fiscal backstop is absent. As equity turned more deeply negative, the Bank passed the point of policy insolvency (Stella and Lönnberg (2008)), at which

\textsuperscript{11}Cryptocurrencies form a very poor substitute to fiat money, given their high volatility and primary use as a speculative asset. Experience from El Salvador, where Bitcoin was adopted as legal tender, suggests that there are substantial costs of a move toward cryptocurrencies (Alvarez et al (2022)). For a theoretical discussion of monetary competition in a model with two currencies - the Dollar and Bitcoin - see Schilling and Uhlig (2019).

\textsuperscript{12}A distinction can be made between private digital alternatives and central bank digital currencies (CBDC) that would be a direct liability of the central bank and thus use the existing numeraire. A growing body of literature assesses the design of CBDCs and their potential effect on the monetary system and the structure of financial intermediation. See eg Fernández-Villaverde et al (2021).
point it was no longer able to achieve its policy objectives. To regain policy solvency, recapitalisation by the fiscal authorities is needed. Such recapitalisations can take different forms, including the distribution of negative dividends by a central bank to the government (Archer and Moser-Boehm (2013)). These fresh resources can help the central bank to regain its footing and continue to use its balance sheet for policy purposes.

Yet what if the fiscal authorities are unable to recapitalise? In theoretical models, it is well-known that fiscal policy has an important impact on the overall price level. For certain configurations of fiscal and monetary policy, the price level may be indeterminate, resulting in stochastically fluctuating, explosive inflation (Sims (1994)). In cases where fiscal sustainability is in doubt, and the government has insufficient resources to recapitalise the central bank, there is the risk that trust in both government debt and the fiat money in which it is denominated could break down simultaneously.

Today, central banks in advanced economies face a challenging environment. After the great financial crisis of 2008, and again during the euro area crisis of 2010–12 and the Covid pandemic of 2020, central banks loosened policy for an extended period – buying up assets and issuing new liabilities in the form of modern fiat money. These policies helped to stabilise financial markets. Yet as underscored in our model, tightening policy can be much more difficult than loosening. Moreover, as central banks raise policy interest rates and sell assets, they are incurring large portfolio losses. These losses could become large enough that they erode the equity position of central banks. These considerations make some recent proposals to write off government bonds held on central bank balance sheets particularly dangerous. Indeed, some prominent economists have argued that central banks can simply write off government bonds, or convert them to 0% perpetuals, to provide debt relief (Various authors (2021)).

13Notably, such proposals were made prior to the recent bout of higher inflation around the world.
These proposals would be expected to impose large losses on central banks. Could such losses be sufficient to push central banks over the break point and thus to erode trust in modern fiat money? Could they permanently erode the basis of modern monetary systems?

In practice, there is great uncertainty around where exactly the break point for trust in fiat money would be. Yet given the stakes for the economy as a whole, central banks and governments would do well to stay well clear of this critical threshold. In order to ensure the public good nature of fiat money in the future, sound central bank finances and fiscal backing are key.

5 Conclusion

Money is a social convention. Yet it is also the key yardstick for the value of all manner of goods, services, claims and assets. Sound money allows individuals, firms (including financial firms) and governments to transact, and to record their obligations to one another in a way that binds the economy together. The governance of money is about ensuring a flexible system that meets the needs of the economy and yet is robust enough to ensure confidence. Experience with monetary institutions through the ages has given rise to central banks as the key institution at the heart of the monetary system. This is not to say that central banks always get it right; the Bank of Amsterdam is the poster child of what can go wrong when governance goes awry. Yet the solution to date has been to bolster the mandate and solvency of central banks, whose governance arrangements have continued to evolve with the economic challenges of their time.

In this paper, we have drawn lessons from the experience of this proto-central bank, which issued a highly successful form of fiat money for nearly two centuries. The Bank
of Amsterdam’s shift from a rigid to an elastic structure paved the way for its success in supporting wholesale payments and international trade. Yet after a severe economic shock, and in the absence of fiscal backing, the same network effects that had initially sustained trust in the Bank guilder worked to expedite its ultimate downfall. Our global games model formalises the conditions under which trust in fiat money can evaporate. This shows that there is a unique break point for trust in fiat money, which is more likely to be reached in a severe shock when central bank equity turns deeply negative and fiscal backing is lacking. When uncertainty declines, the move from one regime (high trust) to another (breakdown of trust) becomes a step function, and the downfall can be swift and precipitous.

Overall, our analysis demonstrates the value in reviewing historical precedents for understanding the monetary systems of today. In a context of high inflation, high global uncertainty and competition between both sovereign currencies and now cryptocurrencies, it is particularly relevant to understand monetary competition and the factors that could lead to shifts between different monetary regimes. Finding and analysing the incentives and governance underlying these structures may be a fruitful avenue for further research.
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Appendix: Additional proof

Proof of Lemma 1 (cumulative distribution function):

Under the assumption that all merchants follow the switching around $v^*$, first observe that the posterior about $\theta$, conditional on marginal type $v^*$, is normally distributed with mean $(\alpha y + \beta v^*)/(\alpha + \beta)$ and standard deviation $1/\sqrt{\alpha + \beta}$. Then, let $\theta_0$ be such that if $\theta = \theta_0$, the proportion of merchants with type higher than $v^*$ is exactly $z$. This yields:

$$Pr(v \geq v^* \mid \theta_0) = z \Rightarrow \theta_0 = v^* - \frac{\Phi^{-1}(1 - z)}{\sqrt{\beta}}.$$ 

For the cumulative distribution function, $G(z \mid v^*, y)$, we derive:

$$G(z \mid v^*, y) = Pr(\theta < \theta_0 \mid v^*) = \Phi \left( \frac{\alpha y + \beta v^*}{\alpha + \beta} - \frac{\alpha y + \beta v^*}{\alpha + \beta} \left( \theta_0 - \frac{\alpha y + \beta v^*}{\alpha + \beta} \right) \right)$$

$$= 1 - \Phi \left( \frac{\alpha y + \beta v^*}{\alpha + \beta} \left( \frac{\alpha y + \beta v^*}{\alpha + \beta} - \frac{\alpha y + \beta v^*}{\alpha + \beta} \right) \right)$$

$$= 1 - \Phi \left( \sqrt{\frac{\alpha + \beta}{\alpha + \beta}} \left( \frac{\alpha y + \beta v^*}{\alpha + \beta} - v^* + \frac{\Phi^{-1}(1 - z)}{\sqrt{\beta}} \right) \right)$$

$$= 1 - \Phi \left( \frac{\alpha y + \beta v^*}{\alpha + \beta} - v^* + \frac{\Phi^{-1}(1 - z)}{\sqrt{\beta}} \right)$$

$$= \Phi \left( \frac{\alpha y + \beta v^*}{\alpha + \beta} - v^* + \frac{\Phi^{-1}(1 - z)}{\sqrt{\beta}} \right).$$

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