Risk-taking with Financing Constraints*

Wei Cui†
Richard Cong Xie‡
Renbin Zhang§

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Abstract

We study how liquidity constraints affect firms' risk-taking behavior, influenced by leveraged returns and volatility. We show that looser liquidity conditions may or may not incentivize risk-taking. After an interest rate cut, fewer firms engage in risky behavior if they have access to low leverage, but more firms do so if they have high leverage. Consequently, we observe a non-monotonic, hump-shaped relationship between interest rates and firm-return volatility at the aggregate level. An optimal interest rate level can exist, and the extent of return spillover determines the optimal level. However, a calibrated model suggests a low interest rate is preferred because of constrained allocation efficiency. The model also implies an optimal policy mix of interest rates and non-monotonic leverage adjustments to manage risk spillover effectively.

Key Words: financing constraint; risk-taking; value effect and substitution effect

JEL code: E22; E44; E61; G32

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†University College London and the University of Groningen. Email: w.cui@ucl.ac.uk
‡Shenzhen Finance Institute. Email: richardxie@cuhk.edu.cn
§School of Economics, Shandong University. Email: zhang.renbin.ken@gmail.com

1 Introduction

Risk-taking plays a pivotal role in driving opportunities and progress within society. It fuels the exploration of new ideas and ventures, essential for societal advancement. Yet, the propensity for individuals to embrace risk varies based on numerous factors, such as the nature of a project's risk, the individual's risk tolerance, leverage limits, and prevailing interest rates. In the context of firms, how do financial constraints influence their risk-taking behavior amidst a prolonged period of low (real) interest rates observed since the global financial crisis? Furthermore, does a reduction in interest rates serve to encourage or deter risk-taking? Lastly, should interest rate and leverage policies be synchronized to optimize risk-taking dynamics?

This paper answers these questions by studying the combined impact of interest rates and leverage restrictions on firms' risk-taking incentives. In essence, we find that financial conditions exhibit non-linear effects on risk-taking, suggesting the potential for an optimal blend of interest rate and leverage policies, such as macro-prudential measures.

To begin with, utilizing diverse proxy measures for risk-taking, we empirically assess the impact of interest rate cuts across periods marked by varying interest rate and leverage environments. Our analysis reveals a robust, non-linear, and non-monotonic relationship between risk-taking behavior and interest rates. For instance, despite similar interest rate levels and policy rate cuts in 2001 and 2007, the risk-taking measure declined in 2001 but increased in 2007, which witnessed higher firm leverage. Thus, it appears that financial conditions exert a significant influence on risk-taking.

Next, to understand the impact of financial conditions on incentives for risk-taking, we explore a model scenario featuring three investment options: risk-free savings, a costlier yet risk-free project yielding higher returns, and a riskier project promising the highest expected return. In our model, firms can borrow funds for either type of project. Undertaking a risky project incurs idiosyncratic costs, such as those associated with testing and prototyping in research and development, which can vary significantly across industries and project types. The presence of the risk-free project offers an alternative avenue for firms, influencing their decision to pursue the riskier option based on idiosyncratic costs and borrowing capabilities. Thus, the model incorporates an "extensive margin" of risk-taking.

We highlight two key effects. Firstly, an increase in liquidity available to firms, achieved through either higher leverage or lower interest rates, eases financing constraints and boosts firm profitability or value. This "valuation effect" diminishes the incentive for risk-taking, as firms prioritize the benefits of enhanced liquidity and seek to accumulate capital with greater certainty. However, heightened liquidity also triggers a second, more traditional "searching for yield" effect, making risky projects more appealing—a phenomenon known as the "substitution effect." We demonstrate that the relationship between liquidity conditions and firms' risk-taking behavior can be non-linear and non-monotone, underscoring the complexity of policies influencing liquidity conditions. This complexity is particularly pertinent in the interaction between interest rates and macro-prudential policies, which directly impact the tightness of financing constraints.

In particular, the model establishes a critical cost threshold for executing the risky project, below which firms choose this option. We analyze how leverage constraints and interest rates influence this threshold. Two key factors illustrate these effects: 1) the impact on leverage return, represented by the return ratio between risky and risk-free projects; and 2) the effect on leveraged risk, summarized as an adjusted Sharpe ratio of the leveraged return.

We illustrate the non-linear outcome through the aforementioned "value" and "substitution effects". When firms have limited leverage, a reduction in interest rates primarily enhances firm valuation, resulting in fewer firms taking risks. This occurs when firm equity can only be leveraged to a limited extent, making the accumulation of equity with certainty more appealing. The rationale suggests that lowering the policy rate may not necessarily incentivize firms to undertake socially beneficial yet risky projects when leverage is stringent. Conversely, when firms can leverage extensively following an interest rate cut, the (leveraged) valuation effect is outweighed by the substitution effect stemming from the increased adjusted Sharpe ratio (associated with risky project execution). As a result, reducing the policy rate stimulates greater risk-taking. This scenario suggests that "searching-for-yield" behavior (e.g. Rajan (2006)) in a low interest rate environment is more pronounced when firms or financial institutions are highly leveraged.

The theoretical findings are in line with the empirical evidence. The model quantitatively reproduces the comparable declines in risk-taking witnessed in 2001 and the subsequent increases observed in 2007, despite similar levels of interest rates and cuts. This pattern is notably linked to higher firm leverage during the latter period. Our model provides insights into why interest rate reductions may not always stimulate firms to undertake socially beneficial yet risky projects. This phenomenon could result in persistently decreased equilibrium marginal products of capital and/or interest rates, thanks to more stringent leverage requirements post-global financial crisis. Additionally, our calibration suggests an optimal interest rate level is possible given a specific leverage limit. Importantly, the spillover of risks among projects plays a crucial role in determining this optimal interest rate.

It turns out to be even better to have an optimal mix of interest rate and leverage limit. As project outcomes become more positively correlated, interest rates should be lowered for channeling resources towards firms with productive projects; our calibrated exercises indicate that the optimal interest rate should generally be low (compared to the time preference rate), spanning from 0% to 1.6%. However, to avoid excessive or insufficient risk-taking (compared to the socially desired level), the leverage limit should fall first and then quickly rise with the correlation of projects.

Related literature. While we focus on examining firms' project choices under financial frictions, our theory of option value in the context of financial conditions closely aligns with literature on entrepreneurship within financial development, as seen in works such as Buera and

Shin (2013) and Buera et al. (2015). In these studies, agents face the decision of becoming either an entrepreneur or a worker, and financial frictions influence this choice. At the aggregate level, such decisions under financial frictions can have significant implications for productivity, a point further underscored by Moll (2014) and Buera and Moll (2014). It's worth noting that agents' optimization problems often exhibit non-convexity due to discrete choices, which can intertwine risk-taking with career decisions. For instance, in Vereshchagina and Hopenhayn (2009), where agents make occupational and risky project choices under no-borrowing constraints, those in the intermediate wealth range who opt to be entrepreneurs may exhibit risk-seeking behavior as this "lottery" mechanism can help smooth value functions. In comparison to this literature, our emphasis lies in illustrating how changes in financial conditions impact agents' risk-taking behavior and the macroeconomic effects of interest rate policy, along with welfare implications when projects become correlated.

To illustrate the mechanism at work, we choose a highly tractable framework to deliver closed-form risk-taking policy functions. In the aggregate, the correlation of projects can be considered without complicating the individual problems. This technical aspect may be of independent interest.

Our framework connects naturally to financial frictions[1] and risks in general. Building on Bernanke et al. (1999), Christiano et al. (2014) show that risk shocks in a quantitative business-cycle model with financing constraints generate counter-cyclical spreads and account for a large fraction of macroeconomic fluctuations. Miao and Wang (2010) include long-term defaultable debt in a macroeconomic model with financial shocks to the recovery rate. Cui and Kaas (2020) develop a framework to analyze belief risks in credit contracts. We add to this literature by examining the effect of financing constraint/interest rates on endogenous risk-taking behaviors.

Our work also adds to the strand of literature on active risk management: the seminal paper by Froot et al. (1993) in the static case and Bolton et al. (2011) in the dynamic case show that financing constraints generate the rationale for active risk management. Dell'Ariccia et al. (2017) illustrate the channel of risk shifting of financial intermediaries when the relevant interest rate changes. We illustrate the potential non-monotone effect between the project choice and financing constraint via a project selection problem. This problem becomes important even at the aggregate level since any liquidity-related policy may generate unintended volatilities and welfare effects.

The rest of the paper is organized as follows: We first present stylized facts about the non-monotone firm volatility to variations in interest rates. Section 3 presents a simple two-period model, which illustrates the key effects of interest rate and leverage on risk-taking. Section 4 extends the two-period model into an infinite-horizon economy in which the correlation between project returns is considered. Section 5 shows quantitatively how the interest rate and leverage limit shape the macroeconomic outcomes, followed by the optimal mix of interest rate

[1] The literature on financial frictions is vast, and the seminal contribution at least includes Kiyotaki and Moore (1997), Bernanke et al. (1999), Mendoza (2010), and Brunnermeier and Sannikov (2014).

policy and leverage policy and particularly how correlated projects affect the result. Section 6 concludes.

2 Motivating Evidence

We start with some evidence about the relationship between firms' risk-taking behavior and interest rate.

As a proxy for firm risk-taking behavior, we utilize a metric that captures the variation in individual stock returns (in log) within a given year. When firms undertake riskier ventures, the volatility of stock returns tends to increase (assuming all else remains constant) due to heightened uncertainty. We extract daily stock returns for non-financial firms from CRSP (see Appendix A for data description) spanning the years 1984 to 2020. The inter-quartile range (IQR) of daily returns is then employed to approximate the extent of firms' risk-taking. Unlike standard deviation, IQR offers the advantage of being less influenced by extreme volatility. But the findings below are robust even when employing standard deviation as the measure.

Figure 1 shows the median inter-quartile range (IQR) of individual stock returns plotted against the logarithmic level of the (gross) Cleveland rate, a widely used gauge of real interest rates provided by the Federal Reserve Bank of Cleveland. A quadratic fit between these variables, accompanied by a 95% confidence interval, is also shown. At higher real interest rates, there appears to be a negative correlation between the interest rate and stock return variability. Conversely, this relationship transitions to a positive one as the interest rate declines to lower levels. Notice that this non-monotonic relationship persists across sectors classified by the one-digit SIC code (see Appendix B).

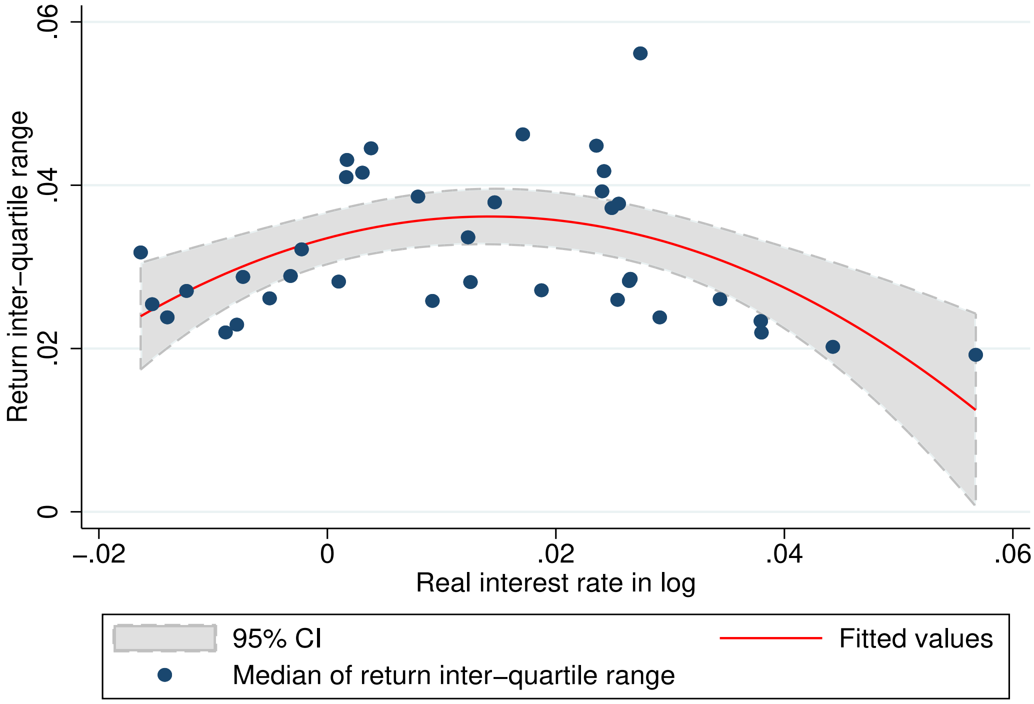
The firm-level data supports the macro evidence above. To measure the impact of interest rates on return variability as a proxy for risk-taking, we adopt the following empirical framework:

$$y_{ij,t} = \beta_0 + \beta_1(R_t)\Delta R_t + X_{ij,t-1} + f_i + h_j + \Gamma_t, \tag{1}$$

where $y_{ij,t}$ is the IQR of the stock return of firm i in sector j at year t in the baseline regression, R_t is the level of interest rate, ΔR_t is a measure of interest-rate shock, Γ_t is an aggregate control which includes the current and lagged terms of inflation, unemployment rate, real GDP growth rate, and the lagged interest rate.[2] The control vector $X_{ij,t-1}$ encompasses various factors, including the firm leverage ratio (measured by long-term debt over fixed assets such as property, plant, and equipment), logarithmic size (total assets), market-to-book ratio, return on assets (ROA), return on equity (ROE), return on investment (ROI), Tobin's Q, and cash holdings (measured by cash and short-term investments over total assets). These firm-level financial data are sourced from US COMPUSTAT from 1984 to 2016. f_i also represents the firm fixed effect,

[2] The results remain robust when we only incorporate current aggregate control or only lagged aggregate control.

Figure 1: Volatility and Interest Rate



while h_j accounts for the sector effect. Standard errors are clustered based on firm ID and year to address the correlation between firms and years.

Our primary coefficient of interest, denoted as β_1, indicates how the level of interest rates influences firms' risk-taking behavior, as measured by $y_{ij,t}$. Notably, $\beta_1(R_t)$ is allowed to vary based on the level of interest rates. Inspired by the insights from Figure 1, we partition the sample periods into two: one characterized by low interest rate levels and the other by high interest rate levels. To determine whether a specific period is associated with low or high interest rates, we use the median interest rate level as the threshold. The period featuring interest rates higher (lower) than the median is designated as the high (low) rate period. Various measures of real interest rates are employed, including the the federal funds rate deflated by past inflation ("ffr" in Table 1) and by expected inflation ("ffr2"), the Cleveland rate, and the real prime rate (prime rate in Table 1). For exogenous interest rate fluctuations, we utilize the monetary policy shock proposed by Jarociński and Karadi (2020) (mpshock) as the source of exogenous interest rate changes.

Table 1 presents the findings derived from estimating the specification outlined in equation (1), employing firms' return IQR as a measure of risk-taking behaviors. The coefficient β_1 exhibits significant positivity when interest rates are low, and conversely, negativity when interest rates are high. An unexpected unit of interest rate cut, estimated to range from 3 to 6 basis points according to Jarociński and Karadi (2020), results in a reduction of the firm's IQR by approximately 80-230 basis points in a low-interest-rate environment. Conversely, in a high-interest-rate environment, the IQR increases by 80-130 basis points, across different measures

Table 1: Variation in IQR with respect to Changes in Interest Rate

VARIABLES	(1) return iqr	(2) return iqr	(3) return iqr	(4) return iqr	(5) return iqr	(6) return iqr	(7) return iqr	(8) return iqr
mpshock	0.023***	-0.013*	0.020***	-0.008	0.008	-0.011	0.018***	-0.011
	(0.005)	(0.007)	(0.005)	(0.014)	(0.004)	(0.010)	(0.004)	(0.008)
Observations	48,297	52,826	47,788	53,602	45,943	54,062	48,297	52,826
R-squared	0.605	0.576	0.531	0.597	0.464	0.635	0.603	0.575
Firm FE	YES	YES	YES	YES	YES	YES	YES	YES
Interest use	ffr	ffr	ffr2	ffr2	Cleveland rate	Cleveland rate	prime rate	prime rate
Interest level	low	high	low	high	low	high	low	high

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Note: Results from estimating equation (1). Alternative measures of the real interest rate are employed to classify the interest rate regime: the real federal fund rate deflated by the past inflation (ffr, columns 1 and 2) and deflated by the expected inflation (ffr2, columns 3 and 4); the Cleveland rate (columns 5 and 6); and the real prime rate (columns 7 and 8).

of interest rates.

It's worth noting that the IQR measure reflects the variation in return on equity (ROE). In line with John et al. (2008) and Boubakri et al. (2013), we also examined variations in return on assets (ROA) and research intensity (RD expenses over total assets) as alternative indicators of risk-taking behaviors. Interestingly, we observe a similar sign switch of β_1, transitioning from positive in a low-interest-rate environment to negative in a high-interest-rate environment.[3]

What might contribute to the non-monotone impact? Firm leverage can be an important factor. To see this, let's examine the years 2001 and 2007, both marking the onset of respective recessions. During both periods, the Cleveland rate was around $1.7-1.8\%$, yet the median firm leverage ratio in 2007 surpassed that of 2001 by approximately 18%. Despite encountering similar interest rate reductions (about 150 basis points) in both years, they triggered differing responses in risk-taking measures. The 5-year moving-average IQR exhibits a decline of 7.5% from 2001 to 2002, whereas it rises by 2% from 2007 to 2008. Using other risk-taking measures leads to similar results, and our model below will shed light on these two periods.

The findings above prompt us to investigate theoretically the influence of interest rates and leverage, or broader financial conditions, on firm risk-taking.

3 A Simple Leveraged Risk-taking Model

In this section, we examine the effect of financing constraints on risk-taking behaviors in a simple two-period model. When the payoff of running a firm is bounded below by risk-free projects, entrepreneurs have incentives to select riskier projects because of an option-value consideration. However, leverage constraints change the option value significantly, and we explicitly model this effect.

[3]Further empirical findings are available upon request.

3.1 The Environment

There are two periods and two types of projects in the economy. A continuum of entrepreneurs, with measure one indexed by $i \in [0, 1]$, pick one of the two types of projects. The risky project has two possible returns: with probability p, the return is Π^h; and with probability $1 - p$, the return is Π^l. The expected return of the risky project is denoted by

$$\Pi = p\Pi^h + (1 - p)\Pi^l.$$

An entrepreneur may choose a safe/risk-free project with return Π^f. When implementing a project, the entrepreneur can borrow at a gross interest rate R.

For exposition reasons, we assume that

$$\Pi^h > \Pi > \Pi^f > R > \Pi^l. \tag{A1}$$

The risky project's expected return Π is higher than the return Π^f of a safe project. Unlike Vereshchagina and Hopenhayn (2009), who study choices of projects with the same expected return,[4] we impose $\Pi > \Pi^f$ so that socially it may be optimal to take risks while it may not be the case at individual levels. The return of the risky project is higher than Π^f when the project turns out to be successful (i.e., with the return Π^h), while the return is lower than Π^f if the project is unsuccessful (i.e., with the return Π^l). In this simple model, we assume that $\Pi^f > R$, so the safe project has a higher return than the risk-free saving rate. This might be natural as running a firm generally should have a higher return lending rate. Finally, our analysis abstracts from saving in risk-free assets (instead of projects) since they are dominated by running safe projects. In the macroeconomic model later, we will consider a case in which an entrepreneur could be indifferent between a safe project and a safe deposit.

In addition, we assume that

$$\Pi^h\Pi^l > \Pi^f\hat{\Pi} \text{ where } \hat{\Pi} \equiv p\Pi^l + (1 - p)\Pi^h \tag{A2}$$

so risk-free project return is not too high, which captures reasonable features in practice and which turns out to be useful in simplifying the analysis.

Let ω denote the wealth level of the entrepreneur each period after repaying previous debt, and let s be the amount invested in a project which can be used as collateral. We assume the utility function of the entrepreneur as

$$u(c) = \log c,$$

where consumption/dividends $c = \omega - s$. It may initially seem that the entrepreneur owns

[4] Their main goal is to answer why entrepreneurs take risks even when expected returns are the same as the safe return.

the firm, but the entrepreneur can also be its manager, so c should be interpreted as a dividend payout. Besides obtaining closed-form solutions, there are several advantages of assuming $u(c) = \log c$. The curvature in $u(.)$ captures the manager's preference for dividend smoothing. Lintner (1956) first showed that managers consider dividend smoothing over time, a fact further confirmed by subsequent studies. In addition, increasing c can be interpreted as equity share repurchases while reducing c can be thought of as sales of new shares. Putting curvature in $u(.)$ is thus a simple way of modeling the speed with which firms can vary the funding source when financial conditions change.[5]

There is a running cost from implementing the risky project, denoted by utility $\eta \in [\underline{\eta}, +\infty)$, which is drawn from an i.i.d. distribution (note: the lower bound does not have to be positive). The value of an entrepreneur with running (utility) cost η and wealth ω can be written as

$$V(\omega, \eta) = \max\{V^r(\omega, \eta), V^f(\omega)\}.$$

Here, we use the realization of η to control for whether the firm chooses a risky or a risk-free project in period t. Using utility cost as in the firm default problem of Cui and Kaas (2020) permits closed-form solutions, and it is equivalent to costs in terms of goods proportional to entrepreneurs' wealth (given the utility and technology in this environment). We abstract from the default issue to focus on the project choices, and firms are required to repay all their debt in both cases.[6]

Let b denote the level of borrowing. The value of an entrepreneur, who implements the risky project, $V^r(\omega, \eta)$ can be written as

$$\begin{aligned} V^r(\omega, \eta) = \max_{s,b} \{ &\log(\omega - s) - \eta + \beta p \log(\Pi^h(s + b) - Rb) \\ &+ \beta(1 - p) \log(\Pi^l(s + b) - Rb)\} \\ \text{s.t. } & 0 \le b \le \bar{\theta} s, \end{aligned}$$

where $\bar{\theta}$ is a parameter that governs the tightness of the borrowing constraint. The entrepreneur can put in internal savings s, together with the borrowing b; next period, the entrepreneur earns either $\Pi^h(s + b)$ if the project turns out to be productive, or $\Pi^l(s + b)$ if the project turns out to be unproductive. The interest payment is naturally Rb. The amount of borrowing, b, is limited due to financial frictions. We use a collateral constraint specification, i.e., the entrepreneur can borrow up to $\bar{\theta}$ fraction of its capital, s at the time of borrowing.

The value function of an entrepreneur who implements the safe project $V^f(\omega)$ can be writ-

[5] Jermann and Quadrini (2012) also assume dividend adjustment costs, supported by empirical evidence cited therein.

[6] As in Cui and Kaas (2020), default can introduce multiple equilibria, which is left for future research.

ten as

$$V^f(\omega) = \max_{s,b} \log(\omega - s) + \beta \log(\Pi^f(s+b) - Rb)$$
$$\text{s.t. } 0 \leq b \leq \bar{\theta}s.$$

Compared to the value function $V^r(\omega, \eta)$, the cost of implementing the risk-free project is normalized to be 0, and the return next period is certain.

The optimization shows that s is linear in the wealth level ω independent of choices of projects. That is, $s = \varphi\omega$, with $\varphi = \frac{\beta}{1+\beta}$. When entrepreneurs choose the risk-free project, their firms will borrow up to the limit $\theta = \bar{\theta}$ since $\Pi^f > R$. When entrepreneurs choose the risky project, depending on the interest rate R, θ can be one of the three, 0, $\bar{\theta}$, and θ^* as the level of the leverage a firm will choose (by ignoring the credit limit) which satisfies:

$$\theta^* \equiv -\frac{\Pi^h\Pi^l - R\hat{\Pi}}{(\Pi^l - R)(\Pi^h - R)} = -\left[p\frac{\Pi^l}{\Pi^l - R} + (1-p)\frac{\Pi^h}{\Pi^h - R}\right]. \tag{2}$$

Assumptions A1 and A2 imply that $\theta^* > 0$. The following proposition determines the threshold cost of taking the risky project, $\tilde{\eta}$.

Proposition 1. *Suppose A1 and A2 hold. The leverage of a firm that implements the risky project satisfies*

$$\theta = \min\{\theta^*, \bar{\theta}\},$$

where θ^* is defined in (2) and θ^* decreases in the interest R. Those entrepreneurs with $\eta \leq \max\{\underline{\eta}, \tilde{\eta}\}$ choose the risky project and the threshold level $\tilde{\eta}$ satisfies

$$\tilde{\eta} = \beta p \log\left(\frac{\Pi^h + (\Pi^h - R)\theta}{\Pi^f + (\Pi^f - R)\bar{\theta}}\right) + \beta(1-p)\log\left(\frac{\Pi^l + (\Pi^l - R)\theta}{\Pi^f + (\Pi^f - R)\bar{\theta}}\right). \tag{3}$$

Proof. See Appendix C. □

3.2 Non-monotone Risk-takings

How does the lending interest rate (or, more generally, financial conditions) affect risk-taking decisions? Specifically, we investigate how borrowing interest rates affect the risk-taking behavior characterized by $\tilde{\eta}$, respectively. First, we show that risk-taking may not be monotone to the interest rate (borrowing cost). A falling borrowing cost does not necessarily induce more risk-taking. Then, we provide a heuristic approach to understand the results.

To simplify the discussion, let us first focus on the case where the financing constraint is binding if a risky project is implemented. We will relax this assumption later. Proposition 1 implies as the interest rate increases, θ^* falls so that the financing constraints for entrepreneurs who choose the risky project might change from binding to non-binding. Notice that when

$\theta = \bar{\theta}$

$$\frac{\partial \tilde{\eta}}{\partial R} = \beta \left[\frac{\bar{\theta}}{\Pi^f + (\Pi^f - R)\bar{\theta}} - p \frac{\bar{\theta}}{\Pi^h + (\Pi^h - R)\bar{\theta}} - (1-p) \frac{\bar{\theta}}{\Pi^l + (\Pi^l - R)\bar{\theta}} \right], \tag{4}$$

and as will become clear the sign of $\partial \tilde{\eta}/\partial R$ depends on the leverage upper bound $\bar{\theta}$. A higher interest rate will reduce the leveraged return of both projects; however, whether the safe project or the risky project is more attractive depends on the maximum leverage $\bar{\theta}$. If $\bar{\theta}$ is low (high) enough, the threshold $\tilde{\eta}$ increases (decreases) in the interest rate R, and the risky (safe) project is thus preferred. Overall, $\tilde{\eta}$ has a hump-shaped relationship with R.

Proposition 2. *Suppose A1 and A2 hold. Assuming the financing constraint is binding when the risky project is implemented (i.e., $\bar{\theta} < \theta^*$). There exists cutoff levels $\bar{\theta}^s_L < \bar{\theta}^s_H$ such that the threshold level $\tilde{\eta}$ increases in R when $0 < \bar{\theta} \leq \bar{\theta}^s_L$, decreases in R when $\bar{\theta}^s_H \leq \bar{\theta} < \theta^*$, and is hump-shaped in R when $\bar{\theta}^s_L < \bar{\theta} < \bar{\theta}^s_H$.*

Proof. See Appendix C. □

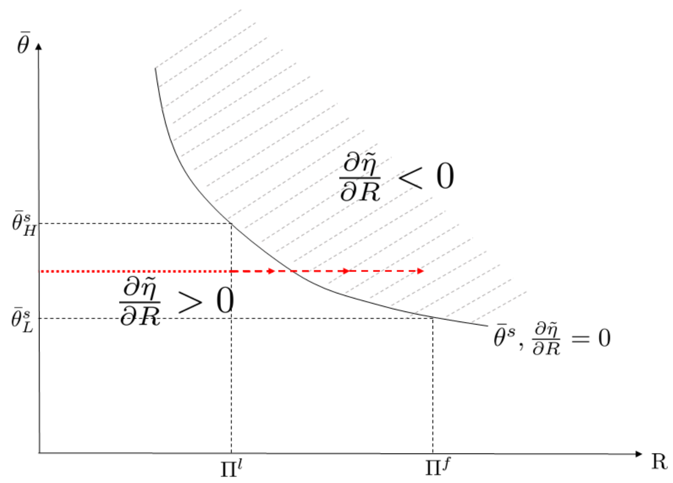
The proposition can be illustrated by Figure 2. The horizontal axis is the interest rate R, and the vertical axis is the leverage upper limit $\bar{\theta}$. The downward-sloping curve is $R^s(\bar{\theta})$, corresponding to the interest rate such that $\partial \tilde{\eta}/\partial R = 0$ (for a given leverage $\bar{\theta}$) and separates the two regions with different signs of the derivative $\partial \tilde{\eta}/\partial R$.[7] The shaded region above the curve has the feature of $\partial \tilde{\eta}/\partial R < 0$, and the blank area under the curve has the feature of $\partial \tilde{\eta}/\partial R > 0$. When $\bar{\theta} < \bar{\theta}^s_L$, as R increases from the low return realization Π^l to the safe return Π^f, $\partial \tilde{\eta}/\partial R$ is always positive. When $\bar{\theta} > \bar{\theta}^s_H$, as R increases from the low return realization Π^l to the safe return Π^f, $\partial \tilde{\eta}/\partial R$ is always negative. In these two cases, the change of the interest rate has a monotone effect on the threshold $\tilde{\eta}$.

However, if $\bar{\theta}$ is middle-ranged i.e. $\bar{\theta} \in (\bar{\theta}^s_L, \bar{\theta}^s_H)$, the effect of interest rate on risk-taking is non-monotone. To see this key result, for the points $(R, \bar{\theta})$ on the red dashed line below the separating curve, $\partial \tilde{\eta}/\partial R$ is positive since $\Pi^l < R < R^s(\bar{\theta})$; for the points $(R, \bar{\theta})$ on the red dashed line above the separating curve, $\partial \tilde{\eta}/\partial R$ turns negative since $R^s(\bar{\theta}) < R < \Pi^f$. In this middle range case, an increase of the interest rate R first encourages more risk-taking behavior and then discourages risk-taking once the interest rate R exceeds $R^s(\bar{\theta})$. This is because an increase in the interest rate will not only increase the cost of borrowing but also affect the trade-off between leveraged return and volatility. Notice that the non-monotonicity effect is not unique to varying interest rate; we also find the non-monotonicity effect of leverage as in Appendix C.2.

To better understand this key result, we approximate the threshold $\tilde{\eta}$ to the second order around a point $\tilde{\eta}_s$ (see below) so we can obtain further analytical insights. First, if we ignore

[7] Alternatively, the downward-sloping curve also represents $\bar{\theta}^s(R)$, which corresponds to the leverage upper bound such that $\partial \tilde{\eta}/\partial R = 0$ for a given interest rate R.

Figure 2: The Hyperplane of $\frac{\partial \tilde{\eta}}{\partial R} = 0$ in $(\bar{\theta}, R)$ Space



the risk involved and imagine there is another hypothetical safe project with the return Π (the same as the expected return of the risky project), the entrepreneur borrows up to the credit limit with this hypothetical safe project (i.e., $\theta = \bar{\theta}$) because the return $\Pi > \Pi^f > R$. Then, the threshold below which an entrepreneur chooses this hypothetical project can be defined as $\tilde{\eta}_s$:

$$\begin{aligned}\tilde{\eta}_s &\equiv \beta \log(\Pi(1+\theta) - R\theta) - \beta \log(\Pi^f(1+\theta) - R\theta) \\ &= \beta \log\left(\Pi - R\frac{\bar{\theta}}{1+\bar{\theta}}\right) - \beta \log\left(\Pi^f - R\frac{\bar{\theta}}{1+\bar{\theta}}\right) \\ &\approx \beta(\Pi - \Pi^f)\frac{1}{\Pi^f - R\frac{\bar{\theta}}{1+\bar{\theta}}},\end{aligned}$$

after we use (3). $\tilde{\eta}_s$ is associated with the ratio between the leveraged return from the risky and the risk-free project; this term represents the "valuation effect" of leverage or interest rate. Now, we expand $\tilde{\eta}$ to the second order around $\tilde{\eta}_s$ by using the cut-off expression (3):

$$\begin{aligned}\tilde{\eta} \approx \quad & \tilde{\eta}_s \quad + \beta p \left[\frac{1+\bar{\theta}}{\Pi(1+\bar{\theta}) - R\bar{\theta}}(\Pi^h - \Pi) - \frac{1}{2}\frac{(1+\bar{\theta})^2}{[\Pi(1+\bar{\theta}) - R\bar{\theta}]^2}(\Pi^h - \Pi)^2\right] \\ & \qquad + \beta(1-p)\left[\frac{1+\bar{\theta}}{\Pi(1+\bar{\theta}) - R\bar{\theta}}(\Pi^l - \Pi) - \frac{1}{2}\frac{(1+\bar{\theta})^2}{[\Pi(1+\bar{\theta}) - R\bar{\theta}]^2}(\Pi^l - \Pi)^2\right] \\ = \quad & \tilde{\eta}_s \quad \underbrace{-\frac{1}{2}\frac{\beta\sigma^2}{\left[\Pi - R\frac{\bar{\theta}}{1+\bar{\theta}}\right]^2}}_{\tilde{\eta}_\sigma},\end{aligned} \tag{5}$$

where $\sigma^2 \equiv p(\Pi^h - \Pi)^2 + (1-p)(\Pi^l - \Pi)^2$ is the variance of the risky project itself. $\tilde{\eta}_\sigma$ is a negative half of the inverse of the leveraged Sharpe ratio after adjusting for discounting; this term reflects the disutility related to the volatility of the leveraged return. It represents the "searching for yield effect" or the substitution effect.

When the interest rate R increases, financing costs increase, and the entrepreneurs' firm will be less valued. This valuation effect makes the risky project more attractive. Taking risks is a gamble entrepreneurs may want to engage in when their firm value is low since taking chances may raise value quickly. Meanwhile, the adjusted Sharpe ratio falls, and the searching for yield effect makes the risky project less attractive so that

$$\frac{\partial \tilde{\eta}_s}{\partial R} = \frac{\beta\bar{\theta}(1+\bar{\theta})(\Pi - \Pi^f)}{[\Pi^f(1+\bar{\theta}) - R\bar{\theta}]^2} > 0 \text{ and } \frac{\partial \tilde{\eta}_\sigma}{\partial R} = \frac{-\beta\bar{\theta}(1+\bar{\theta})^2}{[\Pi(1+\bar{\theta}) - R\bar{\theta}]^3}\sigma^2 < 0.$$

Depending on parameters, we thus know that a rise in the interest rate R can have a non-monotone effect on $\tilde{\eta}$ because the valuation effect and the substitution effect work in the opposite directions, which explains Proposition 2. In fact, for the interest rate $R \in (\Pi^l, \Pi^h)$, a trade-off between the two effects arises when $\bar{\theta}$ is middle ranged.

Notice that the effect of interest rate on risk-taking depends on the leverage constraint measured by $\bar{\theta}$. Rearranging the terms in (5) and $\tilde{\eta}_s$, we obtain

$$\Pi - \Pi^f = \left(\Pi^f - R\frac{\bar{\theta}}{1+\bar{\theta}}\right)\left[\frac{\tilde{\eta}}{\beta} + \frac{1}{2}\left(\frac{\sigma}{\Pi - R\frac{\bar{\theta}}{1+\bar{\theta}}}\right)^2\right],$$

where $\Pi - \Pi^f$ is the expected excess return of the risky project. Therefore, we have a natural asset pricing interpretation of risk management. For entrepreneurs who are indifferent between a safe project and a risky project, the excess (leveraged) return compensates for the risk and the cost of implementing the risky project. That is, on top of the safe leveraged return, the compensation for the risk is $0.5\sigma^2/\left[\Pi - R\bar{\theta}/(1+\bar{\theta})\right]^2$, while the compensation for the cost is simply $\tilde{\eta}/\beta$.

Therefore, an increase in the interest rate makes the portfolio with longing the risky project and shorting bonds riskier. Keeping all other parameters Π, Π^f, σ, and $\bar{\theta}$ the same, this increase will reduce the incentive of taking the risk, i.e., putting downward pressure on $\tilde{\eta}$. Nevertheless, an increase of R also makes the safe leveraged return smaller, and to justify the excess return $\Pi - \Pi^f$ for the *marginal* entrepreneur, it must be that the cost of implementing the risky project, $\tilde{\eta}$, for an indifferent entrepreneur goes up.

So far, we have found the non-monotonicity of risk-taking when the liquidity constraint is binding. That is, $\bar{\theta} < \theta^*_{min}$, where

$$\theta^*_{min} \equiv \frac{\Pi^h\Pi^l - \Pi^f\hat{\Pi}}{(\Pi^h - \Pi^f)(\Pi^f - \Pi^l)}$$

is minimal leverage when entrepreneurs are not constrained, which is obtained by assuming $R = \Pi^f$ in (2). We also generalize that the non-monotonicity can persist even when the liquidity constraint is not binding (i.e. $\bar{\theta} \geq \theta^*_{min}$).

Proposition 3. *Suppose A1 and A2 hold. Let $\bar{R}(\bar{\theta})$ be such that $-\left[p\frac{\Pi^l}{\Pi^l-\bar{R}} + (1-p)\frac{\Pi^h}{\Pi^h-\bar{R}}\right] = \bar{\theta}$ when entrepreneurs just become unconstrained. If $\bar{\theta} \geq \theta^*_{min}$, there exist an interest rate level R^u such that the risk-taking for unconstrained entrepreneurs is decreasing in R when $\bar{R}(\bar{\theta}) \leq R < R^u(\bar{\theta})$ and increases in R when $R^u(\bar{\theta}) \leq R < \Pi^f$.*

Proof. See Appendix C. □

3.3 Numerical Illustrations

Now, we illustrate the non-monotone risk-taking result with numerical examples. An entrepreneur may or may not be financially constrained when implementing the risky project because of the risk profile of the project and the entrepreneur's risk preference.

Figure 3 is a numerical illustration where the parameterization for the exercise is as follows: $\Pi^f = 1.1$, $\Pi^h = 1.3$, $\Pi^l = 0.94$, $p = 0.65$, and $\beta = 0.96$.[8] We do not have to specify $F(\cdot)$ for individual decision problems given the interest rate R. However, it will be used in the analysis later with credit market equilibrium.

The figure delivers a crucial message of the model: a cut in the interest rate reduces risk-taking when the leverage limit $\bar{\theta}$ is low, while it encourages risk-taking when $\bar{\theta}$ is high. When the leverage is low, a cut in interest rate R mainly works through the "leveraged valuation effect" since the debt servicing cost is low; entrepreneurs prefer taking less risk.

When the leverage is high, however, the leveraged valuation effect is dominated by the substitution effect with the same amount of interest rate cut since the high leverage amplifies the rise in the Sharpe ratio of the portfolio with the leveraged risky project. "Searching for yield" behaviors in a low-interest rate environment are more likely when a firm is highly leveraged. In other words, the ease of credit condition encourages risk-taking by reducing the funding cost per unit of capital and increases the Sharpe ratio of the leveraged risky project.

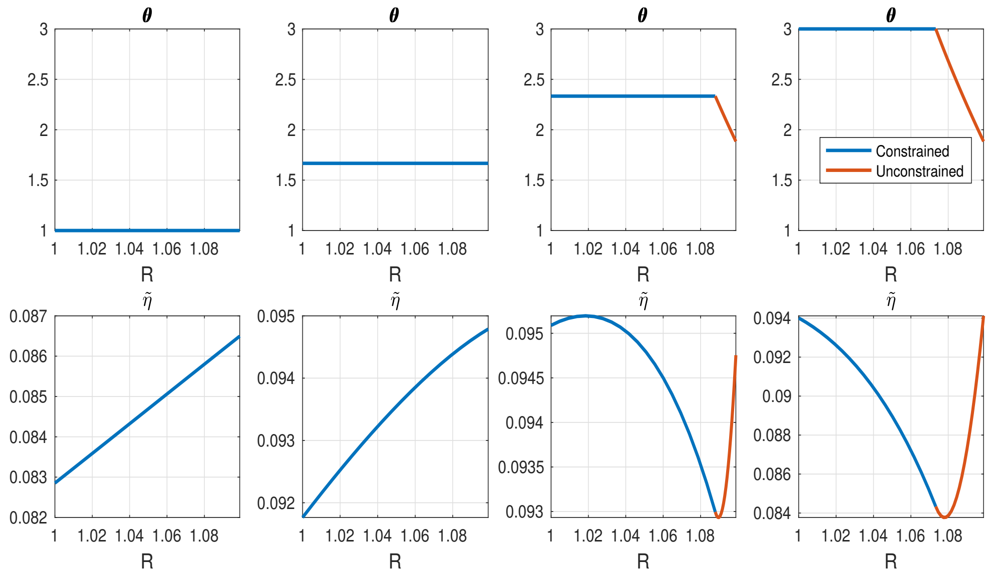
Note that the financing constraint might become slack for high leverage limits and interest rates, as shown by the orange curve in the third and fourth columns. Consistent with Proposition 3, the risk-taking first decreases and then increases as R rises in this case. Overall, these columns exhibit a complex pattern of risk-taking incentives.

Finally, Figure 4 further shows that the effect of borrowing cost R on the risk-taking behavior also depends on the profile of the risky project return Π^l and Π^h. In this experiment, we fix $\bar{\theta} = 1.5$. The first column uses the value of Π^h and Π^l previously specified and shows that a rise in R makes the risky project more appealing as a result of the dominating valuation effect.

Now, we increase Π^h and decrease Π^l simultaneously to keep the mean return of the risky project Π unchanged. For simplicity, let's first look at the case when the financing constraint is binding. If the project becomes riskier, as the interest rate R increases, the adjusted Sharpe

[8] This specification satisfies $\frac{1}{\Pi^f} > \frac{p}{\Pi^h} + \frac{1-p}{\Pi^l}$.

Figure 3: Effect of R on $\tilde{\eta}$ for Alternative $\bar{\theta}$



Note: The top panel of each column shows the leverage associated with the risky project; the bottom panel of each column shows the threshold cost for risk-taking. Each column corresponds to a different leverage upper bound (from the left to the right: $\bar{\theta} = 1.00, 1.67, 2.33,$ and 3.00).

ratio falls more, and the risky project becomes less attractive from the volatility concern. In other words, the substitution effect dominates when the risky project becomes riskier. This result is intuitive because when risk goes up, the leveraged volatility is sensitive to interest rate changes.

We also note that as the project becomes riskier, the leverage constraint can cease binding for a high interest rate R. Then, the substitution effect will eventually be dominated again by the valuation effect as R increases further.

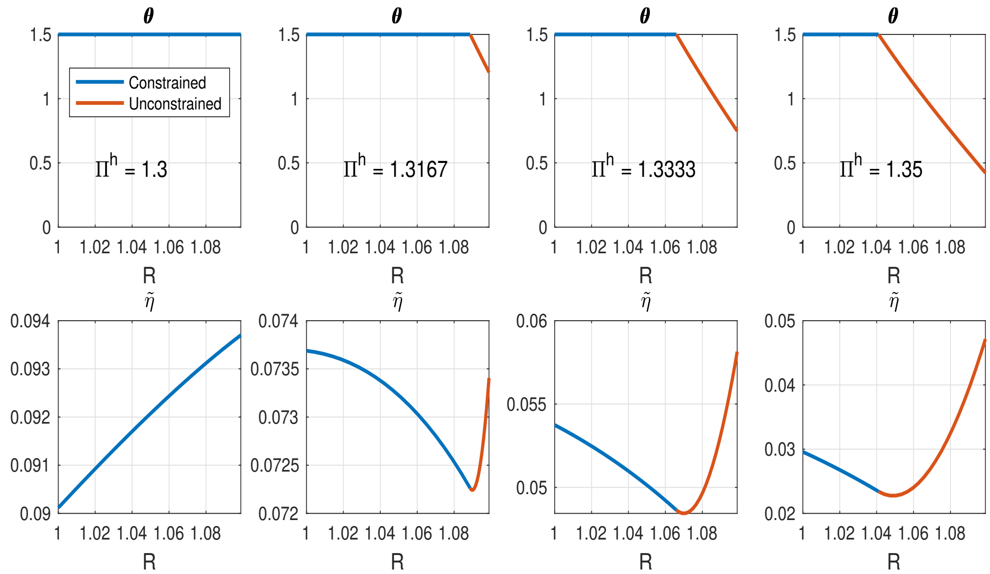
4 Infinite-horizon Model with Risk Spillover

Now we extend the previous two-period model into an infinite-horizon general-equilibrium model. We also introduce the correlation of project success/failure without changing the key ingredients of the model. The next section will use the equilibrium model with the potential spillover effect of risk-taking for policy discussion.

4.1 The Environment

Time is discrete and infinite. We use recursive notations, i.e., let variable x denote x_t and let x_+ to denote x_{t+1}. The economy is populated by a unit measure of entrepreneurs who may run firms. The economy also has a household sector and a government which runs flat-rate

Figure 4: Effect of R on $\tilde{\eta}$ under Different Risk Profiles



Note: The top panel of each column shows the leverage associated with the risky project; the bottom panel of each column shows the threshold cost for risk-taking. Each column corresponds to a different risk profile (from the left to the right: $\Pi^h = 1.3, 1.32, 1.33,$ and 1.35), while the expected return $\Pi = p\Pi^h + (1-p)\Pi^l$ is kept the same across all panels.

value-added tax policy[9] and credit policy.

Entrepreneurs Entrepreneurs can choose among the risk-free project, the risky project, and risk-free savings. Furthermore, similar to the cost of running the risky project η, we assume a fixed (utility) cost η^f associated with running the risk-free project.

The firm technology is represented by

$$y = (zk)^{\alpha} (\ell)^{1-\alpha},$$

where k is capital input, ℓ is labor input, z is risky if an entrepreneur chooses a risky project, and z is a constant number if the entrepreneur chooses the safe project. Consider a firm with capital k. In the labor market, the firm hires workers at the competitive wage rate w, which leads to labor demand proportional to the firm's effective capital input. The firm maximizes its capital return by solving

$$\max_{\ell}\{(1-\tau^y)(zk)^{\alpha}(\ell)^{1-\alpha} - w\ell\}.$$

where τ^y is the VAT tax rate set by the government. The optimal labor choice is:

$$\ell = zk \left(\frac{(1-\alpha)}{w/(1-\tau^y)} \right)^{\frac{1}{\alpha}}. \tag{6}$$

[9] VAT tax policy is equivalent to tax labor income and capital income at the same rate. We also experimented with a lump-sum tax policy; the qualitative results are very similar.

Therefore, the firm's capital return (but before debt repayment) is rzk, where r can be shown as

$$r \equiv (1-\tau^y)\alpha \left[\frac{(1-\alpha)}{w/(1-\tau^y)}\right]^{\frac{1-\alpha}{\alpha}}. \tag{7}$$

Denote the depreciation rate of capital as δ, and we specify $\{z^f, z^h, z^l\}$ as

$$\Pi^f = rz^f + 1 - \delta, \quad \Pi^h = rz^h + 1 - \delta, \text{ and } \Pi^l = rz^l + 1 - \delta. \tag{8}$$

In this way, the decision problem introduced below will be similar to the problems discussed in the simple model.

Let $V(\omega)$ be the expected value of an entrepreneur with wealth ω before knowing the cost of implementing the risky project. Conditioning on having good production technology and the implementing cost, we let $V^r(\omega, \eta)$ and $V^f(\omega)$ (as before) denote the value for an entrepreneur who chooses the risky and risk-free projects, respectively. Let $V^d(\omega)$ be the value of an entrepreneur who chooses to save in deposits with a return R^d. We have the following relationship:

$$V(\omega) = \mathbb{E}\left[\max\{V^r(\omega,\eta), V^f(\omega), V^d(\omega)\}\right],$$

where the expectation is taken over the distribution of η. The value function of an entrepreneur who takes the risky project can be rewritten as

$$\begin{aligned} V^r(\omega,\eta) = \max_{s,b} \Big\{ &(1-\beta)\log(\omega - s) - \eta + \beta p V(\Pi^h(s+b) - Rb) \\ &+ \beta(1-p)V(\Pi^l(s+b) - Rb)\Big\} \\ \text{s.t. } & b \leq \bar{\theta}s, \end{aligned}$$

where $(1-\beta)$ serves as a normalization. As before, the entrepreneur can invest with internal savings s, together with the borrowing b; next period, the entrepreneur earns either $\Pi^h(s+b)$ (if the project turns out to be productive) or $\Pi^l(s+b)$ (if the project turns out to be less productive). The interest payment is naturally Rb. The value function of an entrepreneur who takes a safe project or saves in deposits can be rewritten as

$$\begin{aligned} V^f(\omega) &= \max_{s,b} \Big\{(1-\beta)\log(\omega - s) - \eta^f + \beta V(\Pi^f(s+b) - Rb)\Big\} \\ &\text{s.t. } b \leq \bar{\theta}s; \\ V^d(\omega) &= \max_{s} \Big\{(1-\beta)\log(\omega - s) + \beta V(R^d s)\Big\}, \end{aligned}$$

respectively. When entrepreneurs choose the risk-free project, they will borrow up to the limit $\theta = \bar{\theta}$ if $\Pi^f > R$. Notice that the deposit rate R^d may differ from the lending rate R. We allow a constant $\tau \geq 0$ to be the difference between the deposit rate R^d and the lending rate R:

$$R = R^d (1 + \tau) . \tag{9}$$

The parameter τ represents the intermediation costs or interest rate markup. It turns out that it matters for the equilibrium outcome (see below).

Appendix 4 shows that the value functions satisfy the following forms $V^r(\omega, \eta) = \log(\omega) - \eta + v^r$, $V^f(\omega) = \log(\omega) - \eta^f + v^f$, and $V^d(\omega) = \log(\omega) + v^d$ for some endogenous v^r, v^f, and v^d. As in the two-period model, the saving function is linear in wealth $s = \beta\omega$.

Proposition 4. *Define $l^d \equiv \log R^d$ and $l^f \equiv \log\left(\Pi^f + \bar{\theta}\left(\Pi^f - R\right)\right) - \beta^{-1}\eta^f$. Those entrepreneurs with $\eta \leq \max\{\underline{\eta}, \tilde{\eta}\}$ choose the risky project, where $\tilde{\eta}$ satisfies,*

$$\begin{aligned}\tilde{\eta} = {} & \beta p \log\left(\Pi^h + (\Pi^h - R)\theta\right) + \beta(1-p)\log\left(\Pi^l + (\Pi^l - R)\theta\right) \\ & - \beta \max\{l^f, l^d\},\end{aligned} \tag{10}$$

and the leverage $\theta = \min\{\theta^*, \bar{\theta}\}$. Additionally, let ϕ be the fraction of entrepreneurs who choose to save in deposits, we have the following condition

$$\begin{cases} \phi = 1 & \text{if } l^d > l^f \\ \phi \in (0,1) & \text{if } l^d = l^f \\ \phi = 0 & \text{if } l^d < l^f \end{cases} . \tag{11}$$

Proof. See Appendix C. □

In equilibrium with $\phi \in (0, 1)$, those entrepreneurs who choose the safe project should be indifferent between saving via the risk-free deposits and their firm technology. That is, $l^d = l^f$ and

$$\Pi^f = \frac{1}{1+\bar{\theta}}\left(\frac{e^{\eta^f/\beta}}{1+\tau} + \bar{\theta}\right) R,$$

Notice that if $\Pi^f > R$, $\eta^f > \beta\log(1+\tau)$ has to be true when the equilibrium features $\phi \in (0, 1)$ so that some entrepreneurs choose to save in risk-free deposits. Since τ is non-negative, the equilibrium may feature $\Pi^f \leq R$ so that $\phi = 1$.

Households Households are hand-to-mouth consumers and they supply labors to firms.[10] To focus on the firm side, we assume a Greenwood-Hercowitz-Huffman log utility function, and a

[10]The assumption is less strong than it seems. In incomplete-market models, the interest rate is lower than the time-preference rate. Households would like to borrow, but (unlike entrepreneurs) households do not have collaterals, so they will not borrow in equilibrium.

representative household maximizes

$$(1-\beta)\sum_{t=0}^{\infty}\beta^t \log\left(C_t - \frac{\kappa L_t^{1+\gamma}}{1+\gamma}\right),$$

where C_t is households' consumption level, L_t is the labor supply, $\kappa > 0$ is disutility parameter, and γ is the inverse of the Frisch labor elasticity. The household consumption and labor supply decision satisfies (where again we ignore the time subscript):

$$C = wL; \tag{12}$$

$$w = \kappa L^{\gamma}. \tag{13}$$

Spillovers In the previous section, we consider there are infinitely many $i.i.d.$ risky projects. Now, we relax the independent assumption and allow the project realizations to be correlated. Once the risky project is chosen, each agent will be randomly assigned to one of them. At the aggregate level, how many risky projects will achieve high realization may exhibit correlations. The correlation between projects may matter for the authority in formulating policies. To characterize the dependence structure in Bernoulli distribution, we follow the approach of Conway-Maxwell Binomial/Poisson (CMB/CMP) distribution as in Shmueli et al. (2005) and Kadane (2016). To the best of our knowledge, this specification is new in a macroeconomic setup, and it modifies the baseline model with minimal departure because the individual problem stays the same. The specification also allows a straightforward aggregation.

Specifically, we characterize the overall dependence structure through an exogenous parameter ν. For CMB distribution:

$$Prob(m=k) = \frac{\binom{n}{k}^{\nu} p^k (1-p)^{n-k}}{\sum_{j=0}^{n}\binom{n}{j}^{\nu} p^j(1-p)^{n-j}} \equiv \frac{\binom{n}{k}^{\nu} p^k(1-p)^{n-k}}{D(\nu,p,n)},$$

where m is the total number of successes in n trials, $\binom{n}{k} \equiv \frac{n!}{k!(n-k)!}$, and we have used

$$D(\nu,p,n) \equiv \sum_{j=0}^{n}\binom{n}{j}^{\nu} p^j(1-p)^{n-j}.$$

The expression stands for the probability of k realization of high returns among n risky projects where the probability of realizing high return Π^h for each project is p. If $\nu = 1$, it becomes the standard binomial distribution. When $0 < \nu < 1$, risky projects' returns are positively correlated, and when $\nu > 1$, risky projects' returns are negatively correlated.[11] To see this, denote X_i as the random variable of result at i^{th} position. Consider $P(X_2 = z^l | X_1 = z^l)$, the conditional probability of the second project achieving a low return conditional on the low

[11] See Kadane (2016) for more detailed discussions.

return of the first project is[12]

$$\frac{P(X_2 = z^l, X_1 = z^l)}{P(X_1 = z^l)} = \frac{P(X_2 = z^l, X_1 = z^l)}{P(X_2 = z^l, X_1 = z^l) + P(X_2 = z^h, X_1 = z^l)}$$
$$= \frac{\frac{1}{D(\nu,p,2)}(1-p)^2}{\frac{1}{D(\nu,p,2)}(1-p)^2 + \frac{1}{2}\frac{1}{D(\nu,p,2)}\binom{2}{1}^\nu p(1-p)}$$
$$= \frac{(1-p)}{(1-p) + 2^{\nu-1}p}, \tag{14}$$

If $\nu < 1$, we have $2^{\nu-1} < 1$, then $P(X_2 = z^l | X_1 = z^l) > 1 - p$ implying a positive correlation (positive spillover). If $\nu > 1$, we have $2^{\nu-1} > 1$, then $P(X_2 = z^l | X_1 = z^l) < 1 - p$ implying a negative correlation (negative spillover). Similarly, $P(X_2 = z^h | X_1 = z^h) > p$ if $\nu < 1$ and $P(X_2 = z^h | X_1 = z^h) < p$ if $\nu > 1$.

Denote the overall proportion of success to be $p^s(\nu)$ when the number of projects n goes to infinity. The aggregate expected return of risky projects $\Pi(\nu)$ becomes:

$$\Pi(\nu) = p^s(\nu)\Pi^h + (1 - p^s(\nu))\Pi^l.$$

The aggregate expected return will matter for the social welfare as we aggregate entrepreneurs' choices. The key feature of CMB is that when individuals choose the project at a given interest R, the probability of Π^h is always p. This preserves the solution of the individual optimization problem. For the social planner, the correlation is crucial in determining social welfare. Our modeling choice captures a pecuniary externality where individuals ignore aggregate correlations, which influence the aggregate liquidity demand and, thus, the interest rate that clears the credit market.

To understand how this correlation affects the overall success probability $p^s(\nu)$ and the aggregate expected return of risky projects $\Pi(\nu)$, let's consider a special case which we can trace analytically. When n goes to infinity, and p is small/large enough, we know that the CMB distribution converges to the CMP distribution (see Shmueli et al. (2005) and Daly and Gaunt (2016)):

$$Pr(m = k) = \frac{\frac{(\lambda)^k}{(k!)^\nu}}{\sum_{j=0}^{j=n} \frac{(\lambda)^j}{(j!)^\nu}},$$

where $\lambda = n^\nu p$ when p is small and $\lambda = n^\nu(1-p)$ when p is large. And the overall success probability $p^s(\nu)$ satisfies the following:[13]

$$p^s(\nu) = \begin{cases} p^{\frac{1}{\nu}} & \text{if } p < p^* \\ (1 - (1-p)^{\frac{1}{\nu}}) & \text{if } p \geq p^{**}, \end{cases} \tag{15}$$

[12]The result uses the property of the exchangeability: $P(X_2 = z^h, X_1 = z^l) = P(X_2 = z^l, X_1 = z^h)$.
[13]see Appendix D.1 for more details,

for some small $p^* \in (0,1)$ and large $p^{**} \in (0,1)$, respectively. When p is small, high returns are relatively more scarce. The number of high return realization converges to Conway-Maxwell Poisson distribution. Taking into account the correlation, the social planner realizes that the overall probability of Π^h is $p^{1/\nu}$. When p is large, $1-p$ is small and counting the low realizations Π^l will converge to the Conway-Maxwell Poisson distribution with $\lambda = n^\nu(1-p)$. The social planner realizes that the overall probability of Π^l is $(1-p)^{1/\nu}$. Finding p^* and p^{**} is a numerical question, but one may not even need to find the p^* and p^{**} for the numerical analysis below, where we obtain $p^s(\nu)$ via simulating CMB samples with a large n.

Therefore, the aggregate expected return of the risky projects $\Pi(\nu)$ becomes

$$\Pi(\nu) = \begin{cases} p^{\frac{1}{\nu}}\Pi^h + (1 - p^{\frac{1}{\nu}})\Pi^l & \text{if } p < p^* \\ (1-(1-p)^{\frac{1}{\nu}})\Pi^h + (1-p)^{\frac{1}{\nu}}\Pi^l & \text{if } p \geq p^{**}. \end{cases} \tag{16}$$

To understand how spillover affects the overall return, we can consider $\Pi = \Pi(1)$ (i.e., without the spillover effect) and suppose there is a positive correlation, i.e., $\nu < 1$. When $p > p^{**}$, $p^s(\nu) = 1 - (1-p)^{\frac{1}{\nu}} > p$ and $\Pi(\nu) > \Pi$. Intuitively, when p is large, high returns are relatively more abundant, the positive spillover strengthens the dominance of high returns, and the overall success probability viewed by the social planner becomes higher. That is why the overall aggregate expected return is above $\Pi(1)$. Our numerical results below calibrate for a relatively large p, leading to a social return $\Pi(\nu)$ exceeding the private return $\Pi(1)$. This finding aligns with a large literature on $R\&D$, which demonstrates that the social rate of return to $R\&D$ is typically higher than the private rate of return (see Griffith (2000)). When $p < p^*$, $\Pi(\nu) < \Pi$ holds similarly.

The (consolidated) government agency We consider a consolidated government which conducts joint monetary and fiscal policies with the following budget constraint:

$$G + R^d B_{-1} = T + B \tag{17}$$

The expenditure side includes government spending G and debt repayment $R^d B_{-1}$ accumulated previously. For simplicity, we assume that the government can borrow without the intermediation cost so that the repaying interest rate is also R^d because of no-arbitrage. The revenue side includes tax $T = \tau^y Y$ (where Y is the output to be more specifically defined below) and the newly issued debt B.

B represents a broad liquidity policy, as we consider the government agency as a consolidated identity that includes a monetary authority, a fiscal authority, and financial intermediaries, which lends to the firm sector. Therefore, B is the net debt position of the joint agency; when $B < 0$, the consolidated identity lends to the firm sector. Alternatively, if we interpret the consolidated agency with only monetary and fiscal authorities, B can be considered an outcome of the quantitative easing (QE) policy. That is, we simplify the institutional details of liquidity policy to focus on the equilibrium effect of the interest rate on entrepreneurs' risk-taking and how the correlation of projects affects the optimal interest rate.

The government agency determines (G, R^d) every period. Then, B will be determined in the credit market, while tax rate τ^y will be the residual of the government budget constraint.

4.2 Equilibrium

Denote the total wealth of private agents as Ω. The dynamics of Ω can be derived from the wealth accumulation:

$$\Omega_+ = \beta\Omega \left\{ \phi \left[1 - F(\tilde{\eta})\right] R^d + (1-\phi) \left[1 - F(\tilde{\eta})\right] \left[\Pi^f (1+\bar{\theta}) - R\bar{\theta}\right] + F(\tilde{\eta}) \left[\Pi(\nu)(1+\theta) - R\theta\right] \right\}, \tag{18}$$

where again ϕ is the probability of saving entrepreneurs among those who do *not* run risky projects (note: they save with the rate $R^d \equiv R/(1+\tau)$ with τ denoting the intermediary markup). The next period's wealth Ω_+ will come from three sources. All entrepreneurs put a β fraction of their wealth (which explains $\beta\Omega$) aside. The group of entrepreneurs that do not take risks have ϕ fraction of savers (who has a return R^d) and $(1-\phi)$ fraction implementing the safe project with a return $\Pi^f(1+\bar{\theta}) - R\bar{\theta}$. The remaining entrepreneurs take risks, and the project return after leverage is $\Pi(\nu)(1+\theta) - R\theta$.

In the aggregate, capital productivity is endogenous because of credit-induced investment in different technologies. That is, capital reallocation linked with credit conditions determines the endogenous productivity of capital. Define Z as the endogenous productivity that takes into account the technology choices of entrepreneurs:

$$Z = \left[z^f(1-F)(1-\phi)(1+\bar{\theta}) + \bar{z}F(1+\theta)\right], where$$

$$\bar{z} = p^s(\nu)z^h + (1 - p^s(\nu))z^l.$$

Then, the aggregate output can be written as

$$Y = \int (z_i k_i)^\alpha \ell_i^{1-\alpha} di = (Z\beta\Omega)^\alpha L^{1-\alpha}. \tag{19}$$

Notice that each firm produces $z_i k_i/\alpha$ in which $z_i k_i$ is retained. Additionally, $Z\beta\Omega$ can be regarded as the effective capital stock used in production,

To define equilibrium, we look at market clearing conditions for labor and for credits. For the labor market, hours of work L should be the labor supply from households, so that the "rental rate" r can be written in a conventional way, which is the marginal product of effective capital stock (using L in the product)

$$r = \alpha (Z\beta\Omega)^{\alpha-1} L^{1-\alpha}. \tag{20}$$

For the credit market, the consolidated agency conducts the liquidity policy that determines the market-clearing interest rate. As mentioned before, we can think of the consolidated policy maker conducting an interest rate policy by choosing R^d. An interest rate R^d corresponds to a particular debt/liquidity policy B according to the market clearing condition. The total liquidity provided by savers at period t is $(1 - F(\tilde{\eta}))\phi\beta\Omega$. The total credit demanded by entrepreneurs who borrow is $\left[(1 - F(\tilde{\eta}))(1 - \phi)\bar{\theta} + F(\tilde{\eta})\theta\right]\beta\Omega$: a fraction $[1 - F(\tilde{\eta})](1 - \phi)$ of whom use the safe technology and borrow to the credit limit implied by $\bar{\theta}$ and a fraction $F(\tilde{\eta})$ of whom use the risky technology and leverage to $\theta \leq \bar{\theta}$. Therefore, the clearing of credit market implies

$$\left[(1 - F(\tilde{\eta}))(1 - \phi)\bar{\theta} + F(\tilde{\eta})\theta\right]\beta\Omega = [1 - F(\tilde{\eta})]\,\phi\beta\Omega - B, \tag{21}$$

which then determines the level of B to implement the deposit interest rate R^d.

Definition. Given states including the entrepreneurs' wealth Ω, a government spending G, and an interest-rate policy target R^d, a recursive competitive equilibrium is a collection of variables $\{L, C, \tilde{\eta}, \theta, \phi, \Pi^h, \Pi^l, \Pi^f, r, w, \tau^y, B, R, \Omega_+\}$ such that

- Households supply labor according to (13) ; their budget constraint (12) holds;
- $\tilde{\eta}$ and θ solve the entrepreneurs' problem with risk-taking choice and leverage choice as shown in Proposition 4;
- project returns (8) are satisfied, with r given by (7);
- The wealth dynamics (18) holds;
- The consolidated government budget constraint is satisfied, i.e., (17) holds with tax revenue $T = \tau^y Y$ where output is defined in (19);
- The credit market clears, i.e., (21) holds with (11) satisfied, and the lending rate R is determined by (9);
- The labor market clears, i.e., (20) holds.

Importantly, one can immediately see that the aggregate correlation, measured by ν, matters. It affects the market demand for liquidity and therefore the tax revenue T. When $\upsilon < 1$ and p is small, according to (16) we know that $\Pi(\nu) < \Pi(1) = \Pi$ and the required tax revenue T is lower. We will also show how the spillover affects other equilibrium variables and social welfare.

5 Macroeconomic Effects of Interest-rate Policy

Using the macroeconomic environment introduced above, we now analyze the aggregate long-run effects of interest rate policy, including whether the policy should incentivize risk-taking and how the optimal policy is affected by the leverage level and the degree of spillover ν.

5.1 Parameterization

It becomes too complex to study the optimal interest rate fully analytically. We choose to calibrate the model first and assess the policy effects. Table 2 reports the calibrated parameters.

Some parameters are exogenous. The capital share and the discount factor are set to conventional values $\alpha = 0.33$ and $\beta = 0.95$. The inverse of the Frisch elasticity of labor supply γ is set to 1, also a conventional number for macroeconomic models. The discussion below illustrates the key steps in calibrating other parameters.

Table 2: Calibration

	Value	Explanation/Target		Value	Explanation/Target
β	0.95	Discount factor	z^f	1	Normalization
γ	1	Inverse Frisch elasticity	z^h	1.8152	S&P index return top 90th per.
κ	2.3611	Hours 0.33	z^l	0.1270	S&P index return bottom 10th per.
α	0.33	Capital share	p	0.5953	Annual stock volatility
δ	0.0884	Investment-to-output 16%	μ	-3.0483	Debt-to-output ratio: 65%
τ	0.0208	Interest-rate spread	σ	0.0161	Elasticity of stock volatility: 2.98
$\bar{\theta}$	0.5600	Median leverage ratio	G	0.0843	Gov-spending-to-output 18%
η^f	0.0360	Median S&P index return	R	1.0330	Prime rate

κ, which governs the disutility of labor, is calibrated to hit the labor hours such that $L = 0.33$ after normalizing hours to unity. The model targets government spending to GDP ratio G/Y as 18%, which pins down G. The capital depreciation rate δ is calibrated so that the investment-to-output ratio is 16%.[14]

We choose the prime rate to be the status quo interest rate R, since there is no default in the model and the prime rate is a benchmark interest rate used for high-quality borrowers. We use the period 1954-2018 (see Appendix A for data description). The average annualized gross prime rate during this period is 1.033, while the average annualized gross federal funds rate during this period is 1.012. Therefore, the interest-rate markup τ is set to 2.08%.

The baseline sets $\nu = 1$ (no correlation among projects), and we will illustrate the macro effects of different spillovers. We choose no spillover as the benchmark because little evidence shows that firm returns are positively or negatively correlated stably over time. Additionally,

[14] In the model, we derive the investment output ratio as $\frac{I}{Y} = \frac{\delta\left(1+\left[(1-F(\tilde{\eta}))(1-\phi)\bar{\theta}+F(\tilde{\eta})\bar{\theta}\right]-(1-F(\tilde{\eta}))\phi\right)}{\frac{r}{\alpha}\left[z^f(1-F)(1-\phi)(1+\bar{\theta})+\bar{z}F(1+\bar{\theta})\right]}$.

even if we calibrate ν, e.g. by matching the average difference in private and social returns, we find that both the qualitative and quantitative conclusions from the comparative analysis below are similar.

Then, we use the top $90th$ percentile return (Π^{90th}) and the bottom $10th$ percentile return of the S&P 500's (Π^{10th}) to represent the net leveraged return for high realization and low realization, i.e.,

$$\Pi^{90th} \equiv (1+\bar{\theta})\Pi^h - R\bar{\theta}; \quad \Pi^{10th} \equiv (1+\bar{\theta})\Pi^l - R\bar{\theta}.$$

As a normalization, we map the S&P 500's median return to the net leveraged risk-free return $\Pi^m \equiv (1+\bar{\theta})\Pi^f - R\bar{\theta}$. The top $90th$ percentile, bottom $10th$ percentile, and median net return for the period of 1954-2018 are 22%, -13%, and 5%, respectively, so that $\Pi^{90th} = 1.22$, $\Pi^{10th} = 0.87$, and $\Pi^m = 1.05$. The benchmark exercise normalizes $z^f = 1$ and assumes that some entrepreneurs save. Then, we obtain η^f from the indifference condition $\log R^d = \log(\Pi^m) - \eta^f/\beta$, and we obtain r as a function of $\bar{\theta}$:

$$r = \frac{\Pi^m + R\bar{\theta}}{1+\bar{\theta}} - (1-\delta).$$

Additionally, z^h and z^l are also functions of $\bar{\theta}$ by substituting the above expression for r to the expression for Π^{90th} and Π^{10th}:

$$z^h = \frac{1}{r}\left[\frac{\Pi^{90th} + R\bar{\theta}}{1+\bar{\theta}} - (1-\delta)\right]; \quad z^l = \frac{1}{r}\left[\frac{\Pi^{10th} + R\bar{\theta}}{1+\bar{\theta}} - (1-\delta)\right].$$

Notice that the leverage parameter $\bar{\theta}$ is needed for r, z^h, and z^l. It is set according to the firm leverage ratio used in Section 2. We drop the observations of firms whose return standard deviation is higher than the $95th$ percentile or lower than the $5th$ percentile at the one-digit SIC industry level, and we only consider firms whose leverage ratio is positive. The median of the leverage ratio is 0.56. Then, r, z^h, and z^l are identified immediately as shown above.

We assume the cost distribution follows a log-normal distribution with the mean as μ and variance σ^2 after log transformation.[15] Therefore, the remaining key parameters are: $\{\mu, \sigma, p\}$. They are chosen to match the three moments below.

(1). We target the debt-to-output ratio[16] 0.65 as in the data[17] after we use the total debts of the non-financial businesses.

(2). Assuming risk-taking firms are financially constrained in the status quo, we have the

[15] we tried other distributions, which are not crucial for the qualitative conclusion shown below.

[16] $\tilde{\eta}$ is solved by (10) and ϕ is solved by (18). After simplification $\frac{D}{Y} = \frac{\left[(1-F(\tilde{\eta}))(1-\phi)\bar{\theta}+F(\tilde{\eta})\bar{\theta}\right]}{\frac{r}{\alpha}\left[z^f(1-F)(1-\phi)(1+\bar{\theta})+\bar{z}F(1+\bar{\theta})\right]}$.

[17] Total debts of the non-financial businesses over the total output of the non-financial businesses.

standard deviation of the overall return in the model

$$\sigma_I = \sqrt{F(\tilde{\eta})(1+\bar{\theta})}\sqrt{p(1-p)}\left(\Pi^h - \Pi^l\right) = \sqrt{F(\tilde{\eta})}\sqrt{p(1-p)}\left(\Pi^{90th} - \Pi^{10th}\right),$$

corresponding to the standard deviation of S&P 500's annual return 0.132.

(3). Most importantly, its sensitivity to the interest rate becomes

$$\frac{\partial \sigma_I}{\partial R}\frac{R}{\sigma_I} = \left[\frac{1}{2}\frac{f(\tilde{\eta})}{F(\tilde{\eta})}\frac{\partial \tilde{\eta}}{\partial R} + \frac{1}{\Pi^h - \Pi^l}\left(\frac{\partial \Pi^h}{\partial R} - \frac{\partial \Pi^l}{\partial R}\right)\right] R, \tag{22}$$

corresponding to 2.98 in the data. Note that when the financing constraint is binding, the derivative $\partial\tilde{\eta}/\partial R$ can be computed as

$$\frac{\partial \tilde{\eta}}{\partial R} = \begin{cases} \beta p \frac{1}{\Pi^{90th}}\left(\frac{\partial \Pi^h}{\partial R} + \left(\frac{\partial \Pi^h}{\partial R} - 1\right)\bar{\theta}\right) \\ +\beta\left(1-p\right)\frac{1}{\Pi^{10th}}\left(\frac{\partial \Pi^l}{\partial R} + \left(\frac{\partial \Pi^l}{\partial R} - 1\right)\bar{\theta}\right) \\ -\beta\frac{1}{\Pi^m}\left(\frac{\partial \Pi^f}{\partial R} + \left(\frac{\partial \Pi^f}{\partial R} - 1\right)\bar{\theta}\right) \end{cases}.$$

Since this outcome is unique to our model, we elaborate the procedure. First, as in the calibration above, those entrepreneurs who do not take risks are indifferent between saving in safe assets and implementing the safe project. That is, we have the second branch of (11) so that

$$\Pi^f = \frac{\frac{e^{\beta^{-1}\eta^f}}{1+\tau} + \bar{\theta}}{1+\bar{\theta}} R \rightarrow \frac{\partial \Pi^f}{\partial R} = \frac{\frac{e^{\beta^{-1}\eta^f}}{1+\tau} + \bar{\theta}}{1+\bar{\theta}},$$

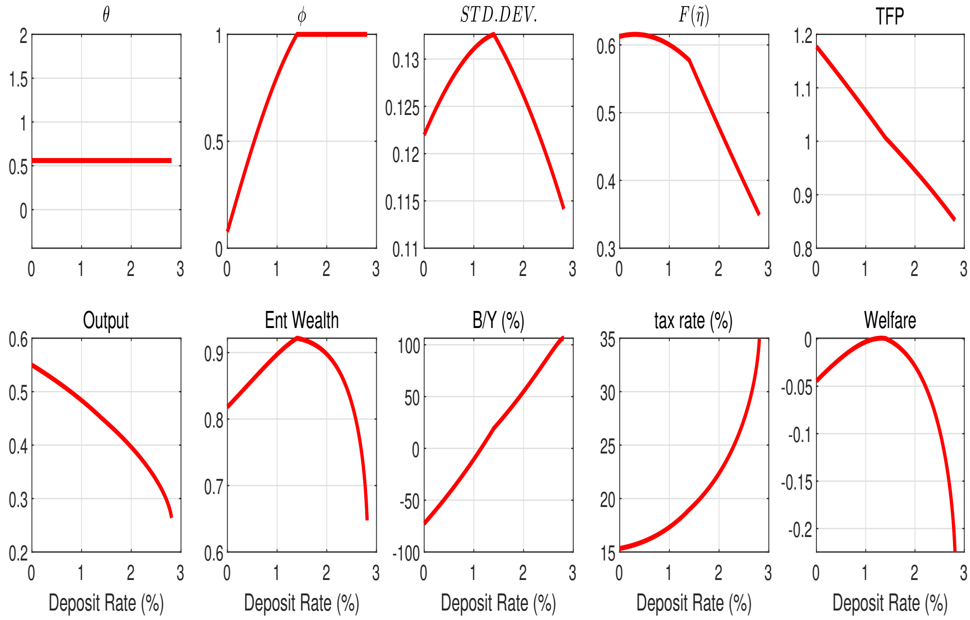
and $\partial\Pi^h/\partial R$ and $\partial\Pi^l/\partial R$ follow immediately. Otherwise, one needs to solve the whole equilibrium to examine the effect of interest rate R on the risk-taking threshold $\tilde{\eta}$ and eventually the volatility measure σ_I, which brings the calibration overly relying on the general equilibrium effect. Second, we also examine the case when risk-taking entrepreneurs may not be financing-constrained, but the final result points to the constrained scenario. It turns out p is estimated to be about 0.6, meaning the high return is relatively more abundant.

5.2 The Effects of Interest Rate Policy

Suppose the policymaker implements different interest rate levels by adjusting B. Figure 5 shows how key variables change with the safe interest rate R^d.

This numerical illustration reveals a modestly curved relationship between the risk-taking threshold $\tilde{\eta}$ and the interest rate. However, the model exhibits a pronounced hump-shaped curve in the standard deviation of investment returns as the interest rate fluctuates. As the interest rate increases, saving in secure deposits becomes more appealing. The proportion of savers among non-risk-takers providing liquidity, denoted as ϕ, rises with the deposit rate R^d and reaches 100% at the upper limit if R^d is sufficiently high. It's worth noting that the total

Figure 5: The Effects of Interest Rate Policy



Note: This figure plots equilibrium variables as functions of the (net) deposit rate $R^d - 1$ at the calibrated parameter values. The variables shown are the optimal leverage θ, the share of saving firms among non-risk-taking firms ϕ, standard deviation of project returns, the share of risk-taking firms $F(\tilde{\eta})$, TFP, aggregate output, aggregate entrepreneurs' wealth (Ent Wealth), and the consumption-equivalent (in the calibrated economy) welfare (see section 5.3 for more details).

factor productivity (TFP), represented as

$$TFP \equiv Z^{\alpha} = \left[z^f(1 - F(\tilde{\eta}))(1 - \phi)(1 + \bar{\theta}) + \bar{z}F(\tilde{\eta})(1 + \theta)\right]^{\alpha},$$

decreases with the interest rate because a higher interest rate predominantly deters risk-taking within the analyzed range. Consequently, reduced risk-taking exerts downward pressure on TFP.

Entrepreneurs' wealth initially rises with R^d but then declines as R^d continues to increase. When R^d is low, changes in the interest rate have minimal impact on risk-taking behavior (note: as the interest rate rises, F experiences slight fluctuations). Consequently, the effect on the wealth of entrepreneurial individuals remains largely unaffected by interest rate hikes. The upsurge in entrepreneurs' wealth primarily stems from the elevated deposit rate, benefiting those who save in deposits. However, equilibrium risk-taking diminishes rapidly when R^d is high. At the aggregate level, as risk-taking typically yields higher returns than safe projects and deposits, entrepreneurs' wealth decreases when fewer decide to take risks.

Notice that aggregate output measured by

$$Y = \frac{r\beta\Omega Z}{\alpha} = \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} L,$$

decreases in r and increases in L. Given that r naturally rises with the deposit rate R^d while labor does the opposite, output thus drops when the government runs a higher interest rate. Hence, we observe falls in investment and consumption when R^d is higher (not plotted). Thus, a downward-sloping long-run "IS" relationship emerges from the risk-taking channel.

At base line B/Y, the gap between saving and borrowing of entrepreneurs denominated by output, is 3.6% while that of the data is around 2.8%[18].

The Role of Leverage Limit. As shown in the simple model, risk-taking behavior depends on leverage, which in turn affects the aggregate. How does leverage limit influence the macroeconomic effects of interest rate policy?

When $\bar{\theta}$ declines by 50% from the calibrated value of $\bar{\theta} = 0.56$ to $\bar{\theta} = 0.28$ (illustrated by the blue dash-dotted lines in Figure 6), the proportion of risk-taking entrepreneurs, $F(\tilde{\eta})$, becomes sensitive to changes in the interest rate. It rises with low interest rates and declines with high interest rates. This finding reaffirms the earlier observation that the impact of interest rate valuation prevails, particularly when leverage is low. As a result, the equilibrium proportion of savers, ϕ, predominantly decreases as the demand for liquidity diminishes due to the reduced external financing limit, $\bar{\theta}$. Simultaneously, total factor productivity (TFP) uniformly decreases with declining leverage, resulting in a downward shift in output. However, for low interest rate levels, entrepreneurs' wealth experiences an upward shift due to a stronger incentive for saving when borrowing capacity is constrained. As interest rates rise, driven by government borrowing, the necessity for higher taxation dampens project returns and subsequently diminishes entrepreneurs' wealth.

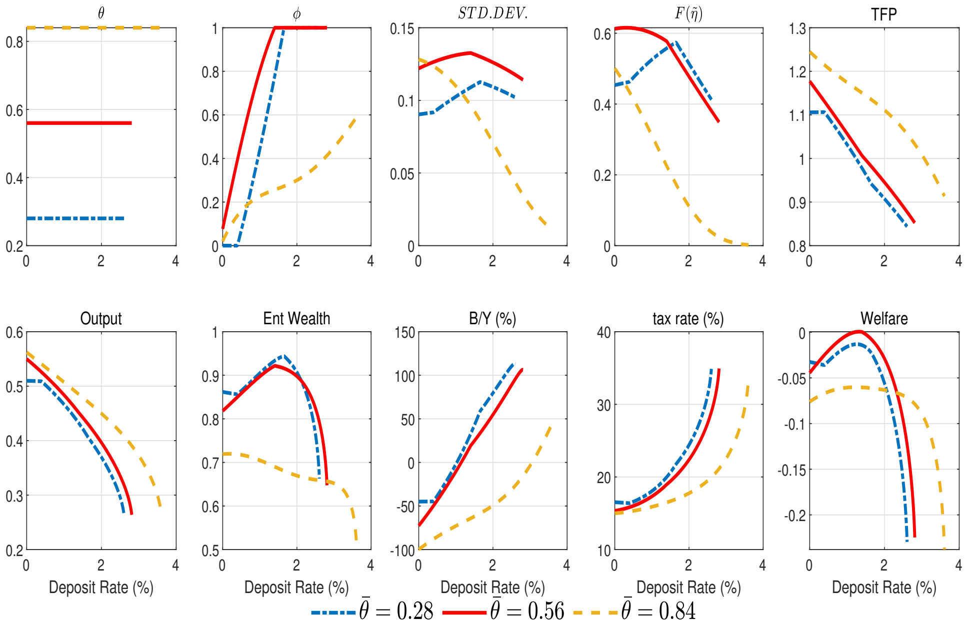
For comparison, we consider an increase of 50% in $\bar{\theta}$ to 0.84 (depicted by the yellow dashed lines). Now, $F(\tilde{\eta})$ exhibits a downward slope because the substitution effect prevails when leverage is high. In other words, when $\bar{\theta}$ is sufficiently high, an increase in the interest rate discourages risk-taking, as indicated by Proposition 2. Moreover, with $\bar{\theta}$ at a higher level, the incentives for risk-taking decrease for any given interest rate. This is evident in the downward shift of the $F(\tilde{\eta})$ curve, attributable to the valuation effect resulting from the elevation of $\bar{\theta}$ while holding the interest rate constant. This finding aligns with the observations made in the corollary of Proposition 2 (see Corollary C.2 in the Appendix).

In general, total factor productivity (TFP) increases with higher leverage, allowing more resources to be channeled to entrepreneurs engaged in productive projects, whether risky or safe. Consequently, there is a reduced necessity for entrepreneurs to save, leading to a decline in the wealth (Ω) of entrepreneurs.

The model's quantitative implications regarding varying leverage exhibit alignment with the data. Figure 7 illustrates this comparison for the years 2001 and 2007 with comparative-

[18] The saving is measured by total asset excluding: net trade receivable,private foreign assets, securities for repurchase, commercial paper,municipal securities,consumer credit, corporate equities,US foreign direct investment, money market fund

Figure 6: Interest Rate Effects under Alternative Financing Constraints



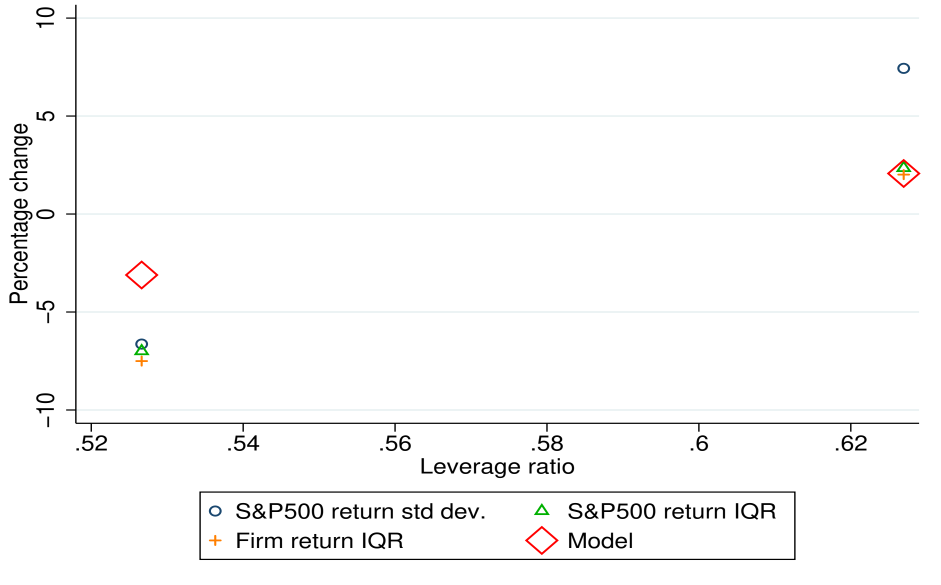
Note: This figure shows the equilibrium outcomes under three possible leverage limits of $\bar{\theta}$. Variables are the same as in Figure 5.

static analysis. Given that we only fit the model with relevant interest rates and leverages, keeping other parameters fixed, a perfect fit should not be expected. However, the results below demonstrate that the impact of an interest rate cut significantly relies on the leverage limit.

In Section 2, we observed that in 2001, leverage stood at approximately 0.527, with the Cleveland rate at around 1.7%. Over the period from 2001 to 2002, the interest rate decreased by 142 basis points (bps). Applying the steady-state model with the same leverage and interest rate levels and the observed cut in interest rate suggests a standard deviation drop of 3.1%. On the data side, focusing on medium-run volatility, we examined the five-period moving average of the standard deviation of the S&P 500 return, which decreased by 6.6%; So, the model can account for about 41% variation. Additionally, we investigated the moving averages of the IQR of the S&P 500 return and the median firm-stock-return IQR, which decreased by 7% and 7.5%, respectively.

Moving to 2007, leverage was around 0.627, while the Cleveland rate was approximately 1.8%. From 2007 to 2008, the interest rate dropped by 150 bps. Applying these figures to the steady-state model suggests a 2.1% increase in the standard deviation. Empirically, the moving average of the standard deviation of the S&P 500 return increased by 7.4%, so the model can account for about 29%. Additionally, the moving averages of the IQR of the S&P 500 return and the median firm-stock-return IQR increased by 2.3% and 2%, respectively, and the model's prediction is very close to the data.

Figure 7: Comparison between data and model for the years 2001 and 2007



Note: The model results are calculated with leverage and interest rates from the data, keeping other parameters fixed. The left points are for 2001 (with a lower leverage ratio), and the right points are for 2007 (with a higher leverage ratio). The data counterparts are 5-year moving averages.

5.3 Optimal Interest Rate

What is the deposit rate R^d that maximizes the joint social welfare of entrepreneurs V and households W Given the value functional form and the nature of the steady state economy, the social welfare measure, which is related to present value of utility of consumption from all groups, thus becomes (see Appendix D for details) :

$$V(\Omega,\phi)+W=\log(\Omega)+\tilde{V}(\phi)+\log\left(C^h-\frac{\kappa L^{1+\gamma}}{1+\gamma}\right)+\text{constants},$$

and $\tilde{V}(\phi)$ is the value of the entrepreneurs depending on the specific value of ϕ:

$$\tilde{V}(\phi)=(1-\beta)^{-1}\left(\beta\max\{l^d,l^f\}+\int^{\tilde{\eta}}F(\eta)d\eta\right).$$

This term includes the base return (either risk-free return or safe deposit return, i.e., the first term in the second bracket of $\tilde{V}(\phi)$) and the relative value gain from choosing the risky project.

When assessing the policy effect on the social welfare of a target economy, we compute the corresponding consumption equivalence measure ψ of the baseline calibrated economy. Given that entrepreneurs' consumption is $(1-\beta)$ fraction of their wealth, we calculate the gain/loss of wealth of entrepreneurs (Ω_{base}) and the consumption of households (C^h_{base}) such that the baseline economy with $(1+\psi)\Omega_{base}$ and $(1+\psi)C^h_{base}$ (and everything else stays as their calibrated level) has the same welfare measure as in the target economy.

Notice that a natural upper bound of R^d is β^{-1} as otherwise the household would save.[19] However, the effective upper bound may be lower than this level because the implied consequence on the tax rate is not feasible. We also assume a lower bound of R^d. This could be because of liquidity traps and/or adverse expectations when the interest rate becomes ultra-low. The numerical examples below set the lower bound to be 1, but having 0.98 or 0.97 (i.e., -2% or -3% net interest rate) does not change the result. Additionally, implementing persistent -2% or -3% interest rate in the model requires the government to hold privately-issued assets close to 100% of aggregate output (that is , $B/Y = -100\%$), which is unrealistic. For these reasons, we set the lower bound to be unity.

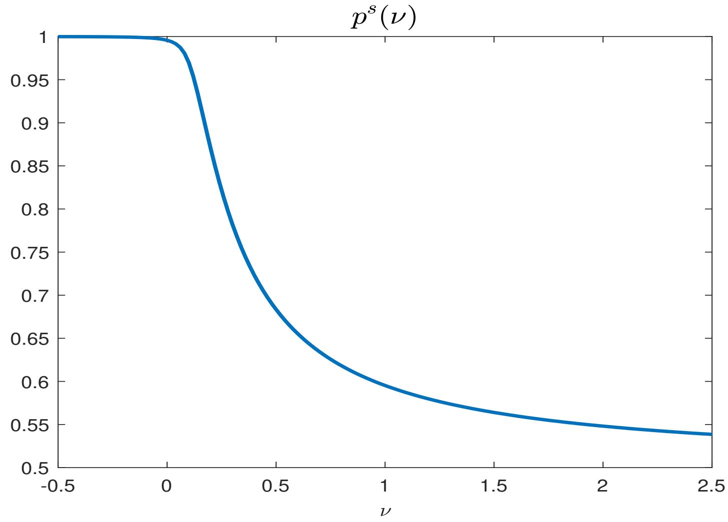
Figure 5 shows entrepreneurs' welfare closely tracks their wealth. When R^d rises, the government authority borrows more (or saves less, as seen by the debt-to-output ratio B/Y) since firms save more. As explained above, initially, risk-taking behavior increases with the interest rate, but the government has to raise more taxes from the private agents to satisfy the government budget constraint. Risk-taking behavior also falls significantly when R^d becomes high enough. Therefore, the welfare of everyone eventually falls, and the social welfare displays a hump shape, suggesting the optimal deposit interest rate being around 1.3%.

Note that the optimal interest rate falls slightly even if $\bar{\theta}$ is halved (Figure 6). There are two competing forces behind this insensitivity of optimal interest rate. On the one hand, a lower interest rate can compensate for the adverse effect of tougher financial conditions when $\bar{\theta}$ falls; on the other hand, given that the valuation effect of risk-taking is dominating, raising the interest rate encourages risk-taking when leverage is low. An optimal interest rate level still exists to balance the incentives of those entrepreneurs who are borrowers to implement projects and those who are savers. The optimal level, however, does not seem to have a clear relationship with leverage, consistent with the non-monotone results we obtained previously.

The Role of Spillover. We experiment with different scenarios by having positive correlation (i.e., $\nu = 0.85$) and negative correlation (i.e., $\nu = 1.15$). Notice that when projects exhibit positive correlation and p is relative high, as discussed in Section 4, we know that $\Pi(\nu) > \Pi$. The reason is that the positive spillover of the higher return dominates the positive spillover effect of the low return when the high return is relatively more abundant. The opposite is true if project outcomes are negatively correlated. The "social" probability of success, $p^s(\nu)$ is thus a decreasing function of ν, as shown by the simulation result in Figure 8 using the benchmark $p = 0.5953$. It is worth noting that, although for ease of communication, we refer to $\nu < 1$ as a positive correlation and $\nu > 1$ as a negative correlation, strictly speaking, they are not true correlations in the conventional sense. Kadane (2016) refers to them as positive and "negative associations".

[19] We check this type of equilibrium in which households save. The welfare is dominated by cases in which the deposit rate is lower, i.e., $R^d < \beta^{-1}$. The reason is that the planner is incentivised to lower the interest rate and redistribute resources towards entrepreneurs with productive projects.

Figure 8: "Social" Probability of Success under Different ν.



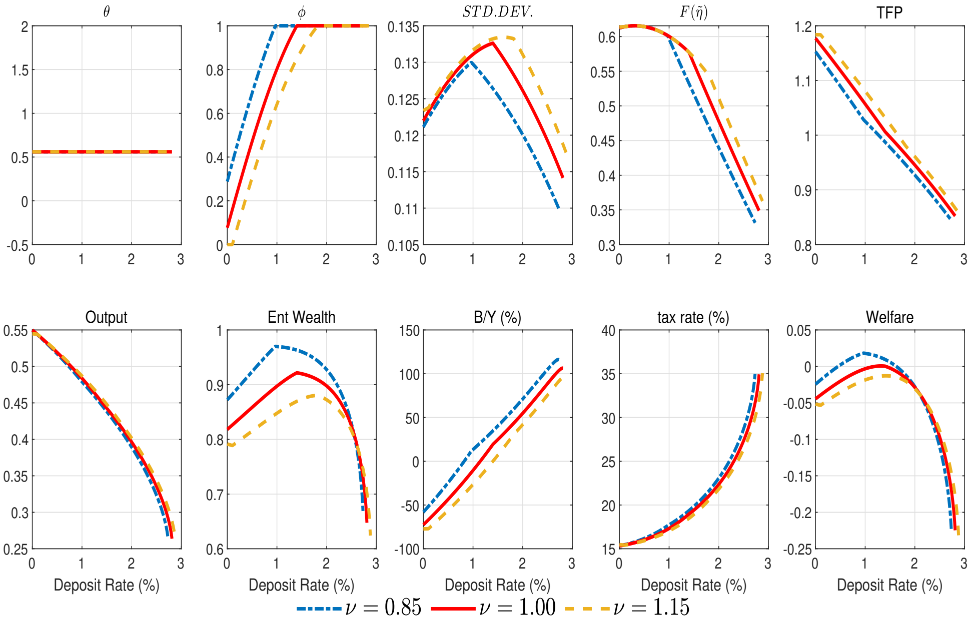
Note: The simulated probability of success $p^s(\nu)$ from CMB as a function of ν.

To understand this observation, we obtain the correlation of two consecutive draws and compute the correlation of the two draws being z^l (see Appendix D.2 for more details). It turns out that under benchmark parameters, $\nu = 0.85$ corresponds to a correlation of 0.05, and $\nu = 1.15$ corresponds to a correlation of -0.05.[20] For $\nu = 0.85$, the pair-wise correlation (0.05) may appear small initially, but the cumulative effect (or the spillover effect) across millions of projects cannot be seen from that. The "social" probability of success thus better reflects this cumulative effect and $p^s(0.85) = 0.6550$, about 10% more than the independent probability p. Another way to see this is to examine the conditional probability. When $\nu = 0.85$, (14) implies that the probability of the failing second project conditioning on the failing first project is about 0.43, about 6.25% more than the independent probability of failing $1 - p$. Therefore, pair-wise correlation does not serve the purpose of measuring the overall spillover effect. Thus, we use ν, and notice that the social probability $p^s(\nu)$ is close to unity when ν approaches zero according to Figure 8.

When project outcomes become positively correlated (the blue dashed-dotted lines in Figure 9), the social return assigns a higher probability $p^s(\nu) > p$ to Π^h. However, individuals still maintain belief in the probability p. Consequently, capital accumulation increases (along with entrepreneurs' wealth) across most interest rate levels, causing a decline in the net return of effective wealth r for a given deposit rate. As a result, the gross returns of risky projects Π^h and Π^l, and consequently the risk-taking measure $\tilde{\eta}$, decrease. With a lower r, the gross return of the safe project Π^f also diminishes, making it less appealing. We thus observe a modestly higher ϕ across various interest rate levels. In conjunction with the lower $\tilde{\eta}$ and higher ϕ, total factor productivity (TFP) declines, leading to increased precautionary savings and higher entrepreneurs' wealth.

[20]The correlation further increases as ν decreases. For instance, $\nu = 0.4$ yields a correlation of 0.2, $\nu = 0.2$ yields a correlation of 0.26, and $\nu = 0$ results in a correlation of 0.32.

Figure 9: Macroeconomy under Different ν.



Note: This figure plots the equilibrium outcomes for different levels of project correlation. $\nu = 1$ is the benchmark case (no correlation) as shown in Figure 5. $\nu < 1$ means project outcomes are positively correlated; $\nu > 1$ means project outcomes are negatively correlated.

Note that the positive spillover effect results in higher entrepreneurs' welfare compared to the baseline scenario. With more firms opting to save in safe deposits, greater government borrowing is necessary to maintain equilibrium in the credit market. This necessitates higher taxation to balance the government budget constraint, ultimately lowering social welfare for sufficiently high interest rates compared to the case of no correlation (the red solid lines).

Regarding the optimal interest rate, the level is lower than that of the baseline scenario. The optimal rate should always strike a balance between the welfare of entrepreneurs engaged in projects and those saving in deposits. However, risk-taking behavior becomes more responsive to interest rate changes when ν decreases (as evidenced by the disparity between the blue and red lines in the $F(\tilde{\eta})$ panel). Hence, a reduction in the interest rate is necessary to utilize the heightened sensitivity of socially beneficial risk-taking behaviors. It's noteworthy that the optimal welfare surpasses that of scenarios with no correlation (illustrated by the red solid line) and negative correlation (depicted by the yellow dashed line) because the social return is highest in this case, i.e., $\Pi(0.85) > \Pi(1) > \Pi(1.15)$.

Naturally, the results are reversed when project outcomes become negatively correlated. The policymaker should increase the interest rate to deal with the negative correlation, consistent with the previous analysis of positive correlation (see the peaks of curves in the welfare panel). But one common feature of the optimal interest rate across the three cases of correlation can be summarized as follows. The optimal interest rate should be generally low to encourage resources used by productive entrepreneurs. However, the optimal rate should be set so the economy is at the steep branch of the $F(\tilde{\eta})$ curve. The steep branch of the curve means that the

Table 3: Optimal (Net) Interest Rate in %

$\bar{\theta}$	ν		
	0.85	1.00	1.15
0.28	1.20	1.26	1.30
0.56	0.97	1.32	1.42
0.84	0.77	1.22	1.79

risk-taking behavior is sensitive to interest rate variations.

Finally, Table 3 presents the optimal interest rate for various levels of $\bar{\theta}$ and spillovers. When risky project returns are positively (negatively) correlated, the optimal deposit interest rate consistently appears lower (higher). This pattern emerges because a positive correlation implies higher $\Pi(\nu)$ compared to Π. Consequently, the planner should take into account the overall positive spillover externality and maintain a lower interest rate. Conversely, for negative correlations, the opposite holds.

Table 3 underscores the significance of striking a proper balance between interest rate and macro-prudential policies, which typically directly affect $\bar{\theta}$. The impact of the spillover effect on the optimal interest rate is contingent upon the leverage limit. Specifically, when risky project returns are positively (negatively) correlated, the optimal deposit interest rate decreases (increases) with $\bar{\theta}$, further underscoring the necessity for a well-balanced approach. In scenarios with no correlation ($\nu = 1$), the optimal rate initially rises and then declines with the leverage limit $\bar{\theta}$. The discussion below delves into the optimal mix of policies.

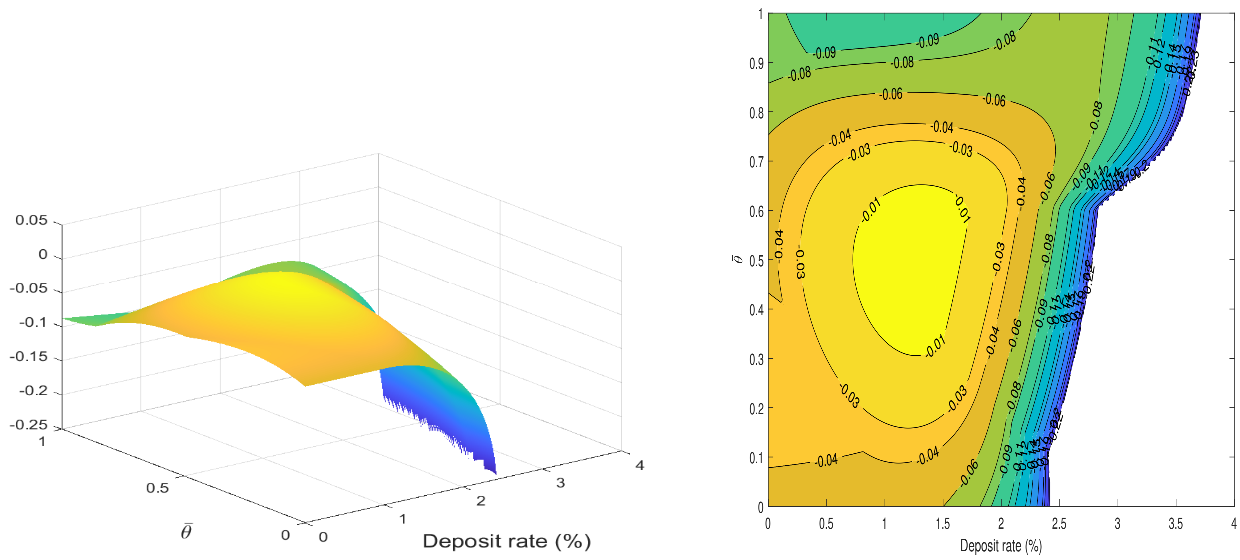
5.4 Optimal Mix of Interest Rate and Leverage Limit

Given a leverage limit $\bar{\theta}$, we have shown that there can be an optimal interest rate level. We further explore the possibility of a joint pair of $(R, \bar{\theta})$ that maximizes social welfare. In this regard, we are searching for an optimal mix of policies, i.e., interest-rate policy and (macro-)prudential/leverage policy.

We start by examining the economy with calibrated parameters. When projects exhibit no correlation ($\nu = 1$), the optimal interest rate and leverage pair are $(1.25\%, 0.4972)$, as shown by Figure 10. The optimal rate is only five basis points above the calibrated level of 1.20%, while the leverage limit is below the calibrated level of 0.56 by about 6.3 percentage points.

Several comparisons can be drawn from the exercise in the previous section. When the deposit rate surpasses 2% (the lending rate exceeds 4.1% due to the banking markup), the tax rate escalates rapidly, leading to distortions in project return rates. The optimal policy should circumvent such a scenario. Furthermore, when the leverage limit reaches between 0.7 and 0.8, social welfare experiences a sharp decline if leverage continues to increase. This outcome can be illustrated by the valuation effect outlined in the theoretical framework; entrepreneurs prefer taking less risk if leverage is already high and continues to rise. Considering that a reduction in leverage limits resource allocation and that a decrease in interest rates adversely affects savers,

Figure 10: The Welfare Consequence of Interest Rate and Leverage Limit



Note: The left panel shows the consumption-equivalent loss as a function of deposit rate $R^d - 1$ and leverage limit $\bar{\theta}$. The right panel is the corresponding contour plot of the 3-dimensional plot on the left.

the social planner may seek an interior optimal blend of interest rate and leverage limit to maximize welfare.

Finally, we assess how spillover affects the optimal pair of interest rates and leverage limits. Figure 11 shows how optimal interest rate and leverage limit pairs respond to variations in the correlation of project outcomes. As outcomes of projects shift from negatively correlated to positively correlated (i.e., as ν falls), the economy calls for a monotone fall in interest rates because more resources should be given to firms. Notice that the firms are more productive at the aggregate level with more positively correlated outcomes. For example, when ν drops from unity to 0.5, the "social" probability increases from $p = 0.5953$ to around 0.67 (according to Figure 8), and the optimal interest rate is almost zero; when ν increases to 1.5, the optimal rate is still below 1.6%.

Therefore, the planner chooses a low interest rate (relative to the time preference rate) since it benefits resource allocation. However, the private economy may have too little or too much risk-taking, thanks to individual entrepreneurs not considering the externality.

To address the externality issue, the leverage limit arrives at the center stage. When project outcomes shift from no correlation to negative correlation, ν increases from unity, and the leverage limit falls to prevent excess risk-taking. However, when project outcomes become more positively correlated, ν falls from unity, and the optimal leverage shape becomes non-monotonic. The leverage limit initially rises gradually, then falls, and rises sharply when the interest rate is binding.

Figure 11: Optimal Policy Mix

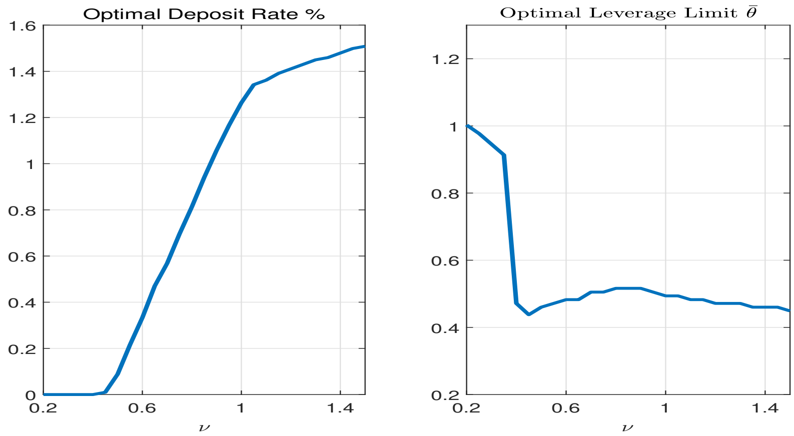
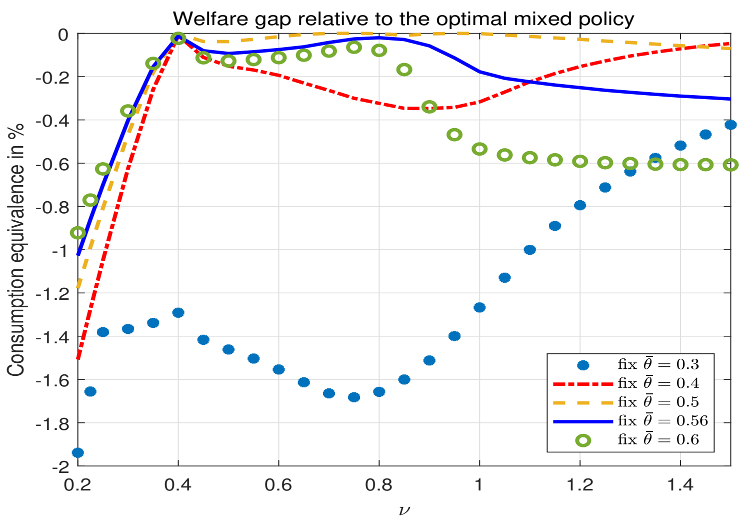


Figure 12 illustrates the welfare loss when only monetary policy is employed, without leverage policy, relative to adopting the optimal policy mix. The leverage limit is fixed at different constant levels. Overall, the optimal monetary policy performs well when projects are moderately correlated and leverage limit is close to the optimal level. However, the welfare loss can be considerable, up to more than 1%, when the interest rate is constrained at the lower bound or the leverage limit is far from the optimal leverage. This emphasizes the importance of proper coordination between monetary policy and leverage policy. In Appendix E we also examine the comparison when both the leverage level $\bar{\theta}$ and interest rate R are fixed at the optimal level for $\nu = 1$. The welfare gap relative to adopting the optimal policy mix is significantly larger.

Figure 12: Welfare Gap against Optimal Policy Mix



6 Conclusion

In this paper, we illustrate the impact of financing constraints on firms' risk-taking behavior. The effect of interest rates on risk-taking could be non-monotonic overall, which may also depend on leverage conditions. Specifically, when firm leverage is low (high), a cut in the interest rate discourages (encourages) risk-taking. When leverage is moderate, an interest rate cut encourages risk-taking only when the interest rate is high and discourages risk-taking otherwise. Our analysis may explain why earlier studies found mixed results regarding the relationship between liquidity and firm volatility. Therefore, with a further cut of interest rate in a low-interest-rate environment, firms may not pursue risky but socially desirable projects.

Apart from leverage conditions, we also show that whether an interest rate policy incentivizes firm risk-taking depends crucially on the spillover effects among risk-takers. The analysis highlights the need for a careful mix of interest rate and macro-prudential policies. In future research, we expect taxations that alter projects' risk profiles should also be carefully mixed with interest rate policy.

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A Data Source

We use daily stock returns from CRSP to calculate the inter-quartile range (IQR) of the daily returns and the standard deviations of annualized stock returns. CRSP is a database that maintains the most comprehensive collection of security price, return, and volume data for the NYSE, AMEX, and NASDAQ stock markets. Other financial data is from US COMPUSTAT, and firms from the financial sector are excluded. We also excluded the observations with independent variables that are below $1st$ percentile or above $99th$ percentile.

We use the real federal fund rate as the benchmark rate to measure the interest rate (results are similar when switched to the prime rate). The federal funds rate, inflation, the Federal Reserve Bank of Cleveland's real interest rate and expected inflation, and the prime rate are all from FRED (maintained by the Federal Reserve Bank of St. Louis).

B Further Empirical Evidence

This section shows that the non-monotone relationship also holds within each sector classified by the one-digit SIC code (see Figure **??**).

C Derivations and Proofs

C.1 Proof of Proposition 1

If the entrepreneur chooses a risky project, the value function becomes:

$$
\begin{aligned}
V^r(\omega,\eta) = & \max_{s,0\le b\le\bar{\theta}s} \Big\{ \log(\omega - s) - \eta + \beta p \log(\Pi^h(s+b) - Rb)\} \\
& + \beta(1-p)\log(\Pi^h(s+b) - Rb)\Big\} \\
= & \max_{s,0\le\theta\le\bar{\theta}} \Big\{ \log(\omega - s) + \beta\log s - \eta + \beta p \log\Big(\Pi^h(1+\theta) - R\theta\Big) \\
& + \beta(1-p)\log\Big(\Pi^l(1+\theta) - R\theta\Big)\Big\}.
\end{aligned}
$$

Through first-order conditions, the optimal saving rate is $s = \varphi\omega$ with $\varphi = \frac{\beta}{1+\beta}$. Denote θ^* as the level of the unconstrained optimal leverage a firm will optimally choose were they not facing the financing constraint, then θ^* solves the first-order condition (ignoring $\theta > 0$ but we will come back to that):

$$
p\frac{\Pi^h - R}{\Pi^h(1+\theta) - R\theta} + (1-p)\frac{\Pi^l - R}{\Pi^l(1+\theta) - R\theta} = 0.
$$

Therefore, the cutoff interest rate level $\Pi^h\Pi^l/\hat{\Pi}$ is obtained by setting $\theta = 0$ above and it is straightforward to verify that $\Pi^h\Pi^l/\hat{\Pi} \in \left(\Pi^l, \Pi^h\right)$. If $R > \Pi^h\Pi^l/\hat{\Pi}$, then the interest rate is too high to justify borrowing. If $R \le \Pi^h\Pi^l/\hat{\Pi}$, then we can express $\theta^* > 0$ as

$$
\theta^* \equiv -\frac{\Pi^h\Pi^l - R(p\Pi^l + (1-p)\Pi^h)}{(\Pi^l - R)(\Pi^h - R)} = -\left[p\frac{\Pi^l}{\Pi^l - R} + (1-p)\frac{\Pi^h}{\Pi^h - R}\right], \tag{23}
$$

proving the first part. Notice that $\theta^* > 0$ because $R < \Pi^f \leq \Pi^h\Pi^l/\hat{\Pi}$ under Assumption A2. The optimal leverage $\theta = \min\{\theta^*, \bar{\theta}\}$. We now show that θ^* decreases in R. Notice that

$$\begin{aligned}\frac{\partial\theta^*}{\partial R} &= \frac{\hat{\Pi}(\Pi^l - R)(\Pi^h - R) + (-\Pi^h\Pi^l + R\hat{\Pi})(+\Pi^h + \Pi^l - 2R)}{(\Pi^l - R)^2(\Pi^h - R)^2} \\ &= \frac{\Pi^l\Pi^h(R - \Pi) + (\Pi^h\Pi^l - \hat{\Pi}R)R}{(\Pi^l - R)^2(\Pi^h - R)^2}.\end{aligned}$$

The numerator is quadratic and concave in R. With some algebra, the maximum value of the numerator is $\frac{\Pi^h\Pi^l(\Pi^h\Pi^l - \hat{\Pi}\Pi)}{\hat{\Pi}}$, which is negative. To see this, we can use the convexity, $\frac{p}{\Pi^h} + \frac{1-p}{\Pi^l} > \frac{1}{p\Pi^h + (1-p)\Pi^l} = \frac{1}{\Pi}$ which implies $\Pi^h\Pi^l < \hat{\Pi}\Pi$. Then the numerator is less than 0 and $\frac{\partial\theta^*}{\partial R} < 0$.

If the entrepreneur chooses a safe project, we have:

$$\begin{aligned}V^f(\omega) &= \max_{s, 0 \leq b \leq \bar{\theta}s} \{\log(\omega - s) + \beta\log(\Pi^f(s + b) - Rb) \\ &= \max_{s, 0 \leq \theta \leq \bar{\theta}} \{\log(\omega - s) + \beta\log s + \beta\log(\Pi^f(1 + \theta) - R\theta)\}.\end{aligned}$$

This means that the optimal saving rate is $s = \varphi\omega$ again. Since $\Pi^f > R$, a firm will borrow up to the limit $\theta = \bar{\theta}$.

Finally, we determine the threshold of taking risky project $\tilde{\eta}$. Taking the difference between $V^r(\omega, \eta)$ and $V^f(\omega)$ above, we obtain (3).

C.2 Proof of Proposition 2

Assuming $\theta^* > \bar{\theta}$, then $\theta = \bar{\theta}$ when an entrepreneur chooses the risky project. Define $x \equiv R\frac{\bar{\theta}}{1+\bar{\theta}}$ as the debt servicing cost per unit of capital used in production, then $\tilde{\eta}$ in (3) can be rewritten as

$$\tilde{\eta} = \beta p \log\left(\frac{\Pi^h - x}{\Pi^f - x}\right) + \beta(1 - p)\log\left(\frac{\Pi^l - x}{\Pi^f - x}\right).$$

Then, we have

$$\frac{\partial\tilde{\eta}}{\partial x} = \beta\frac{\Pi^h\Pi^l - \Pi^f\hat{\Pi} + (\Pi^f - \Pi)x}{(\Pi^h - x)(\Pi^l - x)}.$$

where we use $\hat{\Pi} \equiv (1 - p)\Pi^h + p\Pi^l$ and $\Pi + \hat{\Pi} = \Pi^h + \Pi^l$. Under Assumptions A1 and A2, one can verify that

$$\Pi^h > \Pi^l > R\frac{\theta^*}{1 + \theta^*} > R\frac{\bar{\theta}}{1 + \bar{\theta}}.$$

Then the denominator is positive. Also, $(\Pi^f - \Pi)$ is negative. We can thus reach $\partial\tilde{\eta}/\partial x = 0$ if $x = x^s$, where

$$x^s \equiv \frac{\Pi^h\Pi^l - \Pi^f\hat{\Pi}}{\Pi - \Pi^f}.$$

And $\partial\tilde{\eta}/\partial x > 0$ if $x < x^s$, and $\partial\tilde{\eta}/\partial x < 0$ if $x > x^s$.

Notice that

$$\frac{\partial\tilde{\eta}}{\partial R} = \frac{\partial\tilde{\eta}}{\partial x}\frac{\partial x}{\partial R}.$$

Since $\partial x/\partial R > 0$, the sign of $\partial\tilde{\eta}/\partial R$ depends on the sign of $\partial\tilde{\eta}/\partial x$. Let R^s be the interest rate such that $x = x^s$, i.e., $R^s = R^s(\bar{\theta}) = x^s(1/\bar{\theta} + 1)$. Note that R^s depends on the leverage upper bound $\bar{\theta}$

and the superscript s indicates that the debt servicing cost is x^s (a parameter given the project returns as shown above). Therefore,

$$\bar{\theta}_H^s \equiv \frac{\Pi^h\Pi^l - \Pi^f\hat{\Pi}}{(\Pi - \Pi^f)\Pi^l - (\Pi^h\Pi^l - \Pi^f\hat{\Pi})}$$

is the bound such that $R^s(\bar{\theta}_H^s) = \Pi^l$ and

$$\bar{\theta}_L^s \equiv \frac{\Pi^h\Pi^l - \Pi^f\hat{\Pi}}{(\Pi - \Pi^f)\Pi^f - (\Pi^h\Pi^l - \Pi^f\hat{\Pi})}$$

is the bound such that $R^s(\bar{\theta}_L^s) = \Pi^f$. Using the two bounds, we can reach the following conclusions: 1). if $\bar{\theta} > \bar{\theta}_H^s$, then $R^s(\bar{\theta}) < \Pi^l$. For the range of interest rate under assumptions, $R > \Pi^l > R^s(\bar{\theta})$, implying that the debt servicing cost x is above x^s. In this case, $\partial\tilde{\eta}/\partial x < 0$ and thus $\partial\tilde{\eta}/\partial R < 0$; 2). if $\bar{\theta} < \bar{\theta}_L^s$, then $R^s(\bar{\theta}) > \Pi^f$. For the range of interest rate under assumptions, $R < \Pi^f < R^s(\bar{\theta})$, implying that the debt servicing cost x is below x^s. In this case, $\partial\tilde{\eta}/\partial x > 0$ and thus $\partial\tilde{\eta}/\partial R > 0$; 3) otherwise, $\tilde{\eta}$ is hump-shaped in R and $\Pi^l < R^s(\bar{\theta}) < \Pi^f$ when $\bar{\theta}_L^s < \bar{\theta} < \bar{\theta}_H^s$.

The above reasoning can also be applied if we vary the leverage limit and fix the interest rate R. Notice that

$$\frac{\partial\tilde{\eta}}{\partial\bar{\theta}} = \frac{\partial\tilde{\eta}}{\partial x}\frac{\partial x}{\partial\bar{\theta}}.$$

We know that $\partial x/\partial\bar{\theta} > 0$. Therefore the sign of $\partial\tilde{\eta}/\partial\bar{\theta}$ depends on the sign of $\partial\tilde{\eta}/\partial x$. Let $\bar{\theta}^s$ be the leverage upper bound such that $x = x^s$, i.e.,$\bar{\theta}^s \equiv \bar{\theta}^s(R) = \frac{x^s}{R - x^s}$. One can show that $0 < \bar{\theta}^s < \theta^*$. Therefore $\tilde{\eta}$ is hump-shaped in $\bar{\theta}$. Moreover, $\bar{\theta}^s$ decreases in R. Finally, we have the following corollary:

Corollary. *Suppose A1 and A2 hold. Assuming the financing constraint is binding when the risky project is implemented (i.e., $\bar{\theta} < \theta^*$). $\bar{\theta}^s(R) \in (0, \theta^*)$, the leverage upper bound* such that $\partial\tilde{\eta}/\partial R = 0$, *decreases in R. When $0 < \bar{\theta} < \bar{\theta}^s(R)$, the cutoff $\tilde{\eta}$ is increasing in $\bar{\theta}$; when $\bar{\theta}^s(R) < \bar{\theta} < \theta^*$, the cutoff $\tilde{\eta}$ is decreasing in $\bar{\theta}$.*

Intuitively, when leverage $\bar{\theta}$ is low, the risk-free project is not preferred since its (leveraged) return is smaller than the risky project's. As the leverage rises, the ratio between the leveraged return from the risky and the risk-free projects rises while the returns on both risky and risk-free projects increase. However, the (leveraged) volatility associated with the risky asset increases as leverage increases. If the leverage ratio is above some threshold $\hat{\theta}$, the disutility related to volatility dominates, and the risky project becomes less appealing. Notice that unlike in Proposition 2, which presents three scenarios based on the level of $\bar{\theta}$, we do not have multiple cases here. This is because R^s can vary, being lower than the lower bound Π^l, higher than the upper bound Π^f, or in between, depending on the level of $\bar{\theta}$. However, $\bar{\theta}^s$ is always between the lower and upper bounds, i.e., $0 < \bar{\theta}^s < \theta^*$.

C.3 Proof of Proposition 3

Observe that when leverage is not constrained,

$$\tilde{\eta} = \beta p \log\left(\frac{\Pi^h + (\Pi^h - R)\theta^*}{\Pi^f + (\Pi^f - R)\bar{\theta}}\right) + \beta(1-p)\log\left(\frac{\Pi^l + (\Pi^l - R)\theta^*}{\Pi^f + (\Pi^f - R)\bar{\theta}}\right),$$

where $\theta^* = -\left[p\frac{\Pi^l}{\Pi^l - R} + (1-p)\frac{\Pi^h}{\Pi^h - R}\right]$. Then, we have:

$$\begin{aligned}\tilde{\eta} =& \beta[p\log p + (1-p)\log(1-p) + \log R + \log(\Pi^h - \Pi^l) \\ & - p\log(R - \Pi^l) - (1-p)\log(\Pi^h - R) - \log(\Pi^f + (\Pi^f - R)\bar{\theta})]\end{aligned}$$

Take the derivative with respect to R, we have

$$\frac{\partial \tilde{\eta}}{\partial R} = \beta \left[\frac{\bar{\theta}}{\Pi^f + (\Pi^f - R)\bar{\theta}} - \frac{\theta^*}{R} \right]. \tag{24}$$

To prove the non-monotone behavior of the cutoff level for unconstrained entrepreneurs in Proposition 3 we proceed in two steps: 1) we show that $\tilde{\eta}$ is convex in the interest rate R when the financing constrained is slack; 2) if $\bar{\theta}$ is large enough we will see $\tilde{\eta}$ deceases and then increases in the interest R for unconstrained entrepreneurs.

Lemma 5. *For financially unconstrained entrepreneurs, the cutoff level $\tilde{\eta}$ is convex in R.*

Proof. With the expression for θ^* equation (23), we can rewrite $\frac{\partial \theta^*}{\partial R}$ as

$$\frac{\partial \theta^*}{\partial R} = \frac{\theta^*}{R\hat{\Pi} - \Pi^h\Pi^l} \left[\frac{\Pi^h\Pi^l + \theta^*(\Pi^h\Pi^l - R^2)}{R} \right].$$

From Proposition 1, we know $\frac{\partial \theta^*}{\partial R} < 0$. With this inequality and Assumption A2 ($\Pi^h\Pi^l > \Pi^f\hat{\Pi}$) indicating that $\Pi^h\Pi^l - R\hat{\Pi} > \Pi^h\Pi^l - \Pi^f\hat{\Pi} > 0$, we have $\Pi^h\Pi^l + \theta^*(\Pi^h\Pi^l - R^2) > 0$. Then, it follows that

$$-\frac{\partial \frac{\theta^*}{R}}{\partial R} = \frac{\theta^* \left[\Pi^l\Pi^h + \theta^*(\Pi^h\Pi^l - R^2)\right] + \theta^*(\Pi^h\Pi^l - R\hat{\Pi})}{R^2(\Pi^h\Pi^l - R\hat{\Pi})} > 0$$

Thus, we have the cutoff level $\tilde{\eta}$ is convex in the interest rate R for unconstrained entrepreneurs as

$$\frac{\partial^2 \tilde{\eta}}{\partial R^2} = \frac{\partial \frac{\bar{\theta}}{\Pi^f + (\Pi^f - R)\bar{\theta}}}{\partial R} - \frac{\partial \frac{\theta^*}{R}}{\partial R} > 0.$$

□

Lemma 6. *At the interest rate $\bar{R}(\bar{\theta})$, when the financial constraint changes from binding to non-binding, $\tilde{\eta}$ decreases in the interest rate R, i.e., $\frac{\partial \tilde{\eta}}{\partial R}|_{R=\bar{R}(\bar{\theta})} < 0$.*

Proof. To show this we use the non-monotone result from the binding case in Proposition 2. Since when interest rate is higher than $R^s(\bar{\theta})$, which is the interest rate such that $\tilde{\eta}$ takes its maximum when the financial constraints are binding, $\frac{\partial \tilde{\eta}}{\partial R} < 0$. If $\bar{R}(\bar{\theta}) > R^s(\bar{\theta})$ then $\frac{\partial \tilde{\eta}}{\partial R}|_{R=\bar{R}(\bar{\theta})} < 0$ follows.

To show $\bar{R}(\bar{\theta}) > R^s(\bar{\theta})$[21], it would be more convenient to compare the debt servicing cost, $R\frac{\bar{\theta}}{1+\bar{\theta}}$ in both cases: from the proof of Proposition 2 in Appendix C.2

$$R^s(\bar{\theta})\frac{\bar{\theta}}{1+\bar{\theta}} \quad = \quad x^s = \frac{\Pi^h\Pi^l - \Pi^f\hat{\Pi}}{\Pi - \Pi^f} = g(\Pi^f),$$

where $g(s) \equiv \frac{\Pi^h\Pi^l - s\hat{\Pi}}{\Pi - s}$. Meanwhile

$$\bar{R}(\bar{\theta})\frac{\bar{\theta}}{1+\bar{\theta}} = \frac{-p\Pi^l(\Pi^h - \bar{R}(\bar{\theta})) - (1-p)\Pi^h(\Pi^l - \bar{R}(\bar{\theta}))}{-p(\Pi^h - \bar{R}(\bar{\theta})) - (1-p)(\Pi^l - \bar{R}(\bar{\theta}))} = \frac{\Pi^h\Pi^l - \bar{R}(\bar{\theta})\hat{\Pi}}{\Pi - \bar{R}(\bar{\theta})} = g(\bar{R}(\bar{\theta})).$$

We will show that $\bar{R}(\bar{\theta}) < \Pi^f$ and $g(.)$ is decreasing. Together these will imply $\bar{R}(\bar{\theta}) > R^s(\bar{\theta})$.

Notice that if we observe entrepreneurs become unconstrained then it must be that $\bar{R}(\bar{\theta}) < \Pi^f$. To see this claim, first observe that θ^* is a decreasing function of R and $\bar{R}(\bar{\theta})$ is a function of $\bar{\theta}$. By the definition of $\bar{R}(\bar{\theta})$, we have $\theta^*(R = \bar{R}(\bar{\theta})) = \bar{\theta}$. By the definition of θ^*_{min}, we have $\theta^*(R = \Pi^f) = \theta^*_{min}$. Second, if $\bar{R}(\bar{\theta}) \geq \Pi^f$ that is opposite of the claim, then $\bar{\theta} = \theta^*(R = \bar{R}(\bar{\theta})) \leq \theta^*(R = \Pi^f) = \theta^*_{min}$ as θ^* decreases in R, which contradicts with our assumption that $\bar{\theta} > \theta^*_{min}$ and we prove the claim.

[21] With some algebra, we can verify that $\theta^*_{min} = \bar{\theta}^s_L$ because $\Pi + \hat{\Pi} = \Pi^h + \Pi^l$. The proof in Proposition 2 has shown that when $\bar{\theta} > \bar{\theta}^s_L$, we have $R^s(\bar{\theta}) < \Pi^f$.

By convexity, $\frac{p}{\Pi^h} + \frac{1-p}{\Pi^l} > \frac{1}{p\Pi^h + (1-p)\Pi^l} = \frac{1}{\Pi}$ which implies $\Pi^h \Pi^l < \hat{\Pi}\Pi$. Then, the expression $g(s)$ is decreasing in s.[22] This implies $\bar{R}(\bar{\theta})\frac{\bar{\theta}}{1+\bar{\theta}} > R^s(\bar{\theta})\frac{\bar{\theta}}{1+\bar{\theta}}$ and thus $\bar{R}(\bar{\theta}) > R^s(\bar{\theta})$. □

Finally, using Lemma 5 and Lemma 6, we can finish the proof of Proposition 3, the non-monotonicity of $\tilde{\eta}$ in the interest R for unconstrained case. To show $\tilde{\eta}$ changes from decreasing to increasing in R, we need to further find an interest rate level such that $\frac{\partial \tilde{\eta}}{\partial R} > 0$.

In the proof of (C.2), the derivative of $\tilde{\eta}$ with respect to the debt servicing cost x is zero when $x = x^s$; we see that $\frac{\partial \tilde{\eta}}{\partial R}|_{R=\Pi^f} = 0$ when $\bar{\theta}$ takes the value of $\bar{\theta}^s_L$, because the debt servicing cost is indeed $x = x^s$ when $\bar{\theta} = \bar{\theta}^s_L$ and $R = \Pi^f$. It is straightforward to verify that $\frac{\partial^2 \tilde{\eta}}{\partial R \partial \theta} > 0$ according to (24), which means that $\frac{\partial \tilde{\eta}}{\partial R}$ increases in $\bar{\theta}$. Thus $\frac{\partial \tilde{\eta}}{\partial R}|_{R=\Pi^f} > 0$ when $\bar{\theta} > \theta^*_{min}$ since we already know that $\theta^*_{min} = \bar{\theta}^s_L$. Then by Lemma 5, Lemma 6, and the mean value theorem, when $\bar{\theta} > \theta^*_{min}$ there exists an interest rate $R^u(\bar{\theta})$ where $\bar{R}(\bar{\theta}) < R^u(\bar{\theta}) < \Pi^f$ such that $\frac{\partial \tilde{\eta}}{\partial R}|_{R=R^u(\bar{\theta})} = 0$, as $\frac{\partial \tilde{\eta}}{\partial R}|_{R=\bar{R}(\bar{\theta})} < 0$ (from Lemma 6), $\frac{\partial \tilde{\eta}}{\partial R}|_{R=\Pi^f} > 0$, and $\frac{\partial \frac{\partial \tilde{\eta}}{\partial R}}{\partial R} > 0$ (from Lemma 5). The implied risk taking threshold for unconstrained entrepreneurs decreases with R first and increases with R later, i.e., $\frac{\partial \tilde{\eta}}{\partial R} < 0$ if $\bar{R}(\bar{\theta}) \le R \le R^u(\bar{\theta})$and $\frac{\partial \tilde{\eta}}{\partial R} > 0$ if $R^u(\bar{\theta}) < R(\bar{\theta}) \le \Pi^f$.

C.4 Proof of Proposition 4

Guess that $V^r(\omega, \eta) = \log(\omega) + v^r - \eta$, $V^f(\omega) = \log(\omega) + v^f - \eta^f$, $V^d(\omega) = \log(\omega) + v^d$. Plugging these guessed forms into the entrepreneur's Bellman equation, we have the following

$$\begin{aligned} v^r =& B + \beta p \left[\log\left(\Pi^h(1+\theta) - R\theta\right)\right] + \beta(1-p)\left[\log\left(\Pi^l(1+\theta) - R\theta\right)\right] \\ &+ \beta \mathbb{E}\max\{v^r - \eta', v^f - \eta^f, v^d\}; \quad (25) \\ v^f =& B + \beta\left[\log(\Pi^f(1+\bar{\theta}) - R\bar{\theta})\right] + \beta\mathbb{E}\max\{v^r - \eta', v^f - \eta^f, v^d\}; \quad (26) \\ v^d =& B + \beta\log(R^d) + \beta\mathbb{E}\max\{v^r - \eta', v^f - \eta^f, v^d\}, \quad (27) \end{aligned}$$

where $B \equiv (1-\beta)\log(1-\beta) + \beta\log\beta$. One can solve for the three unknown v^r, v^f, and v^d from the above three equations, and verify the guess. They are, however, not important for our purposes. The choice (between the risk-free project and deposits) depends on the maximal value of the two, $v^f - \eta^f$ and v^d. Using v^f and v^d just above, we then have

$$\phi = \begin{cases} 0 & \text{if } \log(\Pi^f(1+\bar{\theta}) - R\bar{\theta}) - \eta^f/\beta > \log R^d \\ (0,1) & \text{if } \log(\Pi^f(1+\bar{\theta}) - R\bar{\theta}) - \eta^f/\beta = \log R^d \\ 1 & \text{if } \log(\Pi^f(1+\bar{\theta}) - R\bar{\theta}) - \eta^f/\beta < \log R^d. \end{cases}$$

Notice that when $\phi = 0$, the risk-free project strictly dominates; when $\phi = 1$, the risk-free deposits strictly dominates; when $0 < \phi < 1$, an entrepreneur is indifferent between implementing the risk-free project and the risk-free deposits.

The choice of leverage θ is identical to the two-period model and the proof has been provided before. For the cutoff, there exists a level such that below the level, entrepreneurs choose the risky project and above the level, entrepreneurs choose the risk-free project or deposits. Similar to the proof in the 2-period model, we have $\tilde{\eta}$ as:

$$\begin{aligned} \tilde{\eta} &= v^r - \max\{v^f - \eta^f, v^d\} \\ &= \beta p \log\left(\Pi^h + (\Pi^h - R)\theta\right) + \beta(1-p)\log\left(\Pi^l + (\Pi^l - R)\theta\right) \\ &\quad -\beta\max\{\log(\Pi^f(1+\bar{\theta}) - R\bar{\theta}) - \eta^f/\beta, \log R^d\}, \end{aligned}$$

[22]The first order condition of $g(s) = \frac{\Pi^h\Pi^l - s\hat{\Pi}}{\Pi - s}$ with respect to s is $\frac{-\hat{\Pi}(\Pi - s) + \Pi^h\Pi^l - s\hat{\Pi}}{(\Pi - s)^2} = \frac{\Pi^h\Pi^l - \hat{\Pi}\Pi}{(\Pi - s)^2} < 0$.

where we use the value functions at the beginning of the proof and $\theta = \min\{\theta^*, \bar{\theta}\}$ (again θ^* is the unconstrained solution given by (23)). Of course, if the right-hand side is below $\underline{\eta}$, $\tilde{\eta} = \underline{\eta}$. Therefore, those entrepreneurs with $\eta \leq \max\{\underline{\eta}, \tilde{\eta}\}$ choose the risky project.

D Omitted Details in the Macroeconomic Model

D.1 The average return in the society

Let m denote number of realizations of Π^h in the economy, the proportion of success $p^s(\nu) = \mathbb{E}\left[\frac{m}{n}\right]$ as n goes to infinity, which is the share of the risky projects with high return Π^h as n goes to infinity.

It has been shown in Shmueli et al. (2005) that for the Conway-Maxwell Poisson distribution, $\mathbb{E}[m]$ converges to $\lambda^{1/\nu} - \frac{\nu-1}{2\nu}$ when n goes to infinity and p is small i.e. $p < p^*$. This suggests that $p^s(\nu)$ converges to $p^{1/\nu}$ when n goes to infinity once substituting $\lambda = n^\nu p$ as illustrated in Daly and Gaunt (2016). Note that $p^{1/\nu}$ is the probability of high realizations assessed by the social planner, which differs from the individual's assessment p whenever there is correlation in the projects (i.e., $\nu \neq 1, \nu > 0$). Then the average return from the society's view becomes $p^{\frac{1}{\nu}}\Pi^h + (1 - p^{\frac{1}{\nu}})\Pi^h$.

The case when p is large i.e. $p > p^{**}$ is similar if we replace p by $1 - p$ in the argument above.

D.2 Pairwise correlation for spillovers

Let $\mathbb{I}_{x_1=l}$ be the indicator function of $x_1 = l$ (the first trial fails) and $\mathbb{I}_{x_2=l}$ be the indicator function of $x_2 = l$ (the second trial fails), respectively.

$$
\begin{aligned}
Cov(\mathbb{I}_{x_1=l}, \mathbb{I}_{x_2=l}) &= \mathbb{E}[\mathbb{I}_{x_1=l}\mathbb{I}_{x_2=l}] - \mathbb{E}[\mathbb{I}_{x_1=l}]\mathbb{E}[\mathbb{I}_{x_2=h}] \\
&= Pr(x_1 = l, x_2 = l) - Pr(x_1 = l)P(x_2 = l) \\
&= Pr(x_1 = l, x_2 = l) - Pr(x_1 = l)\left[Pr(x_1 = l, x_2 = l) + Pr\left(x_1 = h, x_2 = l\right)\right] \\
&= \left[1 - Pr(x_1 = l)\right] Pr(x_1 = l, x_2 = l) - Pr(x_1 = l)Pr(x_1 = h)Pr(x_2 = l|x_1 = h) \\
&= Pr(x_1 = h)Pr(x_1 = l)\left[Pr(x_2 = l|x_1 = l) - Pr(x_2 = l|x_1 = h)\right],
\end{aligned}
$$

is the covariance of the two indicator functions. With $\sigma(\mathbb{I}_{x_1=l}) = \sqrt{Pr(x_1 = h)Pr(x_1 = l)}$ and $\sigma(\mathbb{I}_{x_2=l}) = \sqrt{Pr(x_2 = h)Pr(x_2 = l)}$ being the standard deviation of these two (indicator functions, then the correlation coefficient ρ_ν becomes:

$$
\begin{aligned}
\rho_\nu &= \frac{Cov(\mathbb{I}_{x_1=l}, \mathbb{I}_{x_2=l})}{\sigma(\mathbb{I}_{x_1=l})\sigma(\mathbb{I}_{x_2=l})} \\
&= \frac{\sigma(\mathbb{I}_{x_1=l})}{\sigma(\mathbb{I}_{x_2=l})}\left(Pr(x_2 = l|x_1 = l) - Pr(x_2 = l|x_1 = h)\right) \\
&= Pr(x_2 = l|x_1 = l) - Pr(x_2 = l|x_1 = h) \\
&= \frac{Pr(x_2 = l, x_1 = l)}{Pr(x_1 = l)} - \frac{Pr(x_2 = l, x_1 = h)}{Pr(x_1 = h)} \\
&= \frac{\frac{1}{D(\nu,p,2)}(1-p)^2}{\frac{1}{D(\nu,p,2)}(1-p)^2 + \frac{1}{2}\frac{1}{D(\nu,p,2)}\binom{2}{1}^\nu p(1-p)} - \frac{\frac{1}{2}\frac{1}{D(\nu,p,2)}\binom{2}{1}^\nu p(1-p)}{\frac{1}{D(\nu,p,2)}p^2 + \frac{1}{2}\frac{1}{D(\nu,p,2)}\binom{2}{1}^\nu p(1-p)} \\
&= \frac{1-p}{1-p+2^{\nu-1}p} - \frac{2^{\nu-1}(1-p)}{p+2^{\nu-1}(1-p)},
\end{aligned}
$$

where the second line uses the property of exchangeability (which implies $\sigma(\mathbb{I}_{x_1=l}) = \sigma(\mathbb{I}_{x_2=l})$) and the fifth line uses the definition of CMB distribution and the property of exchangeability.

D.3 Welfare function

We first derive entrepreneurs' welfare, which comes from three types: those who take risks, those who implement the risk-free project, and those who save in safe deposits:

$$V_e(\omega) = \int^{\tilde{\eta}} V^r(\omega,\eta)dF(\eta) + (1-\phi)\int_{\tilde{\eta}} V^f(\omega,\eta^f)dF(\eta) + \phi\int_{\tilde{\eta}} V^d(\omega)dF(\eta). \qquad (28)$$

We have two scenarios below, each of which determines the expected value $\mathbb{E}\max\{v^r-\eta', v^f-\eta^f, v^d\}$ in (25) - (27) differently. For exposition simplicity, $\tilde{\eta}$ is assumed to be above $\underline{\eta}$, which is always verified in numerical exercises. In addition, the lower bound $\underline{\eta}$ is set to zero since it simplifies the derivation and it does not affect welfare comparison.

Scenario 1: When $\phi = 0$, no one chooses the safe deposit option. That is, $v^f - \eta^f > v^d$. Then, we can explicitly express $\mathbb{E}\max\{v^r-\eta', v^f-\eta^f, v^d\}$ as follows

$$\begin{aligned}
\mathbb{E}\max\{v^r-\eta', v^f-\eta^f, v^d\} &= v^d + \mathbb{E}\max\{v^r - v^d - \eta', v^f - v^d - \eta^f, 0\} \\
&= v^d + v^f - v^d - \eta^f + \mathbb{E}\max\{\tilde{\eta}-\eta', 0\} \\
&= v^f - \eta^f + \int^{\tilde{\eta}} F(\eta)d\eta,
\end{aligned}$$

since $v^f - v^d - \eta^f > 0$ and $\tilde{\eta} = v^r - v^f + \eta^f$. This means that we can explicitly solve the values in (25) - (27). It turns out that we only need to express

$$\begin{aligned}
v^f &= B + \beta\left[\log(\Pi^f(1+\bar{\theta}) - R\bar{\theta})\right] + \beta\left[v^f - \eta^f + \int^{\tilde{\eta}} F(\eta)d\eta\right] \\
&= \frac{B}{1-\beta} + \frac{\beta}{1-\beta}\left[\log(\Pi^f(1+\bar{\theta}) - R\bar{\theta})\right] + \frac{\beta}{1-\beta}[\int^{\tilde{\eta}} F(\eta)d\eta - \eta^f],
\end{aligned}$$

where the second equality rearranges the v^f in the first equality on the right-hand side and divides $(1-\beta)$ on both sides. Therefore, we rewrite (28) as

$$\begin{aligned}
V_e(\omega) &= \int^{\tilde{\eta}} (V^r(\omega,\eta))dF(\eta) + \int_{\tilde{\eta}} V^f(\omega,\eta^f) \\
&= \log(\omega) + v^r F(\tilde{\eta}) - \int^{\tilde{\eta}} \eta dF(\eta) + [1 - F(\tilde{\eta})](v^f - \eta^f) \\
&= \log(\omega) + \tilde{\eta}F(\tilde{\eta}) + (v^f - \eta^f) - F(\tilde{\eta})\tilde{\eta} + \int^{\tilde{\eta}} F(\eta)d\eta \\
&= \log(\omega) + \frac{B}{1-\beta} + \frac{\beta}{1-\beta}\left[\log(\Pi^f(1+\bar{\theta}) - R\bar{\theta})\right] \\
&\quad + \frac{1}{1-\beta}\left[\int^{\tilde{\eta}} F(\eta)d\eta - \eta^f\right],
\end{aligned}$$

where the third equality uses $v^r = v^f - \eta^f + \tilde{\eta}$ and the fourth equality uses the expression for v^f above.

Scenario 2: When $0 < \phi \leq 1$, the risk-free deposits option is weakly preferred. That is, $v^f - \eta^f \leq v^d$, with "=" when $0 < \phi < 1$. Then, we can explicitly express $\mathbb{E}\max\{v^r-\eta', v^f-\eta^f, v^d\}$ as

$$\begin{aligned}
\mathbb{E}\max\{v^r-\eta', v^f-\eta^f, v^d\} &= v^d + \mathbb{E}\max\{v^r - v^d - \eta', 0\} \\
&= v^d + \mathbb{E}\max\{\tilde{\eta} - \eta', 0\} \\
&= v^d + \int^{\tilde{\eta}} F(\eta)d\eta.
\end{aligned}$$

Using the result above and following a similar procedure in the Scenario 1, we simplify the values in (25) - (27). It turns out that we only need to express

$$\begin{aligned} v^d &= B + \beta \log R^d + \beta(v^d + \int^{\tilde{\eta}} F(\eta) d\eta) \\ &= \frac{B}{1-\beta} + \frac{\beta \log(R^d)}{1-\beta} + \frac{\beta}{1-\beta} \int^{\tilde{\eta}} F(\eta) d\eta, \end{aligned}$$

where we use the fact that $v^f - \eta^f \leq v^d$ in the first equality and rearrange v^d in the second equality. Therefore, we can rewrite (28) as

$$\begin{aligned} V_e(\omega) &= \int^{\tilde{\eta}} (V^r(\omega, \eta)) dF(\eta) + \int_{\tilde{\eta}} V^d(\omega) \\ &= \log(\omega) + v^r F(\tilde{\eta}) - \int^{\tilde{\eta}} \eta dF(\eta) + (1 - F(\tilde{\eta})) v^d \\ &= \log(\omega) + v^d + F(\tilde{\eta})(v^r - v^d) - \int^{\tilde{\eta}} \eta dF(\eta) \\ &= \log(\omega) + v^d + F(\tilde{\eta})\tilde{\eta} - F(\tilde{\eta})\tilde{\eta} + \int^{\tilde{\eta}} F(\eta) d\eta \\ &= \log(\omega) + \frac{B}{1-\beta} + \frac{\beta \log(R^d)}{1-\beta} + \frac{1}{1-\beta} \int^{\tilde{\eta}} F(\eta) d\eta, \end{aligned}$$

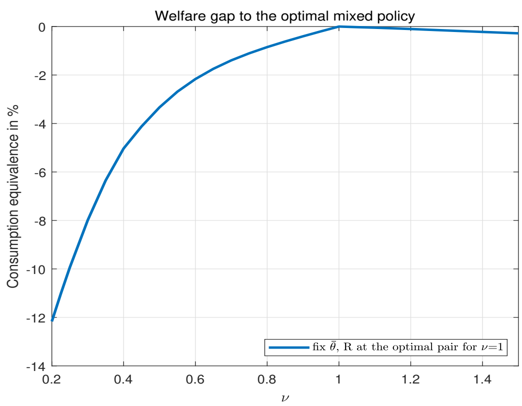
where we used $v^d \geq v^f - \eta^f$ in the first equality, $v^r - v^d = \tilde{\eta}$ (note: in this case $l^d \geq l^f$ and entrepreneurs choose to deposit) in the fourth equality, and the expression for v^d in the last equality.

Remark: In a steady-state economy, project choice is independent of the wealth level. The distribution of the wealth level ω is always preserved. Denote V as the value function of the entrepreneurs. Thus, $\omega = \Omega$ and $V = V_e(\Omega)$. We, therefore, finish showing that entrepreneurs' welfare can be represented by $\log(\Omega) + \tilde{V}(\phi) + constants$ as shown in the main text, since the results in Scenarios 1 and 2 above imply the constant term is $\frac{B}{1-\beta}$ and

$$\tilde{V}(\phi) = (1-\beta)^{-1} \left(\beta \max\{l^d, l^f\} + \int^{\tilde{\eta}} F(\eta) d\eta \right).$$

E Optimal Policy

Figure 13: Welfare Gap against Optimal Policy Mix



In this case both the leverage level $\bar{\theta}$ and interest rate R are fixed at the optimal level for $\nu = 1$. The welfare gap against the optimal policy mix is significantly larger.